Project 1

In the following Maple project, let $L_n$, $R_n$, $M_n$, $T_n$ and $S_n$ represent the left sum, right sum, midpoint rule, trapezoid rule and Simpson's rule with $n$ cells, respectively. In addition, set $I = \int_0^1 f(x) \, dx$.

1. Consider the function $f(x) = \sin(x^2)$ on the interval $0 \leq x \leq 1$.

(a) Draw a graph of the function.

(b) Use the graph to decide whether $L_2$, $R_2$, $M_2$, and $T_2$ underestimate or overestimate $I$.

(c) Compute $L_5$, $R_5$, $M_5$ and $T_5$. From the graph which do you think gives the best estimate of $I$? Why?

2. Consider the function $f(x) = \sin(\pi x)$ on the interval $0 \leq x \leq 1$. Using the Maple command provided in the forth section of Lab 3 (see below), determine the best fit for the error for $L_n$, $T_n$ and $S_n$. State explicitly the function you use for the fit. Does the rate of convergence agree with the theory?

3. Consider the function $f(x) = \sin(10x)$ on $0 \leq x \leq 1$. Compute the error using $L_4$, $T_4$ and $S_4$.

The command

\[
abs(evalf(int(sin(10*x), x=0..1)-trapezoid(sin(10*x), x=0..1, 4)))
\]

for example, will be useful. Which scheme is most accurate in this case. Does this contradict any of the theory?

(Section 4 of Lab 3)

Verify the rates of convergence using the following command

\[
> z:= [[n,abs(evalf(int(x^2,x=0..2)-trapezoid(x^2,x=0..2,n)))]$n=8..64]$;
\]

\[
> plot([[z,1.3/x^2],x=8..64,style=line,symbol=circle]);
\]

The function $1.3/x^2$ will need to be adjusted to fit the data for each case and for each integration method.