1. Verify the divergence theorem for the field \( \mathbf{F} = (xy^2, -yz, x^2z^2) \), where \( V \) is the cube bounded by the planes \( x = 1, y = 1, z = 1 \). (The answer is \( \frac{1}{6} \).)

2. Verify Stokes theorem for the field \( \mathbf{F} = (-2z, 3x, 4y) \), where \( S \) is parabolic cap \( z = 1 - x^2 - y^2, \ z \geq 0 \). (The answer is \( 3\pi \).)

3. Let \( f(x) \) be a piecewise smooth function defined on the interval \( -L < x < L \). Write a general expression for the Fourier series of the periodic extension of \( f \) and the Fourier coefficients \( a_0, a_n, b_n \) for \( n = 1, 2, 3 \ldots \). You need not derive the expression for the Fourier coefficients.

4. Let \( f(x) \) be defined by \( f(x) = x \) for \( 0 < x < \pi \).

(a) Sketch at least two periods of the odd periodic extension.

(b) Sketch at least two periods of the even periodic extension.

(c) Find the Fourier series of the even periodic extension.

(d) Sketch at least two periods of the Fourier series of part (a) (you need not find the Fourier series to accomplish this).

(e) Use part (c) to find the value of the sum \( 1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \ldots \).