\[ a_n = 2n - 3 \]
\[ a_1 = 2(1) - 3 = -1 \]
\[ a_2 = 2(2) - 3 = 1 \]
\[ a_3 = 2(3) - 3 = 3 \]
\[ a_4 = 2(4) - 3 = 5 \]
\[ a_5 = 2(5) - 3 = 7 \]
\[ \{ -1, 1, 3, 5, 7, \ldots \} \]
DIVERGES (GOES TO \( \infty \))
\[ \lim_{n \to \infty} (2n-3) = \infty \]

\[ b_n = \cos(n) \]
\[ b_1 = \cos(1) \]
\[ b_2 = \cos(2) \]
\[ b_3 = \cos(3) \]
\[ \{ \cos(1), \cos(2), \cos(3), \cos(4), \cos(5), \ldots \} \]
DIVERGES (ALWAYS ABOUNDS)
\[ \lim_{n \to \infty} \cos(n) = \text{Does not exist} \]

\[ c_n = \left(\frac{2}{3}\right)^n \]
\[ c_1 = \left(\frac{2}{3}\right)^1 = \frac{2}{3} \]
\[ c_2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9} \]
\[ c_3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27} \]
\[ c_4 = \left(\frac{2}{3}\right)^4 = \frac{16}{81} \]
\[ c_5 = \left(\frac{2}{3}\right)^5 = \frac{32}{243} \]
\[ \{ \frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \frac{32}{243}, \ldots \} \]
CONVERGES TO \( 0 \)
\[ \lim_{n \to \infty} \left(\frac{2}{3}\right)^n = 0 \]

\[ d_n = \left(\frac{-3}{2}\right)^n \]
\[ \{ \frac{-3}{2}, \frac{9}{4}, \frac{-27}{8}, \frac{81}{16}, \frac{-243}{32}, \ldots \} \]
DIVERGES (THE ABSOLUTE VALUE OF)
TAKE TERMS APPROACHING \( \infty \)

\[ e_n = \frac{5-n^2}{2n^2 + 5n - 8} \]
\[ e_1 = \frac{5-(1)^2}{2(1)^2 + 5(1) - 8} = -4 \]
\[ e_2 = \frac{5-(2)^2}{2(2)^2 + 5(2) - 8} = \frac{1}{10} \]
\[ \{ -4, \frac{1}{10}, \frac{-4}{5}, \frac{-4}{5}, \frac{-20}{67}, \ldots \} \]
\[ \lim_{n \to \infty} \frac{5-n^2}{2n^2 + 5n - 8} = \lim_{n \to \infty} \frac{\frac{5}{n^2} - \frac{n^2}{n^2}}{\frac{2n^2}{n^2} + \frac{5n}{n^2} - \frac{8}{n^2}} = \frac{0 - 1}{2 + 0 + 0} = \frac{-1}{2} \]
Every time $n$ increases by 1, $g_n$ increases by 5.

This is similar to a line, "y" = "x".

So, 
\[ \text{slope} = \frac{9-4}{2-1} = \frac{5}{1} = 5 \]

Or, 
\[ g_n - 4 = 5(n-1) \]

\[ g_n = 5n - 1 \]

The next term is found by multiplying by $\frac{2}{3}$.

So we can include $\left(\frac{2}{3}\right)^n$ in the formula, but $\left(\frac{2}{3}\right)^7 \neq 5$.

So, the formula may look like $A \left(\frac{2}{3}\right)^n$.

\[ A_1 = A \left(\frac{2}{3}\right)^1 = 3 \]

So, 
\[ \left(\frac{2}{3}\right)^1 = \frac{3}{A} \]

So, 
\[ 2A = 9 \]

\[ A = \frac{9}{2} \]

Answer: 
\[ \frac{9}{2} \left(\frac{2}{3}\right)^n = g_n \]

\[ \frac{9}{2} \cdot \frac{2}{3} \cdot \left(\frac{2}{3}\right)^{n-1} = 3 \left(\frac{2}{3}\right)^{n-1} \]

Which is a form that we will see more of in the future.
\[
\begin{align*}
\{ \frac{3}{2}, 4 \} & \quad \{ -1, -3, -5 \} \\
3, 1, -1, -3, -5 & \text{ is "linear" (if the ratio were TR)} \\
\text{"slope"} & = \frac{1 - 3}{2 - 1} = -2 \\
\alpha_n - 3 & = -2(n - 1) \\
\alpha_n - 3 & = -2n + 2 \\
\alpha_n & = -2n + 5 \\
\text{NUM} & \\
2, 4, 8, 16, 32 & \text{ powers of 2} \\
\alpha_n & = 2^n \\
\alpha_1 & = \alpha \cdot 2 = 2 \cdot 2 = 2 \\
\therefore \alpha & = 1 \\
2^n & = 2^n \\
\text{DENOM} & \\
\text{So overall we can put the two above together at} \\
\frac{\alpha_n}{\text{den}} & = \frac{-2n + 5}{2^n} \\
\end{align*}
\]
\{2, -2, 2, -2, 2, -2, ... \} \quad 3

**METHOD #1**

\[ G_n = (-1)^n = \{ -1, 1, -1, 1, -1, 1, ... \} \]

So \[ 2(-1)^{n+1} = \{ 0, -2, 2, -2, 2, -2, ... \} \]

\[ G_n = 2(-1)^{n+1} \]

**METHOD #2**

\[ \cos(n\pi) = \{ -1, 1, -1, 1, -1, 1, ... \} \]

\[ -\cos(n\pi) = \{ 1, -1, 1, -1, 1, -1, ... \} \]

So \[ 2(-\cos(n\pi)) = \{ 2, -2, 2, -2, 2, -2, ... \} \]

\[ G_n = -2\cos(n\pi) \]

\{10, 4, 10, 4, 10, 4, ... \} \quad 3

**THINK OF THIS AS**

\{7+3, 7-3, 7+3, 7-3, 7+3, 7-3, ... \} \quad 3

So we need a sequence that looks like \{3, -3, 3, -3, ... \}

And that would be either \[ 3(-1)^{n+1} \] or \[ -3\cos(n\pi) \]

So

\[ G_n = 7 + 3(-1)^{n+1} \quad \text{or} \quad G_n = 7 - 3\cos(n\pi) \]
\[ \{ 4, -9, 14, -19, 24, -29, \ldots \} \]

Note that this is an alternating series \( \{ +1, -1, +1, -1, \ldots \} \)
and it starts with a \(+\). We can take care of that part by including \((-1)^{n+1}\) (rest of sequence)
without the changes of signs, we have
\[ \{ 4, 9, 14, 19, 24, 29, \ldots \} \] which would be \( 5n - 1 \)

So \[ A_n = (-1)^{n+1}(5n - 1) \]

\[ \lim_{n \to \infty} \cos \frac{1}{n} = \cos(0) = 1 \]

So if \( A_n = \cos \frac{1}{n} \) then \( \lim_{n \to \infty} A_n = 1 \)

\[ \lim_{n \to \infty} A_n = \lim_{n \to \infty} \frac{(-1)^n(15n+9)}{n^2} = \lim_{n \to \infty} (-1)^n \left( \frac{\frac{15n}{n^2} + \frac{9}{n^2}}{\frac{n}{n^2}} \right) \]

\[ = \lim_{n \to \infty} (-1)^n \frac{0+0}{1} = \frac{0}{1} = 0 \]
\[
\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{n}{\sin^2(n)} \quad \text{ numerator goes to } \infty \\
\text{ denominator is between 0 and 1} \\
= \infty \quad \text{diverges}
\]

\[
a_n = \frac{n!}{n^n} = \frac{n(n-1)(n-2)(n-3) \ldots 3 \cdot 2 \cdot 1}{n \cdot n \cdot n \cdot n \ldots n \cdot n}\\
= \left\{ 1, \frac{2}{4}, \frac{6}{27}, \frac{24}{256}, \frac{120}{3125}, \frac{720}{96656}, \frac{5040}{835571} \right\}, \ldots \]
\[
\begin{align*}
\lim_{n \to \infty} a_n &= 0 \\
\text{denominator } &\quad \text{"grows much faster"} \\
\text{"} n^n \quad \text{is much bigger than } \quad n! \quad \text{as } n \text{ get large} \\
\end{align*}
\]

\[
a_n = \frac{(2n)!}{n^n} = \frac{(2n)(2n-1)(2n-2)(2n-3) \ldots (n+1)(n)(n-1)(n-2)(n-3) \ldots 3 \cdot 2 \cdot 1}{n \cdot n \cdot n \cdot n \ldots n} \\
> 1
\]

so \( a_n > n! \) as \( n \to \infty \)

which means \( a_n \) diverges