Volume – Shell Method

If f(x) $\geq$ 0, then the volume of the object generated by revolving the area between f(x) and g(x) about the line x = k from x = a to x = b is given by

$$V = 2\pi\int_a^b (x-k)h(x)\,dx$$ when $k \leq a < b$ (Use (k – x) if $a < b \leq k$)

Where h(x) is the distance between f(x) and g(x) at location x.
h(x) = f(x) – g(x) if f(x) > g(x) **or** **h(x) = g(x) – f(x) if f(x) < g(x)**

Similarly, If g(y) $\geq$ 0 then the volume of the object generated by revolving the area between f(y) and g(y) about the line y = k from y = a to y = b is given by

$$V = 2\pi\int_a^b (y-k)h(y)\,dy$$ when $k \leq a < b$ (Use (k – y) if $a < b \leq k$)

Where h(y) is the distance between f(y) and g(y) at location y.

Examples

1) Use the Shell method to find the volume of the solid created by rotating the region bounded by y = 2x – 4, y = 0, and x = 3 about the X axis.
2) Use the Shell method to find the volume of the solid created by rotating the region bounded by y = 2x – 4, y = 0, and x = 3 about the Y axis.
3) Use the Shell method to find the volume of the solid created by rotating the region bounded by y = 2x – 4, y = 0, and x = 3 about the line x = 4.
4) Use the Shell method to find the volume of the solid created by rotating the region bounded by y = 2x – 4, y = 0, and x = 3 about the line y = -3.
5) Use the Shell method to find the volume of the solid created by rotating the region bounded by y = $2x^2$ - 3, y = -3, and x = 2 about the line x = -1.
6) Use the Shell method to find the volume of the solid created by rotating the region bounded by y = $2x^2$ - 3, y = -3, and x = 2 about the line y = 7.
7) Use the **Disk/Washer** method to find the volume of the solid created by rotating the region bounded by y = 2x, y = -4, x = 1, and x = 3 about the Y axis.
8) Use the **Shell** method to find the volume of the solid created by rotating the region bounded by y = 2x, y = -4, x = 1, and x = 3 about the Y axis.
9) Use the Shell method to find the volume of the solid created by rotating the region bounded by y = x^2 + 3 and y = 7 about the line x = 4.

Solutions

1) $2\pi\int_0^2 y\left(3-\frac{y+4}{2}\right)dy = \frac{4\pi}{3}$

2) $2\pi\int_2^3 x(2x-4)dx = \frac{16\pi}{3}$

3) $2\pi\int_2^3 (4-x)(2x-4)dx = \frac{8\pi}{3}$

4) $2\pi\int_0^2 (y+3)\left(3-\frac{y+4}{2}\right)dy = \frac{22\pi}{3}$

5) $2\pi\int_0^2 (x+1)\left[(2x^2-3)+3\right]dx = \frac{80\pi}{3}$

6) $2\pi\int_{-3}^5 \left[(7-y)\left(2-\sqrt{\frac{y+3}{2}}\right)\right]dy = \frac{1216\pi}{15}$

7) $\pi\int_{-4}^2 \left(3^2-1^2\right)dy + \pi\int_2^6\left[3^2-\left(\frac{y}{2}\right)^2\right]dy = \frac{200\pi}{3}$

8) $2\pi\int_0^2 x(2x+4)\,dx = \frac{200\pi}{3}$

9) $2\pi\int_{-2}^2 (4-x)\left(7-(x^2+3)\right)dx = \frac{256\pi}{3}$