1. The behavior of a physical system is believed to be described by the ODE

\[ y'' + \alpha y' + y = 0 , \]

for \( \alpha > 0 \). Show that for all initial data

\[ \lim_{t \to \infty} y(t) = 0 . \]

2. Suppose that the ODE

\[ y'' + \gamma y = 0 \]

describes an electrical gadget. The engineers of this gadget hope that the device produces oscillatory behavior. How should they choose \( \gamma \)? Why?

3. Suppose that oscillations in a bridge are modeled by the ODE

\[ y'' + \omega^2 y = \sin(\omega_0 t) . \]

How should the parameter \( \omega_0 \) be chosen so that the solutions remain bounded for all time?

4. A system with friction may be modeled by the ODE

\[ y'' + \gamma y' + \lambda^2 y = 0 , \]

where \( \gamma \geq 0 \) is the friction coefficient. How small must \( \gamma \) be chosen so that the system exhibits oscillatory behavior?

5. The manufacture of bathroom scales wants the dial that indicates the weight of a person not to oscillate. Suppose that the position of the dial satisfies the ODE

\[ my'' + 5y' + y = 0 , \]

where \( mg \) is the person’s weight (\( g = 32.2 \text{ ft/sec}^2 \)). How heavy can a person be and not cause the dial to oscillate?

6. Suppose that some interesting process may be modeled by the ODE

\[ y'' - y = 0 . \]

How should the initial data be chosen so that the solution remains bounded as \( t \) approaches infinity?