MATH 119 Test #2 Solutions

There were two versions of Test #2 written up, and this document provides solutions to both of them. If your test is Form C, you should consult the parts of the solutions labelled [C]; if your test is Form H, consult the solutions labelled [H]. In each of the problems, version [C] of the test had the wording in the upper part of the \{ ∗ \} constructions, and [H] had the lower wording.

(1) Identify the following tableaus as one of the following:

i. A final tableau. In this case, state the solution (values of \( P \) and \( x \)’s), and list the basic variables.

ii. Requires pivoting. Write this and circle the pivot element.

iii. No maximum. What can you say about the feasible region in this case?

<table>
<thead>
<tr>
<th>( P )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( x_5 )</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-2</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-4</td>
<td>0</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>18</td>
</tr>
</tbody>
</table>

Answer: Final tableau. \( x_1 = 3, \ x_2 = 10, \ x_3 = 0, \ x_4 = 4, \ x_5 = 0, \ P = 18 \). The basic variables are \( x_1, \ x_2, \) and \( x_4 \).

<table>
<thead>
<tr>
<th>( P )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( x_5 )</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>-1/4</td>
<td>2</td>
<td>0</td>
<td>1/3</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1/2</td>
<td>3</td>
<td>1</td>
<td>1/3</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>2</td>
<td>-2</td>
<td>0</td>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

Answer: Requires pivoting. The next pivot element is boxed above.

<table>
<thead>
<tr>
<th>( P )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( x_5 )</th>
<th>( x_6 )</th>
<th>( x_7 )</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>1/2</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1/6</td>
<td>14</td>
</tr>
<tr>
<td>0</td>
<td>-1/2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-5/6</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>13/6</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>0</td>
<td>-3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-2</td>
<td>16</td>
</tr>
</tbody>
</table>

Answer: No maximum; the feasible region is unbounded. (All of the entries in the \( x_3 \) column are \( \leq 0 \).)

<table>
<thead>
<tr>
<th>( P )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( x_5 )</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1/3</td>
<td>-3/2</td>
<td>1</td>
<td>0</td>
<td>( 7 )</td>
<td>14</td>
</tr>
<tr>
<td>0</td>
<td>8/3</td>
<td>-1/2</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>-4</td>
<td>25</td>
</tr>
</tbody>
</table>

Answer: Requires pivoting. The next pivot element is boxed above.
H (1) Identify the following tableaus as one of the following:

i. **A final tableau.** In this case, state the solution (values of P and x’s), and list the basic variables.

ii. **Requires pivoting.** Write this and circle the pivot element.

iii. **No maximum.** What can you say about the feasible region in this case?

### a. [5 pts]

\[
\begin{array}{cccccccc|c}
P & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & \text{RHS} \\
0 & 1/2 & 1 & 0 & -2 & 0 & 1 & 3 \\
0 & -1/2 & 0 & 0 & 0 & 1 & -2 & 5 \\
0 & 0 & 0 & 1 & -1 & 0 & -3 & 2 \\
1 & 1 & 0 & 0 & -1 & 0 & -2 & 24
\end{array}
\]

**Answer:** Requires pivoting. The next pivot element is boxed above.

### b. [5 pts]

\[
\begin{array}{cccccccc|c}
P & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & \text{RHS} \\
0 & 1 & 1/2 & 3 & -1/2 & 0 & 1 & 5 \\
0 & 0 & -1/4 & -3 & 2 & 1 & 0 & 2 \\
1 & 0 & 1/2 & 0 & 4 & 0 & 8 & 12
\end{array}
\]

**Answer:** Final tableau. \(x_1 = 5, x_2 = 0, x_3 = 0, x_4 = 0, x_5 = 2, x_6 = 0, P = 12\). The basic variables are \(x_1\) and \(x_5\).

### c. [5 pts]

\[
\begin{array}{cccccccc|c}
P & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & \text{RHS} \\
0 & 2 & 0 & 1 & 0 & 5 & 1/3 & 10 \\
0 & -3 & 1 & 0 & 0 & 1/2 & -1/30 & 1 \\
0 & 1 & 0 & 0 & 1 & -3/2 & 1/10 & 3 \\
1 & 1 & 0 & 0 & 0 & -7/2 & -3 & 9
\end{array}
\]

**Answer:** Requires pivoting. The next pivot element is boxed above.

### d. [5 pts]

\[
\begin{array}{cccccccc|c}
P & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & \text{RHS} \\
0 & 0 & 0 & 2 & 0 & 1 & 3 & 0 & -1 & 6 \\
0 & 0 & 0 & 0 & 0 & 10 & 1 & 4 & 20 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 & -3 & 3 \\
0 & 0 & 3 & 1 & 0 & -8 & 0 & 2 & 18 \\
1 & 0 & -2 & 0 & 0 & -2 & 0 & 4 & 14
\end{array}
\]

**Answer:** Requires pivoting. The next pivot element is boxed above.

**Grading:** For tableaus which required pivoting, +2 points for saying that pivoting was required, +3 points for choosing the pivot element. If the tie-breaking rule was wrong, −1 points were given; for any other entry, −2 points were given. For tableaus which represented final tableaus, +2 points were awarded for saying the tableau was final, +2 points for writing out the complete solution, +1 point for listing the Basic Variables. For tableaus which represented problems with no maximum, +2 points for identification, +3 points for saying the region was unbounded. −2 points for saying the region was “undefined” or “the tableau is unbounded.”

2
A company manufactures three types of toys A, B, and C. Each requires rubber, aluminum, and plastic as listed below.

<table>
<thead>
<tr>
<th>Toy</th>
<th>Rubber</th>
<th>Aluminum</th>
<th>Plastic</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

The company has available 400 units of rubber, 1100 units of aluminum, and 2000 units of plastic. The company makes a profit of $6, $7, and $3 on toys A, B, and C respectively. Assuming all toys that are manufactured are sold, determine how many of each toy should be produced to maximize the profit.

a. [7 pts] Define the variables $x_1$, $x_2$, $x_3$. Write the objective function, and all the constraints.

Solution: The variables are the number of each type of toy to make: $x_1$ is the number of Toy A that is made, $x_2$ the number of Toy B, and $x_3$ is the number of Toy C. The objective value is what is being maximized or minimized; in this case, it is the profit. So $P = 6x_1 + 7x_2 + 3x_3$.

The constraints are conditions on $x_1$, $x_2$, and $x_3$. They are represented by the limited resources of rubber, aluminum, and plastic; you can’t use more than the company has available. So the conditions are

\[
\begin{align*}
    x_1 + x_2 + x_3 & \leq 400 \quad \text{(rubber)} \\
    2x_1 + 2x_2 + 4x_3 & \leq 1100 \quad \text{(aluminum)} \\
    5x_1 + 4x_2 + 4x_3 & \leq 2000 \quad \text{(plastic)}
\end{align*}
\]

Also, each $x_i$ must be nonnegative, so the constraints $x_1 \geq 0$, $x_2 \geq 0$, and $x_2 \geq 0$ need to be added. (See H for the grading scheme.)

b. [8 pts] Set up the initial simplex tableau, circle the pivot element, and do one complete pivot.

Solution:

\[
\begin{bmatrix}
BV & P & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & RHS \\
\hline
x_4 & 0 & 1 & 1 & 1 & 0 & 0 & 400 & 2 - 21 \\
x_5 & 0 & 2 & 2 & 4 & 0 & 1 & 0 & 1100 & 3 - 41 \\
x_6 & 0 & 5 & 4 & 4 & 0 & 0 & 1 & 2000 & 4 + 71 \\
P & 1 & -6 & -7 & -3 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
BV & P & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & RHS \\
\hline
x_2 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 400 \\
x_5 & 0 & 0 & 0 & 2 & -2 & 1 & 0 & 300 \\
x_6 & 0 & 1 & 0 & 0 & -4 & 0 & 1 & 400 \\
P & 1 & 1 & 0 & 4 & 7 & 0 & 0 & 2800 \\
\end{bmatrix}
\]
c. **[5 pts]** Examine the tableau you found by performing one pivot of the initial tableau. If it requires another pivot, circle the pivot element. If it is a final tableau, write the complete solution (slack variables not required) and present your answer in the context of the problem, with units.

**Solution:** The tableau is a final tableau, since there are no negative numbers in the bottom row. The optimal solution is thus \(x_1 = 0, x_2 = 400, x_3 = 0\), or: The company should produce 400 units of Toy B and none of Toy A or Toy C, for a total profit of $2800.

\[H\] (2) A company manufactures three types of toys A, B, and C. Each requires rubber, aluminum, and plastic as listed below.

<table>
<thead>
<tr>
<th>Toy</th>
<th>Rubber</th>
<th>Aluminum</th>
<th>Plastic</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

The company has available 600 units of rubber, 1000 units of aluminum, and 4000 units of plastic. The company makes a profit of $6, $8, and $4 on toys A, B, and C respectively. Assuming all toys that are manufactured are sold, determine how many of each toy should be produced to maximize the profit.

a. **[7 pts]** Define the variables \(x_1, x_2, x_3\). Write the objective function, and all the constraints.

**Solution:** The variables are defined to be the number of each type of toy to be produced, as in \([C]\). The objective function is \(P = 6x_1 + 8x_2 + 4x_3\), and the constraints are

\[
\begin{align*}
2x_1 + x_2 + 3x_3 & \leq 600 \quad \text{(rubber)} \\
3x_1 + 2x_2 + 3x_3 & \leq 1000 \quad \text{(aluminum)} \\
7x_1 + 8x_2 + 6x_3 & \leq 4000 \quad \text{(plastic)}
\end{align*}
\]

and \(x_1, x_2, x_3 \geq 0\).

Grading: +1 point for defining \(x_1, x_2, x_3\); +2 points for finding \(P\); +1 point per constraint; +1 point for all of the nonnegative constraints. Grading for common mistakes: −1 point for equalities instead of inequalities; −1 point for transposing the equations (i.e., \(2x_1 + 3x_2 + 7x_3 \leq 600\) for \([H]\)).

b. **[8 pts]** Set up the initial simplex tableau, circle the pivot element, and do one complete pivot.
Solution:

\[
\begin{array}{c|cccccc|c}
BV & P & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & RHS \\
\hline
x_4 & 0 & 2 & 1 & 3 & 1 & 0 & 0 & 600 \\
x_5 & 0 & 3 & 2 & 3 & 0 & 1 & 0 & 1000 \\
x_6 & 0 & 7 & 8 & 6 & 0 & 0 & 1 & 4000 \\
P & 1 & -6 & -8 & -4 & 0 & 0 & 0 & 0 \\
\end{array}
\]

\[\rightarrow\]

\[
\begin{array}{c|cccccc|c}
BV & P & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & RHS \\
\hline
x_4 & 0 & 2 & 1 & 3 & 0 & 0 & 0 & 0 \\
x_5 & 0 & 3/2 & 1 & 3/2 & 0 & 1/2 & 0 & 500 \\
x_6 & 0 & 7 & 8 & 6 & 0 & 0 & 1 & 4000 \\
P & 1 & -6 & -8 & -4 & 0 & 0 & 0 & 0 \\
\end{array}
\]

\[\rightarrow\]

\[
\begin{array}{c|cccccc|c}
BV & P & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & RHS \\
\hline
x_4 & 0 & 1/2 & 0 & 3/2 & 1 & -1/2 & 0 & 100 \\
x_2 & 0 & 3/2 & 1 & 3/2 & 0 & 1/2 & 0 & 500 \\
x_6 & 0 & -5 & 0 & -6 & 0 & -4 & 1 & 0 \\
P & 1 & 6 & 0 & 8 & 0 & 4 & 0 & 4000 \\
\end{array}
\]

Gradint: +3 points for setting up the initial tableau; +2 points for finding the pivot element, +3 points for the pivoting operations. –2 points, if the pivoting operations were done, but the row operations were not listed. Note that the tableau was used from the Linear Program given in (a), not the “correct” one.

c. [5 pts] Examine the tableau you found by performing one pivot of the initial tableau. If it requires another pivot, circle the pivot element. If it is a final tableau, write the complete solution (slack variables not required) and present your answer in the context of the problem, with units.

Solution: The tableau is a final tableau, since there are no negative numbers in the bottom row. The optimal solution is thus \(x_1 = 0\), \(x_2 = 500\), \(x_3 = 0\), or: The company should produce 500 units of Toy B and none of Toy A or Toy C, for a total profit of $4000.

Grading: +2 points for stating that the tableau was a final one; +3 points for the answer. Grading for common mistakes: –1 point if \(x_1\) and \(x_3\) were not listed, and the answer to the problem was “only make Toy B”; –2 points if \(x_1\) and \(x_3\) were not given; –1 points for not putting the answer in context.

Grading for (c) was based on the last tableau in (b). If a mistake was made earlier, part (c) was graded as follows: If another pivot was needed, +5 points were awarded for finding the correct pivot; +4 points if the incorrect pivot was chosen, but only because of a bad tie-breaking rule; and +3 points if the wrong row or column was chosen, unless the circled entry was \(\leq 0\), in which case +1 point (total for (c)) was awarded.
Rhiannan is looking to purchase a farm. The one she likes best is priced at $125,000. She can afford to make a 25% down payment, and the rest she can borrow with a thirty-five year amortized loan at an annual rate of 6.0% compounded monthly.

a. [5 pts] What is her monthly payment? Show your work.

Solution: Since she is paying off a debt, the formula from 5.4 is to be used:

$$(0.75)(125,000) = A = P \cdot \frac{n}{r} \left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right] = P \cdot \frac{12}{0.06} \left[1 - \left(1 + \frac{0.6}{12}\right)^{-12 \cdot 30}\right]$$

or $$P = \frac{(0.75)(125,000)}{12 \cdot 0.06} = \frac{12 \cdot 0.06 \cdot 12}{(0.06 + 0.6)^{-12 \cdot 30}} = 166.7916,$$

so $$P = \frac{(0.80)(120,000)}{12 \cdot 0.08} = \frac{12 \cdot 0.08 \cdot 12}{(0.08 + 0.8)^{-12 \cdot 35}} = 140.7933,$$

b. [10 pts] Rhiannan purchased the farm. How much equity will she have established for herself after making this monthly payment for twelve years? Don’t forget the down payment, and show all work.

Solution: The equity is equal to the original price minus the present value of $562.08 (for C; $681.85 for H). The present value is calculated using the formula from 5.4. Here $$t = 30 - 12 = 18$$ (for C), because $$t$$ represents the number of years remaining. Then

$$PV = P \cdot \frac{n}{r} \left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right] = (562.08) \frac{12}{0.06} \left[1 - \left(1 + \frac{0.6}{12}\right)^{-12 \cdot 18}\right] = 74,137.16,$$

with an equity of $$125,000 - PV = 50,862.84$$, for C. For H, $$t = 35 - 10 = 25$$, and the present value is

$$PV = P \cdot \frac{n}{r} \left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right] = (681.85) \frac{12}{0.08} \left[1 - \left(1 + \frac{0.8}{12}\right)^{-12 \cdot 25}\right] = 88,343.57,$$

with an equity of $$120,000 - PV = 35,656.43$$.

Grading: +3 points for the formula to use for Present Value (PV), +2 points for finding PV, +3 points for finding the equity. Grading for common mistakes: −1 point for $$t = 12$$ (C) or 10 (H); −1 point for using the formula from 5.3; −2 points for using the formula from 5.2; −2 points for saying the equity is the down payment plus the Present Value.

c. [5 pts]

Not graded. These points were given automatically.
C (4) [10 pts] One day Luc gave Bala $90 as an eighteen month discount loan, at 11% interest. How much does Bala owe Luc at the end of the eighteen months?

Solution: The discounted loan formula is a variation of the formula from 5.1:

\[ P = L(1 - rt) \]
\[ 90 = L(1 - (.11)(1.5)) \]
\[ L = \frac{90}{0.835} = 107.78. \]

Grading: +5 points for the formula, +5 points for substitution and solving for \( L \). Grading for common mistakes: −3 points for the wrong formula, except −2 points for \( L = P(1 + rt) \).

H (4) [10 pts] Danita has $750 to save in a simple interest account yielding 4.5% interest. How much total money would she have after 7 years?

Solution: This is a simple interest problem, which uses the formula from 5.1:

\[ A = P(1 + rt) \]
\[ A = 750(1 + (.045)(7)) = 986.25. \]

Grading: +5 points for the formula, +5 points for substitution. Grading for common mistakes: −3 points for the wrong formula.

C (5) [10 pts] At age 15, Kuang made a deposit to save for his first car. He deposited $5,000 in an account with quarterly compounding at 6\( \frac{3}{4} \)% annual interest. How much does he have available to spend for a car at age 19?

Solution: Since there was a deposit made at the beginning of the time span, and no other money was deposited, the compound interest formula (5.2) is to be used. Then

\[ A = P \left( 1 + \frac{r}{n} \right)^{nt} = 5000 \left( 1 + \frac{.0675}{4} \right)^{4 \cdot 4} = 6535.08. \]

Grading: +5 points for the formula, +5 points for substitution. Grading for common mistakes: −3 points for the wrong formula.

H (5) [10 pts] Knowing the advantage of saving over time, Sasha opened an IRA account at age 22. She deposits $2,000 every year, and plans to continue until she's 65. If the account averages an annual interest rate of 11.5% compounded annually, how much money will she have at age 65? Show your work.

Solution: Since she is adding money periodically to an account, the annuity formula (from 5.3) is to be used:

\[ A = P \cdot \frac{n}{r} \left[ \left( 1 + \frac{r}{n} \right)^{nt} - 1 \right] = 2000 \cdot \frac{1}{.115} \left[ \left( 1 + \frac{.115}{1} \right)^{1-43} - 1 \right] = 1,858,257.19. \]

Grading: +5 points for the formula, +5 points for substitution. Grading for common mistakes: −3 points for the wrong formula.
C (6) [10 pts] Minerva wants to save money to buy a car, so she wants to be sure she has $12,000 in two years when she is ready to buy. She decides to deposit some money each month into an account with an annual percentage rate of 5.75% compounded monthly. How much is her monthly payment to guarantee that she will have $12,000 in two years?

Solution: Since she is adding money periodically to an account, the annuity formula (from 5.3) is to be used:

$$A = P \cdot \frac{n}{r} \left[ \left(1 + \frac{r}{n}\right)^{nt} - 1 \right]$$

$$12,000 = P \cdot \frac{12}{0.0575} \left[ \left(1 + \frac{0.0575}{12}\right)^{12 \cdot 2} - 1 \right]$$

$$12,000 = P \cdot 25.37016$$

$$P = \frac{12,000}{25.37016} = 473.00.$$  

Grading: +5 points for the formula, +5 points for substitution. Grading for common mistakes: −3 points for the wrong formula, except −2 points for the formula from 5.4.

H (6) [10 pts] Dan has accumulated a credit card debt of $425. This card charges a monthly interest rate of 1.7%, and his minimum payment is $25. If he pays the minimum payment each month, and doesn’t use the card anymore, how long will it take him to pay off the entire balance? Show all your work.

Solution: Since he is paying off a debt, the amortization formula (5.4) will be used. Note that the monthly interest rate is given; thus $$i = 0.017$$, or $$r = ni = (12)(0.017) = 0.204$$. Substituting:

$$A = P \cdot \frac{n}{r} \left[ 1 - \left(1 + \frac{r}{n}\right)^{-nt} \right]$$

$$425 = 25 \cdot \frac{12}{0.204} \left[ 1 - \left(1 + \frac{0.204}{12}\right)^{-12t} \right].$$

The goal here would be to solve for $$t$$, using logarithms. Since this is not a compound interest formula, I did not require that you solve for $$t$$. Full credit was given for work up to this point.

Grading: +5 points for the formula, +5 points for substitution. Grading for common mistakes: −3 points for the wrong formula, except −2 points for the formula from 5.2; −2 points for $$r = 0.017$$. 

The tableau to the right was obtained by trying to solve the Linear Program to the left. Does the tableau represent a solution, or was there a mistake made sometime during the Simplex Method? Justify your answer.

MAXIMIZE \( P = 4x_1 + 4x_2 + 5x_3 \)

\[
\begin{align*}
&x_1 + 3x_2 + 2x_3 \leq 30 \\
&2x_1 + x_2 + 3x_3 \leq 12 \\
&x_1, x_2, x_3 \geq 0
\end{align*}
\]

Solution: This is where LP duality appears on the test. Recall that to show that the maximum has been found, you need to check two things: (a) The point given by the tableau is feasible; and (b) The tableau shows that the upper bound is the same as the objective value for the point given by the tableau.

To settle (a), we extract the solution from the tableau: \( x_1 = 0, x_2 = 12, x_3 = 0, x_4 = 6, x_5 = 0, \) and \( P = 24. \) The point \((x_1, x_2, x_3)\) is not feasible, since

\[
x_1 + 3x_2 + 2x_3 = 0 + 36 + 0 = 36 \not\leq 30,
\]

contradicting one of the inequalities given in the original Linear Program.

There are other errors as well: \( 4x_1 + 4x_2 + 5x_3 = 48, \) but \( P = 24. \) Also, to find the upper bound for (b), we multiply the first inequality by 0, multiply the second by 2, and add the results together. This gives \( 4x_1 + 2x_2 + 3x_3 \leq 24, \) but this is not an upper bound on \( P, \) since \( 2 \not\geq 4 \) (the coefficients of \( x_2 \) in this bound, and in \( P, \) are not compatible).

Any one of these mistakes indicates an error in the tableau.

Grading: +10 points for finding any of the mistakes above. Grading for common errors: +3 points for applying the Simplex Method to the Linear Program (the question was whether the tableau represented a maximum, not whether it is the final result of the Simplex Method); +3 points for saying there was a mistake, but with no supporting work; +2 points for saying there was no mistake, with no supporting work; +5 points (total) for saying that all the numbers in the bottom row are at least 0; +9 points for adding inequality 1 to twice inequality 2.
(7) [10 pts] The tableau to the right was obtained by trying to solve the Linear Program to the left. Does the tableau represent a solution, or was there a mistake made sometime during the Simplex Method? Justify your answer.

MAXIMIZE $P = 2x_1 + x_2 + 4x_3 + x_4$

\[
\begin{align*}
2x_1 + 2x_2 + x_3 + 3x_4 & \leq 12 \\
x_2 + 2x_3 + 2x_4 & \leq 20 \\
2x_1 + 4x_2 + x_3 & \leq 16 \\
x_1, x_2, x_3, x_4 & \geq 0
\end{align*}
\]

Solution: We extract the solution that the tableau represents: $x_1 = 1$, $x_2 = 0$, $x_3 = 10$, $x_4 = 0$, $x_5 = 0$, $x_6 = 0$, $x_7 = 4$, $P = 42$, and this time the solution $(x_1, x_2, x_3, x_4) = (1, 0, 10, 0)$ is feasible, and $P = 2x_1 + x_2 + 4x_3 + x_4$. (You should check this by verifying that $(1, 0, 10, 0)$ makes all of the constraints true, and that $2x_1 + x_2 + 4x_3 + x_4 = 2 \cdot 1 + 0 + 4 \cdot 10 + 0 = 42$.)

Now we move to finding an upper bound on the largest possible $P$. The entries in the bottom row ($x_5, x_6, x_7$ columns) say that we need to add the first and second inequalities of the LP together. This gives us:

\[
\begin{align*}
2x_1 + 2x_2 + x_3 + 3x_4 & \leq 12 \\
x_2 + 2x_3 + 2x_4 & \leq 20 \\
2x_1 + 3x_2 + 3x_3 + 5x_4 & \leq 32
\end{align*}
\]

But 32, the number on the right-hand side, is not the value of $P$ for the solution that the tableau represents, and $3 \ngeq 4$ (comparing the coefficient of $x_3$ in the sum of the inequalities and the equality for $P$).

Either mistake indicates an error in the tableau.

Grading: See [C].

MAT 119 Webpage: http://math.la.asu.edu/~checkman/119/