1. (1 pt) It can be helpful to classify a differential equation, so that we can predict the techniques that might help us to find a function which solves the equation. Two classifications are the order of the equation – (what is the highest number of derivatives involved) and whether or not the equation is linear. Linearity is important because the structure of the family of solutions to a linear equation is fairly simple. Linear equations can usually be solved completely and explicitly.

Determine whether or not each equation is linear:

1. \( \frac{d^4y}{dt^4} + \frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} + \frac{dy}{dt} = 1 \)
2. \( y'' - y + y^2 = 0 \)
3. \( \frac{d^3y}{dt^3} + t \frac{dy}{dt} + (\cos^2(t))y = t^3 \)
4. \( (1 + y^2) \frac{d^2y}{dt^2} + t \frac{dy}{dt} + y = e^t \)

**Correct Answers:**
- 4 Linear
- 2 Nonlinear
- 3 Linear
- 2 Nonlinear

2. (1 pt) Find the general solution to the homogeneous differential equation

\[ \frac{d^2y}{dx^2} - 0 \frac{dy}{dx} - 1y = 0 \]

The solution has the form

\[ y = C_1 f_1(x) + C_2 f_2(x) \]

with

\[ f_1(x) = \quad \text{and} \quad f_2(x) = \]

Left to your own devices, you will probably write down the correct answers, but in case you want to quibble, enter your answers so that \( f_1, f_2 \) are normalized with their value at \( x = 0 \) equal to 1.

**Correct Answers:**
- \( \exp(-1x) \)
- \( \exp(1x) \)

3. (1 pt) Find the general solution to the homogeneous differential equation

\[ \frac{d^2y}{dt^2} - 15 \frac{dy}{dt} + 50y = 0 \]

The solution can be written in the form

\[ y = C_1 e^{r_1 t} + C_2 e^{r_2 t} \]

with

\[ r_1 < r_2 \]

Using this form, \( r_1 = \quad \text{and} \quad r_2 = \quad \)

**Correct Answers:**
- 5
- 10
4. (1 pt)
Find the general solution to the homogeneous differential equation
\[
\frac{d^2y}{dt^2} - 9 \frac{dy}{dt} = 0
\]
The solution has the form
\[
y = C_1 f_1(t) + C_2 f_2(t)
\]
with \( f_1(t) = \text{___________} \) and \( f_2(t) = \text{___________} \)
Left to your own devices, you will probably write down the correct answers, but in case you want to quibble, enter your answers so that \( f_1, f_2 \) are normalized with their value at \( t = 0 \) equal to 1.

**Correct Answers:**
- \( \exp(9t) \)
- \( \exp(0t) \)

5. (1 pt)
Find the solution to the boundary value problem:
\[
\frac{d^2y}{dt^2} - 6 \frac{dy}{dt} + 5y = 0, \quad y(0) = 5, y(1) = 7
\]
The solution is __________________________

**Correct Answers:**
- \( 5.04524118668836*\exp(1*t) - 0.0452411866883553*\exp(5*t) \)

6. (1 pt)
Find the general solution to the homogeneous differential equation
\[
\frac{d^2y}{dx^2} + 15 \frac{dy}{dx} + 50y = 0
\]
The solution has the form
\[
y = C_1 f_1(x) + C_2 f_2(x)
\]
with \( f_1(x) = \text{___________} \) and \( f_2(x) = \text{___________} \)
Left to your own devices, you will probably write down the correct answers, but in case you want to quibble, enter your answers so that \( f_1, f_2 \) are normalized with their value at \( x = 0 \) equal to 1.

**Correct Answers:**
- \( \exp(-5*x) \)
- \( \exp(-10*x) \)

7. (1 pt) Find \( y \) as a function of \( t \) if
\[
4y'' - 9y = 0, \quad y(0) = 8, \quad y'(0) = 8.
\]
\( y(t) = \) __________________________

**Correct Answers:**
- \( (4 - 8/3) *\exp((0 - 3/2)*t) + (4 + 8/3) *\exp((0 + 3/2)*t) \)

8. (1 pt) Find \( y \) as a function of \( t \) if
\[
y'' + y' - 12y = 0, \quad y(0) = 8, \quad y(1) = 7.
\]
\( y(t) = \) __________________________

Remark: The initial conditions involve values at two points.

**Correct Answers:**
- \( (8*\exp(-4) - 7)/(\exp(-4) - \exp(3)) *\exp(3*t) + (-8*\exp(3) + 7)/(\exp(-4) - \exp(3)) *\exp(-4*t) \)
9. (1 pt) Find y as a function of t if 

$$2500y'' - 729y = 0$$

with 

$$y(0) = 9, \quad y'(0) = 4.$$ 

\[ y = \] 

**Correct Answers:**

- $$(9/2 - 100/27) \exp((0 - 27/50)*t) + (9/2 + 100/27) \exp((0 + 27/50)*t)$$

10. (1 pt) Determine whether the following pairs of functions are linearly independent or not.

? 1. The Wronskian of two functions is \( W(t) = t \) are the functions linearly independent or dependent?

? 2. \( f(x) = e^{15x} \) and \( g(x) = e^{15(x-1)} \)

? 3. \( f(t) = t^2 + 15t \) and \( g(t) = t^2 - 15t \)

**Correct Answers:**

- Linearly independent
- Linearly dependent
- Linearly independent