Problem 1:
(a) Give the iteration function of the Newton - Gauss - Seidel method (i.e. Newton with one Gauss Seidel step on the linear system) for the nonlinear system
\[ x_1^2 + x_2^2 - 2 = 0, \quad x_1^2 - x_2 = 0. \]
(b) What is the local convergence rate of the resulting nonlinear iteration?

Problem 2:
The matrix \( C \) is given by the product \( C = AB \) with
\[
C = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}
\]
find the eigenvalues and eigenvectors of \( C \).

Problem 3:
Consider the matrix
\[
A = \begin{pmatrix} 51 & -49 & 49 \\ 2 & 0 & -2 \\ 51 & -51 & 49 \end{pmatrix}
\]
(a) Use three steps of power iteration to estimate the largest eigenvalue and eigenvector. (Carry three significant digits.)
(b) Explain how you would go on, using deflation, to compute the next eigenvalue and eigenvector. Give the Householder matrix for the similarity transformation of the matrix \( A \) onto a matrix of the form
\[
A_1 = \begin{pmatrix} \lambda_1 & * & * \\ 0 & * & * \\ 0 & * & * \end{pmatrix}
\]

Problem 4:
Consider the nonlinear iteration \( x \to \phi(x) \) with \( \phi \) given implicitly by
\[
\omega \phi_1 + (1 - \omega) x_1^2 + x_2^2 - 1 = 0, \quad 2\phi_1^2 - 4\phi_1 + [\omega \phi_2 + (1 - \omega) x_2]^2 = 0
\]
What is the optimal relaxation parameter \( \omega \) that minimizes the spectral radius \( \rho[D\phi(x^*)] \)? Can \( \omega \) be chosen such that the method is faster than linearly convergent?