Possible Test I Questions

1. We say a nonempty subset of the real numbers is bounded if there exists a $M \in \mathbb{R}$ such that $|x| \leq M$ for all $x$ in the set. Show that a nonempty set is bounded if and only if it is contained in a bounded interval.

2. Prove that the set $(0, 1)$ has no maximum.

3. Prove that the infimum and supremum of the interval $(a, b)$ are $a$ and $b$ respectively.

4. Find the supremum of the set $\{x \in \mathbb{R} \mid 3x^2 + 3 < 10x\}$.

5. Let $x \in \mathbb{R}$ and $\varepsilon > 0$. Then there exists a rational number $r$ such that $|x - r| < \varepsilon$.

6. Let $A$ and $B$ be nonempty bounded subsets of the real numbers with $A \subset B$. Show that
   $$\inf B \leq \inf A \leq \sup A \leq \sup B.$$ 

7. Let $A$ and $B$ be nonempty bounded subsets of the real numbers. Prove $\inf(A + B) = \inf A + \inf B$.

8. Let $A$ and $B$ be nonempty bounded subsets of the real numbers. Prove
   $$\sup(A \cup B) = \max\{\sup A, \sup B\}$$
   and
   $$\inf(A \cup B) = \min\{\inf A, \inf B\}$$

9. Suppose a nonempty subset of the real numbers is bounded above and the supremum is not in the subset. Show the subset must have an infinite number of elements.

10. Let $f$ and $g$ be bounded real-valued functions on a nonempty set $\mathcal{D}$. Show that
    $$\sup_{x \in \mathcal{D}} (f(x) + g(x)) \leq \sup_{x \in \mathcal{D}} f(x) + \sup_{x \in \mathcal{D}} g(x).$$