\[0.5 \, y'' + y + 5y = -3\cos(2t) + 2\sin(2t)\]

\[Y_p = a \cos(2t) + b \sin(2t)\]

Plug into the original eq, and get \(a = -1, \ b = 0\)

So \(Y_p = -\cos(2t)\). Done with \(Y_p\)!

For \(Y_h\), we solve

\[0.5y'' + y' + 5y = 0\]

Ch. Eq:

\[0.5r^2 + r + 5 = 0\]

Quad. Formula

\[r = -1 \pm 12 - 4 \times 0.5 \times 52 \times 0.5\]

\(r_1 = -1 + 3i, \ r_2 = -1 - 3i\) (Euler’s formula)

\(y_h = d_1 e^{-1+3i} + d_2 \ldots\)

\(= d_1 e^{-t} e^{3it} + d_2 \ldots\)

\(= d_1 e^{-t} \cos(3t) + i \sin(3t) + d_2 \ldots\)

\(y_h = c_1 e^{-t} \cos(3t) + c_2 e^{-t} \sin(3t)\)

Answer

\(Y = Y_p + Y_h = -\cos(2t) + c_1 e^{-t} \cos(3t) + c_2 e^{-t} \sin(3t)\)

As \(t\) approaches infinity
Y(t) behaves just like \(-\cos(2t)\). So, the surviving frequency is \(\omega=2\).

And it “forgets” its own frequency \(\omega=3\).