PDE Problems on Finite Domains

1. Starting from scratch express the solution of the following PDE in closed form (no sums). Check so see that your solution in fact solves the PDE, boundary conditions, and initial conditions.

\[
\frac{\partial u}{\partial t} = u_{xx}, \quad 0 < x < 1,
\]

\[
u(0, t) = 0 = u(1, t), \quad t > 0,
\]

\[
u(x, 0) = \sin(x), \quad 0 < x < 1.
\]

2. (Mixed Boundary Conditions) In this case the heat equation has insulated boundary conditions at \( x = 0 \) and Dirichlet boundary conditions at \( x = 1 \). Express the solution as an infinite sum.

\[
\frac{\partial u}{\partial t} = u_{xx}, \quad 0 < x < 1,
\]

\[
u_{x}(0, t) = 0 = u(1, t), \quad t > 0,
\]

\[
u(x, 0) = f(x), \quad 0 < x < 1.
\]

3. (Nonhomogeneous BC) Find the solution to

\[
\frac{\partial u}{\partial t} = u_{xx}, \quad 0 < x < 1,
\]

\[
u(0, t) = 2, \quad u(1, t) = 1, \quad t > 0,
\]

\[
u(x, 0) = 0, \quad 0 < x < 1.
\]

4. (Nonhomogeneous and Mixed BC) Find the solution to

\[
\frac{\partial u}{\partial t} = u_{xx}, \quad 0 < x < 1,
\]

\[
u(0, t) = 1, \quad u_{x}(1, t) = 0, \quad t > 0,
\]

\[
u(x, 0) = 0, \quad 0 < x < 1.
\]