Volume - Shell Method

If \( f(x) \geq 0 \), then the volume of the object generated by revolving the area between \( f(x) \) and \( g(x) \) about the line \( x = k \) from \( x = a \) to \( x = b \) is given by

\[
V = 2\pi \int_a^b (x - k)h(x) \, dx \quad \text{when} \quad k \leq a < b \quad \text{(Use \( k - x \) if \( a < b \leq k \))}
\]

Where \( h(x) \) is the distance between \( f(x) \) and \( g(x) \) at location \( x \).

\[
h(x) = f(x) - g(x) \quad \text{if} \quad f(x) > g(x) \quad \text{or} \quad h(x) = g(x) - f(x) \quad \text{if} \quad f(x) < g(x)
\]

Similarly, if \( g(y) \geq 0 \) then the volume of the object generated by revolving the area between \( f(y) \) and \( g(y) \) about the line \( y = k \) from \( y = a \) to \( y = b \) is given by

\[
V = 2\pi \int_a^b (y - k)h(y) \, dy \quad \text{when} \quad k \leq a < b \quad \text{(Use \( k - y \) if \( a < b \leq k \))}
\]

Where \( h(y) \) is the distance between \( f(y) \) and \( g(y) \) at location \( y \).

Examples

1) Use the Shell method to find the volume of the solid created by rotating the region bounded by \( y = 2x - 4 \), \( y = 0 \), and \( x = 3 \) about the X axis.

2) Use the Shell method to find the volume of the solid created by rotating the region bounded by \( y = 2x - 4 \), \( y = 0 \), and \( x = 3 \) about the Y axis.

3) Use the Shell method to find the volume of the solid created by rotating the region bounded by \( y = 2x - 4 \), \( y = 0 \), and \( x = 3 \) about the line \( x = 4 \).

4) Use the Shell method to find the volume of the solid created by rotating the region bounded by \( y = 2x - 4 \), \( y = 0 \), and \( x = 3 \) about the line \( y = -3 \).

5) Use the Shell method to find the volume of the solid created by rotating the region bounded by \( y = 2x^2 - 3 \), \( y = -3 \), and \( x = 2 \) about the line \( x = -1 \).

6) Use the Shell method to find the volume of the solid created by rotating the region bounded by \( y = 2x^2 - 3 \), \( y = -3 \), and \( x = 2 \) about the line \( y = 7 \).

7) Use the Disk/Washer method to find the volume of the solid created by rotating the region bounded by \( y = 2x \), \( y = -4 \), \( x = 1 \), and \( x = 3 \) about the Y axis.

8) Use the Shell method to find the volume of the solid created by rotating the region bounded by \( y = 2x \), \( y = -4 \), \( x = 1 \), and \( x = 3 \) about the Y axis.

9) Use the Shell method to find the volume of the solid created by rotating the region bounded by \( y = x^2 + 3 \) and \( y = 7 \) about the line \( x = 4 \).

Solutions

1) \( 2\pi \int_0^2 \left( y \left( 3 - \frac{y + 4}{2} \right) \right) \, dy = \frac{4\pi}{3} \)

2) \( 2\pi \int_2^3 x(2x - 4) \, dx = \frac{16\pi}{3} \)

3) \( 2\pi \int_2^3 (4 - x)(2x - 4) \, dx = \frac{8\pi}{3} \)

4) \( 2\pi \int_0^2 \left( y + 3 \left( 3 - \frac{y + 4}{2} \right) \right) \, dy = \frac{22\pi}{3} \)

5) \( 2\pi \int_0^2 (x + 1)[(2x^2 - 3) + 3] \, dx = \frac{80\pi}{3} \)

6) \( 2\pi \int_0^2 (7 - y) \left( 2 - \sqrt{\frac{y + 3}{2}} \right) \, dy = \frac{1216\pi}{15} \)

7) \( \pi \int_{-4}^2 \left( 3y^2 - 12y + 20 \right) \, dy = \frac{200\pi}{3} \)

8) \( 2\pi \int_0^2 x(2x + 4) \, dx = \frac{200\pi}{3} \)

9) \( 2\pi \int_{-2}^2 (4 - x)(7 - (x^2 + 3)) \, dx = \frac{256\pi}{3} \)