Area Between Two Curves

If two functions, \( f(x) \) and \( g(x) \) are continuous on the interval \([a, b]\) and \( f(x) \geq g(x) \) on \([a, b]\), then the area between \( f(x) \) and \( g(x) \) from \( x = a \) to \( x = b \) is given by

\[
Area = \int_{a}^{b} [f(x) - g(x)] \, dx
\]

Sometimes, though, it may be easier to switch from functions of \( x \) to functions of \( y \) (with respect to \( y \)), such as below

The area between the curves \( f(y) \) and \( g(y) \) to the left is given by

\[
Area = \int_{a}^{b} [f(y) - g(y)] \, dy
\]

Examples

1) Find the area of the region bounded by the curves \( y = x^2 - 8x + 13 \) and \( y = 6 \).
2) Find the integral(s) required to do example 1 with respect to \( y \).
3) Find the area of the region(s) bounded by \( y = x^3 + 2x^2 - 7x + 5 \) and \( y = 2x^2 - 3x + 5 \)
4) Find the area of the region(s) bounded by \( f(x) = \sin x \) and \( g(x) = \tan x \) from \( x = 0 \) to \( x = \frac{\pi}{4} \)
5) Find the area of the region(s) bounded by \( x = y^2 \) and \( y = x^2 \)
6) Find the area of the region(s) bounded by \( x = y^2 - 2 \) and \( x = |y| \)
7) Use technology to estimate the area between the curves \( y = e^{.5x} \) and \( y = x^2 - 5 \) to 4 decimal places. (Note that there are 2 regions).

Solutions to examples

1) 36  2) \( \int_{-3}^{6} \left(4 + \sqrt{y + 3}\right) - \left(4 - \sqrt{y + 3}\right) \, dy \)  3) 8  4) \(-\ln\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2} - 1\)  5) \(\frac{1}{3}\)  6) \(\frac{20}{3}\)  7) 59.0335