Design of Robust Topologies for Logistics Networks

M.P.M. Hendriks\textsuperscript{1}, D. Armbruster\textsuperscript{2,1}, Marco Laumanns\textsuperscript{2,3}, A.A.J. Lefeber\textsuperscript{1}, J.T. Udding\textsuperscript{1}

\textsuperscript{1}Mechanical Engineering, Eindhoven University of Technology
\textsuperscript{2}Mathematics, Arizona State University
\textsuperscript{3}Institute for Operations Research, ETH Zürich

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Introduction

Long-distance distribution networks:

1. Nodes:
   - Manufacturers (stochastic supply, zero inventory)
   - Warehouses (temporary storage)
   - Customers (stochastic demand, zero inventory)

2. Links
   - Roads

A logistics provider takes over the management of the distribution problem.
Introduction

Objectives

Approach

Results

Robustness

Conclusions

Future work

Manufacturers | Warehouses | Customers

Diagram showing connections between manufacturers, warehouses, and customers.
Introduction

Consider distribution by a third-party logistics provider:

- Number + location of manufacturers, warehouses and customers given
- Supplies and Demands:
  - uncontrollable
  - known over restricted time horizon
- Costs associated with storage, transportation, backlog
Introduction

Problem:

• Which transportation links to use?
• How much to ship through them?

s.t.

• Costs are minimized
• Demands are met
Objectives

1. Given the supply $M(t)$, the demand $C(t)$, warehouses $W$ and their locations, find a network topology s.t.
   - Costs are close to minimal,
   - Number of links is close to minimal.

2. Investigate the robustness of this topology to changes in stochastic behavior of $M(t)$ and $C(t)$. 
Approach

1. Optimization
   - Level 1, Model Predictive Control:
     Optimal shipping schedule for certain topology
   - Level 2, Heuristic:
     Close to optimal topology

2. Robustness
   - Evaluate performance of found topology for different supply and demand distributions
Approach

Optimization, level 1 (MPC):

1. Assume a horizon of $\omega$ days, $\omega \ll T$

2. Define $u(k)$ to be the amounts of products shipped through the links in the considered network at day $(k)$

3. Define costs per day $\alpha(u)(k)$

4. Minimize total costs for days $k,...,k + \omega - 1$

   $$\min_{u(k),...,u(k+\omega-1)} \sum_{q=0}^{\omega-1} \alpha(u)(k + q)$$

   This determines $u^*(k)$

5. Total costs $\Omega$ for time period $T$ for certain topology:

   $$\Omega = \sum_{k=1}^{T} \alpha(u^*)(k)$$
Approach

Optimization, level 2 (Heuristic):

1. Start with the fully connected network

2. Until the network is minimally connected:
   (a) Run the MPC scheme for time period $T$ and compute total costs $\Omega$
   (b) Delete one of the least used links

3. Plot total costs $\Omega$ against number of deleted links
Results

$m=6, w=3, c=7$

Number of deleted links

Costs

10^8

10^7

10^6

10^5

10^4

10^3

10^2

0 10 20 30 40 50 60 70

Number of deleted links
Results

$m=3, w=2, c=20$

Number of deleted links

Costs

Number of deleted links
Results

$m=15, w=3, c=20$

Number of deleted links

Costs

Number of deleted links
Results

$m=20, w=4, c=25$

Number of deleted links vs. Costs

Costs

Number of deleted links
Robustness

Delete links in the same order for new $M(t)$ and $C(t)$ out of
Robustness
Delete links in the same order for new $M(t)$ and $C(t)$ out of
1. Same type of distr., same means, same variances.
Robustness

Delete links in the same order for new $M(t)$ and $C(t)$ out of

1. **Same** type of distr., *same* means, *same* variances.
2. **Different** type of distr., *same* means, *same* variances.
Robustness

Delete links in the same order for new $M(t)$ and $C(t)$ out of

1. **Same** type of distr., **same** means, **same** variances.

2. **Different** type of distr., **same** means, **same** variances.

3. **Sinusoidal** time series, **same** means, **same** variances.
Robustness

Delete links in the same order for new $M(t)$ and $C(t)$ out of

1. Same type of distr., same means, same variances.
2. Different type of distr., same means, same variances.
3. Sinusoidal time series, same means, same variances.
4. Same type of distr., same means, different variances.
Robustness

Delete links in the same order for new $M(t)$ and $C(t)$ out of

1. **Same** type of distr., **same** means, **same** variances.
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4. **Same** type of distr., **same** means, **different** variances.
5. **Same** type of distr., **different** means, **same** variances.
Robustness

![Robustness Chart]

- Original result
- Experiment 1
- Experiment 2
- Experiment 3
- Experiment 4
- Experiment 5

Number of deleted links vs Costs

0 10 20 30 40 50 60 70

10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8

Number of deleted links

Costs

/department of mechanical engineering
Conclusions

- The optimization procedure finds a cost-effective topology with only 5% of all possible links. This topology is only sensitive to changes in the 1st moments of supply and demand distributions.
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• The 1st moments suffice to generate a set of stochastic supply and demand distributions from which such a close to optimal topology can be constructed.
Conclusions

• The optimization procedure finds a cost-effective topology with only 5% of all possible links. This topology is only sensitive to changes in the $1^{st}$ moments of supply and demand distributions.

• The $1^{st}$ moments suffice to generate a set of stochastic supply and demand distributions from which such a close to optimal topology can be constructed.

• The optimization procedure finds a cost-effective topology with only 15% of all possible links. This topology is insensitive to changes in supply and demand distributions.
Future work

- Sensitivity analysis of the network performance dependent on changes in the 1st moments of supply and demand distributions.
- Analysis of the network structure dependent on the trade-off between different associated costs.

Link to paper:
se.wtb.tue.nl

⇒ Research
⇒ SE Reports
⇒ SE Report 2006-06
relatively low backlog costs  relatively high backlog costs