Estimating Transport Coefficients in Re-entrant Factory Models

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Structure of the re-entrant manufacturing system

- **G**: Generator
- **M**: Machine
- **E**: Exit

At the machine $M_N$, the particles change their attribute from 1 to 2.
Goal

We have the following conservation law:

\[ \partial_t \rho + \partial_x [V \rho - D \partial_x \rho] = 0 \]

where

- flux function \( f(x) = V \rho - D \partial_x \rho \),
- \( V \): Velocity coefficient,
- \( D \): Diffusion coefficient,
- \( \rho \): Density of particles \( (\rho = \rho(x, t)) \).
Goal (ctd.)

- We will try to find the $V$ and $D$ (velocity and diffusion coefficients of the particles in the production system) from DES (Discrete Event Simulation).
- We will use Chi Simulation-version 0.8
Possibility 1

- $V = V(x)$ and $D = D(x)$
  
  $V$ and $D$ depend on $x$.

- Take a part in DES, say at the attribute $p$, and buffer $B_k$, where we have the following buffer structure:

  $B_1 M_1 \ldots B_N M_N$
Possibility 1 (ctd.)

Here, $x \in [0,1]$ is the position variable for the production process, formulated as follows:

$$x = \frac{k}{2N} + \frac{(p-1)}{2},$$

where

- $k$: buffer stage,
- $p$: attribute of the loop (1 or 2),
- $N$: total # of buffers.
Possibility 1 (ctd.)

- In general, particle $n$ is in buffer $B_k$ with attribute $p$ at time $t_{nk}^p$.
- Similarly, particle $n$ is in buffer $B_N$ with attribute 2 at time $t_{nN}^2$.
- So, $\Delta t = t_{nN}^2 - t_{nk}^p$ is the time that particle $n$ takes from $B_k$ with attribute $p$, to $B_N$ with attribute 2.
Possibility 1 (ctd.)

- Since \( x = \frac{k}{2N} + \frac{(p-1)}{2} \), we get: \( x = 1 \) for \( p = 2 \) and \( k = N \).

And we get: \( \Delta x = 1 - \frac{k}{2N} - \frac{p-1}{2} \).

- Average Velocity = \( \frac{\text{Total Distance}}{\text{Total Time}} = \frac{\Delta x}{\Delta t} \).

So, "estimated velocity" for particle \( n \) is:

\[
V_n\left(\frac{k}{2N} + \frac{p-1}{2}\right) = \frac{1 - \frac{k}{2N} - \frac{p-1}{2}}{t_{nN}^2 - t_{nk}^p}.
\]
Possibility 1 (ctd.)

- Then, for each particle \( n \), we will find the velocity according to its position w.r.t. buffer stage and attribute:

\[
v_1(x_{kp}) \ldots v_{100}(x_{kp}), \text{ where } x_{kp} = \frac{k}{2N} + \frac{(p-1)}{2}.
\]

- Velocity coefficient: \( V(x_{kp}) = \frac{1}{100} \sum_j v_j(x_{kp}) \), from mean formula.

\[
\frac{1}{100} \sum_j (v_j(x_{kp}) - V(x_{kp}))^2
\]

- Diffusion coefficient: \( D(x_{kp}) = \frac{V(x_{kp})^2}{V(x_{kp})^2} \), from variance formula.
Possibility 2

- \( V = V(W) \) and \( D = D(W) \)

- \( V \) and \( D \) depend on Wip level of particles:
  
  \[
  W(t) = \int_0^1 \rho(x, t) dx
  \]
Possibility 3

- $V = V(\rho(x, t))$ and $D = D(\rho(x, t))$.

- $V$ and $D$ depend on density of particles $(\rho(x, t))$. 
Possibility 4

- \( V = V(W_-(x,t), W_+(x,t)) \) and \( D = D(W_-(x,t), W_+(x,t)) \)

- \( V \) and \( D \) depend on right and left Wip levels of particles.

where left Wip level is \( W_-(t) = \int_0^x \rho(y,t)dy \)

and right Wip level is \( W_+(t) = \int_x^1 \rho(y,t)dy \).
Possibility 4 (ctd.)

- We can get the Wip level of each particle at each time.
- $W(t_{nk}^p)$ can be found for $n = 1:100$, $k = 1:N$, $p : 1: 2$.

$$1 - \frac{k}{2N} - \frac{p-1}{2}$$

- We know that $V_n(x_k^p) = \frac{2N}{t_{nN}^2 - t_{nk}^p}$, so we can easily draw $W - V$ graph and see how $V$ depends on $W$. 
Possibility 4 (ctd.)

For example:

\[ V(0.15) = \frac{\sum_{nkp} v_n(x_k^p) \cdot \chi_{(0.1,0.2)}(W(t_{nk}))}{\sum_{nkp} \chi_{(0.1,0.2)}W(t_{nk})} \]
Objectives

- Objective 1: Find the velocity and diffusion coefficients in terms of position \( x \). So, \( V(x) = ? \) \( D(x) = ? \)

- Objective 2: Overall, what do \( V(x) \) and \( D(x) \) depend on?