MAT 270 Derivative Mastery Test 2N
Solutions

Find the derivative of each of the following functions. Take the derivative with respect to the variable which appears in the function (x for problem (1), t for problem (2), etc.)

Note: In addition to the answers, the rules which are used to find the answers are listed, as well as the basic functions whose derivatives were needed. Functions whose derivatives were taken are: E is for exponential functions, P is for power functions \(x^r\), L is for logarithms, T is for trig functions, and I is for inverse trig functions. Transformation rules: q is for Quotient Rule, p is for Product Rule, and c is for the Chain Rule. (For instance, problem #1 uses the chain rule and the derivative of an exponential function.) In some cases, the function can be rewritten to avoid using one or more rules; problem #5 can be rewritten as \(-3x^{-4/3}\) to avoid using the Quotient Rule and Chain Rule.

Solutions to Section S begin on page 4 of this document.

(1) \(\ln(\sin x)\)
   \[Answer: \text{TLC.} \frac{1}{\sin x} \cdot \cos x\]

(2) \(e^{-z} \sin(2z)\)
   \[Answer: \text{ETcp.} e^{-z} \cdot 2 \cos(2z) + \sin(2z) \cdot (-e^{-z})\]

(3) \(8v^3 + 9v - 7\)
   \[Answer: \text{P.} 24v^2 + 9\]

(4) \(\frac{u^3}{u - 3}\)
   \[Answer: \text{Pq.} \frac{(u - 3)3u^2 - u^3(1)}{(u - 3)^2}\]

(5) \((2\pi + x)^e\)
   \[Answer: \text{Pc.} e(2\pi + x)^{e-1}(1)\]

(6) \(\sec(t^2)\)
   \[Answer: \text{TPc.} \sec(t^2) \tan(t^2) \cdot 2t\]

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(7) \( \ln 3^y \)

Answer: ELc. \( \frac{1}{3^y} \cdot \ln 3 \cdot 3^y \)

(8) \( \sin^{-1}(2x) \)

Answer: Ic. \( \frac{1}{\sqrt{1-(2x)^2}} \cdot 2 \)

(9) \( \frac{1+v}{v+3\ln v} \)

Answer: Lq. \( \frac{(v+3\ln v)(1)-(1+v)(1+3/v)}{(v+3\ln v)^2} \)

(10) \( t - t \cos t \)

Answer: Tp. \( 1 - (t(-\sin t) + \cos t) \)

(11) \( u - 8^u \)

Answer: E. \( 1 - \ln 8 \cdot 8^u \)

(12) \( \cos^2(2\phi) \)

Answer: TPc. \( 2\cos(2\phi)(-\sin(2\phi))(2) \)

(13) \( \sin(3x) \)

Answer: Tc. \( \cos(3x)(3) \)
(14) \[ \frac{t - 1}{2t^2 + 1} \]

Answer: Pq. \[ \frac{(2t^2 + 1)(1) - (t - 1)(4t)}{(2t^2 + 1)^2} \]

(15) \ln(e^x + 1)

Answer: LEc. \[ \frac{1}{e^x + 1} \cdot e^x \]

(16) \tan(sin x)

Answer: Tc. \sec^2(sin x)(cos x)

(17) \[ \frac{-4}{\sqrt{y^5}} \]

Answer: P or Pq. \((-4)(-5/2)y^{-7/2}\)

(18) \csc^3 \theta

Answer: PTc. \(3 \csc^2 \theta(-\csc \theta \cot \theta)\)

(19) \[ \frac{1}{1 + 2e^y} \]

Answer: Eq. \[ \frac{-(2e^y)(1)}{(1 + 2e^y)^2} \]

(20) \log_2(5t)

Answer: Lc. \[ \frac{1}{5t \cdot \ln 2} \cdot 5 \]
MAT 270 Derivative Mastery Test 2S
Solutions

See page 1 for notation.

(1) \[ \frac{x^2}{x + \pi} \]

Answer: Pq. \[ \frac{(x + \pi)(2x) - x^2(1)}{(x + \pi)^2} \]

(2) \[ 16u^8 - 9u^5 + 1 \]

Answer: P. \( (16)(8)u^7 - 9(5)u^4 \)

(3) \[ e^{-x} \cos(3x) \]

Answer: TEpc. \( e^{-x}(-\sin(3x))(3) + \cos(3x) \cdot -e^{-x} \)

(4) \[ \ln(\cos \phi) \]

Answer: LTc. \( \frac{1}{\cos \phi} \cdot (-\sin \phi) \)

(5) \[ \log_5(-x) \]

Answer: Lc. \( \frac{1}{-x \cdot \ln 5} \cdot (-1) \)

(6) \[ \frac{1}{2 - 3e^z} \]

Answer: Eq. \( \frac{-1(-3e^z)}{(2 - 3e^z)^2} \)
(7) $\sec^3 x$

*Answer:* TPC. $(3 \sec^2 x)(\sec x \tan x)$

(8) $\frac{3}{\sqrt[5]{y^2}}$

*Answer:* P or Pq. $3(-2/5)y^{-7/5}$

(9) $\sin(\tan \theta)$

*Answer:* Tc. $\cos(\tan \theta) \sec^2 \theta$

(10) $\ln(e^y - 1)$

*Answer:* ELq. $\frac{1}{e^y - 1} \cdot e^y$

(11) $\frac{t + 2}{3t - 1}$

*Answer:* q. $\frac{(3t - 1)(1) - (t + 2)(3)}{(3t - 1)^2}$

(12) $\cos(4t)$

*Answer:* Tc. $-\sin(4t)(4)$

(13) $\tan^2(5x)$

*Answer:* PTc. $2 \tan(5x) \cdot \sec^2(5x) \cdot 5$

(14) $x + 0.5^x$

*Answer:* E. $1 + (\ln 0.5) \cdot (0.5)^x$
(15) $y^2 - ye^y$

Answer: PEp. $2y - (ye^y + e^y)$

(16) $\frac{1 - u}{u + \ln u}$

Answer: Lq. $\frac{(u + \ln u)(-1) - (1 - u)(1 + 1/u)}{(u + \ln u)^2}$

(17) $\tan^{-1}(3y)$

Answer: Ic. $\frac{1}{1 + (3y)^2} \cdot 3$

(18) $\ln 2^x$

Answer: ELc. $\frac{1}{2^x} \cdot \ln 2 \cdot 2^x$

(19) $\cot(4\theta)$

Answer: Tc. $-\csc^2(4\theta)(4)$

(20) $(y - 3e)^{4\pi}$

Answer: Pc. $4\pi(y - 3e)^{4\pi-1}(1)$