Heat Eqn. on Infinite Domains and The Wave Eqn.

1. Solve
\[
\frac{\partial u}{\partial t} = u_{xx}, \quad 0 < x, \\
u(0, t) = 0, \quad t > 0, \\
u(x, 0) = \begin{cases} 
1 & 0 < x < 1 \\
0 & x > 0.
\end{cases}
\]

2. Solve
\[
\frac{\partial u}{\partial t} = u_{xx}, \quad 0 < x, \\
u_x(0, t) = 0, \quad t > 0, \\
u(x, 0) = e^{-x^2}
\]

3. (Wave Equation with free boundary) Solve
\[
u_{tt} = u_{xx}, \quad 0 < x < 1, \\
u(0, t) = 0 = u_t(1, t), \quad t > 0, \\
u(x, 0) = f(x), \quad 0 < x < 1. \\
u_t(x, 0) = 0, \quad 0 < x < 1.
\]

4. (Damped Wave Equation) Solve the following PDE. What happens as \( t \to \infty \)?
\[
u_{tt} = u_{xx} - ku_t, \quad 0 < x < 1, \\
u(0, t) = 0 = u(1, t), \quad t > 0, \\
u(x, 0) = f(x), \quad 0 < x < 1. \\
u_t(x, 0) = 0, \quad 0 < x < 1.
\]