1. (20’) The determinant of a 3-by-3 matrix can be computed fairly easily. Here is the wiki entry [http://en.wikipedia.org/wiki/Determinant#3-by-3_matrices](http://en.wikipedia.org/wiki/Determinant#3-by-3_matrices) and this diagram is also useful [http://en.wikipedia.org/wiki/File:Sarrus_rule.png](http://en.wikipedia.org/wiki/File:Sarrus_rule.png)

Now, consider matrix

\[
A = \begin{pmatrix}
3 & 0 & -3 \\
-3 & 6 & 0 \\
0 & -6 & 3
\end{pmatrix}
\]

(a) Find the characteristic equation \( \det(\lambda I - A) = 0 \) and show that the 3 eigenvalues are 0, 6 + 3i, 6 − 3i. Hint: the calculation can be easier if the characteristic equation is factorized before the \( \lambda \)'s are plugged in.

(b) Among the following 4 vectors, match 3 of them with the associated eigenvectors; and then, show that the remaining one is not an eigenvector.

\[
\begin{pmatrix}
-i \\
1 \\
i - 1
\end{pmatrix}, \quad
\begin{pmatrix}
4 \\
2 \\
4
\end{pmatrix}, \quad
\begin{pmatrix}
4 \\
-2 \\
4
\end{pmatrix}, \quad
\begin{pmatrix}
i \\
1 \\
-i - 1
\end{pmatrix}
\]

(c) Find 3 linearly independent solutions to

\[
\frac{d}{dt} \vec{x}(t) = A\vec{x}(t)
\]

in terms of complex numbers. Explain how would you verify their linear independence, but do not actually compute it!

2. (20’) Find the particular solution, in terms of real numbers, for

\[
\begin{cases}
x'_1 = -2x_1 + 4x_2, & x_1(0) = 0 \\
x'_2 = -4x_1 - 2x_2, & x_2(0) = -1
\end{cases}
\]

3. (20’) Problem 4 on Page 316 in the textbook is

\[
\begin{cases}
x'_1 = 4x_1 + x_2 \\
x'_2 = 6x_1 - x_2
\end{cases}
\]
(a) Find the general solution. You answer is not necessarily the same as given by the textbook on Page 548. **Show all your work.**

(b) Copy the graph on Page 548 and indicate which solution curve corresponds to

\[
\text{initial condition (1): } \begin{align*}
    x_1(0) &= -4 \\
    x_2(0) &= 4
\end{align*}
\]

and which curve corresponds to

\[
\text{initial condition (2): } \begin{align*}
    x_1(0) &= 3 \\
    x_2(0) &= 3
\end{align*}
\]

(c) Find the particular solution satisfying initial condition (2).

4. (20’) Problem 4 on Page 316 in the textbook is

\[
\begin{align*}
    x_1' &= 9x_1 + 5x_2 \\
    x_2' &= -6x_1 - 2x_2
\end{align*}
\]

(a) Find the general solution. You answer is not necessarily the same as given by the textbook on Page 548. **Show all your work.**

(b) Copy the graph on Page 548 and indicate which solution curve corresponds to

\[
\text{initial condition (3): } \begin{align*}
    x_1(0) &= 1 \\
    x_2(0) &= 0
\end{align*}
\]

and which curve corresponds to

\[
\text{initial condition (4): } \begin{align*}
    x_1(0) &= -1 \\
    x_2(0) &= 0
\end{align*}
\]

Are these two curves symmetric in any sense?

(c) Find the particular solutions satisfying initial condition (3); then, find the particular solutions satisfying initial condition (4). Are these two solutions symmetric in any sense?

5. (20’) Problem 8 on Page 316 in the textbook is

\[
\begin{align*}
    x_1' &= x_1 - 5x_2 \\
    x_2' &= x_1 - x_2
\end{align*}
\]
(a) Find the general solution in terms of complex numbers.
(b) Find the general solution in terms of real numbers.
(c) Copy the graph on Page 548 and indicate which solution curve corresponds to

\[
\begin{align*}
\text{initial condition (5): } & \begin{cases} 
    x_1(0) = 0 \\
    x_2(0) = 2
\end{cases}
\end{align*}
\]

and which curve corresponds to

\[
\begin{align*}
\text{initial condition (6): } & \begin{cases} 
    x_1(0) = -4 \\
    x_2(0) = 2
\end{cases}
\end{align*}
\]