Introduction

Two important applications with matrices in MAT 119 are solving a system of linear equations and finding the inverse of a matrix. The TI series of graphing calculators is able to find the inverse and put a matrix in reduced row echelon form automatically, but students still should learn how to do row operations, especially when using the Simplex Method in Chapter 4. The purpose of this paper is to indicate the appropriate steps.

A brief word on notation: When a box is drawn around a symbol, that means to press that key on the TI. Hence \[ \times \] means to push the multiplication button. If two key presses are used, they will be to refer to functions printed in different colored keys on the calculator. The keypresses will be given, along with the operation to be performed, the latter in slanted type and in brackets. For instance, \[ 2\text{nd} \ln [e^x] \] means to press \[ 2\text{nd} \] then \[ \ln \], and this accesses the function \( e^x \); this is in yellow ink on the TI-83. Also, \[ \text{ALPHA} \text{MATH} [A] \] puts the calculator in alphabetic mode and enters the letter \( A \) in the display; these letters are printed in green ink on the TI-83.

Solving A System of Linear Equations

Suppose you want to solve the system:

\[
\begin{align*}
25x + 61y - 12z &= 10 \\
18x - 12y + 7z &= -9 \\
3x + 4y - z &= 12
\end{align*}
\]

Here’s how to solve it.

1. Enter the augmented matrix into the calculator.

Press \[ \text{MATRX} \] to put the calculator in MATRIX mode.

You now see a list of matrix names down the left-hand side of the screen, and the words NAMES, MATH and EDIT across the top, with NAMES currently “highlighted.” Pressing the right-arrow key \[ \rightarrow \] or \[ \leftarrow \] will change which word is highlighted. Pressing \[ 1 \] or \[ 2 \] will select the name of the matrix you want to enter. Let’s enter the system above into the matrix \( A \); move the cursor to the right so that EDIT is highlighted (press \[ \rightarrow \rightarrow \] if you haven’t moved the highlighted part) and press \[ \text{ENTER} \].

You are now prompted to enter the dimensions of the matrix \( A \); enter the number of rows (\[ 3 \] \[ \text{ENTER} \]) and the number of columns (\[ 4 \] \[ \text{ENTER} \]). The TI-83 can handle matrices with up to 99 rows and/or columns, but cannot handle a matrix with both due to limited memory.

NOTE. At any time, if you realize you made a mistake, you can press \[ \text{2nd} \text{MODE} \{QUIT\} \] and then start over.

Now you need to enter the coefficients and numbers on the right hand side. The cursor is placed in the first row, first column of the matrix. If you type in a number (or an expression) and hit \[ \text{ENTER} \], it will put that
number into the matrix and move to the next position, to the right (if you’re not at the right-most column) or down to the first column of the next row. After entering a number in the last row and last column, the cursor stays there.

So we need to put in the coefficients. The keystrokes for this are 2 5 ENTER 8 1 ENTER (-) 1 2 ENTER 1 0 ENTER, etc.¹

2. Perform the row operations.

Now you need to get back to the calculation mode. Press 2nd MODE [QUIT]. You are ready to perform row operations on your matrix A.

The first thing we will do is to swap the first and third rows of A.² Press MATRX → then [rowSwap]. Press ENTER, which will bring you back to the “calculation mode.”³ Now you need to enter the matrix: press MATRX ENTER (If you want to do a row reduction on a matrix other than A, you need to “scroll down” by pressing ↓ repeatedly.) Then press [, 1, 3 ] ENTER.

This gives you the matrix
\[
\begin{pmatrix}
3 & 4 & -1 & 12 \\
18 & -12 & 7 & -9 \\
25 & 61 & -12 & 10
\end{pmatrix}
\]
If you need to see columns further to the right of the screen, press the → key.

NOTE: Doing row operations on A will not change the value of A on a TI-83, so we will need to keep track of the new matrix. We will do this by using the ANS (2nd (-)) command. It is also a good idea to copy onto paper the matrix we get at this point, in case we have to go back to it.

Now we want to divide the first row by 3, so that there is a 1 in the upper left corner. Press MATRX → ALPHA SIN [*row(* (−1) 1 8, 2nd (-) [ANS], 1 ) ENTER. The matrix we get is
\[
\begin{pmatrix}
1 & 1.333333333 & -333333333 & 4 \\
18 & -12 & 7 & -9 \\
25 & 61 & -12 & 10
\end{pmatrix}
\]

(Don’t worry about the decimal approximation; we’ll fix it at the very end.)

Now you need to subtract 18 times Row 1 from Row 2, to turn the entry in the 2nd row, 1st column into a 0. Enter: MATRX → ALPHA COS [*row+( (−1) 2 5 , 2nd (-) [ANS], 1, 3 ) ENTER. The new matrix is
\[
\begin{pmatrix}
1 & 1.333333333 & -333333333 & 4 \\
0 & -36 & 13 & -81 \\
25 & 61 & -12 & 10
\end{pmatrix}
\]

Now, you perform these same types of row operations over and over. The key sequences are below.

1. MATRX → ALPHA COS [*row+( (−1) 2 5 , 2nd (-) [ANS], 1, 3 ) ENTER (subtract 25 times row 1 from row 3).
2. MATRX → ALPHA SIN [*row+( (−1) 1 ÷ 3 6 , 2nd (-) [ANS], 2 ) ENTER (divide row 2 by −36).

¹ And so on for the other rows. Note that you use the unary minus to enter negative numbers.
² You don’t have to do this to solve the system; it’s mainly to show how to do all three types of row operations on the calculator.
³ Note that, instead of pressing ↓ twelve times and pressing ENTER you can just press ALPHA PRGM [C] to get the same command.
3. \[ \text{MATRX} \rightarrow \text{ALPHA} \cos \{ \times \text{row} + (\text{row}) \} \] \[ \text{[ANS]} , 2 , 3 ) \] \text{ENTER} (subtract 27.66666667 times row 2 from row 3). \(^4\)

4. If you press \( \rightarrow \) repeatedly, you will find that the entry in the 3rd row and 3rd column is approximately 6.32407407407. This is what you want to divide row 3 by:

\[ \text{MATRX} \rightarrow \text{ALPHA} \sin \{ \times \text{row} \} \] \[ \text{[ANS]} , 3 , 1 ) \] \text{ENTER}.

5. The matrix is now in row echelon form. To get it into reduced row echelon form, continue with:

\[ \text{MATRX} \rightarrow \text{ALPHA} \cos \{ \times \text{row} + (\text{row}) \} \] \[ \text{[ANS]} , 3 , 1 ) \] \text{ENTER}.

6. \[ \text{MATRX} \rightarrow \text{ALPHA} \cos \{ \times \text{row} + (\text{row}) \} \] \[ \text{[ANS]} , 3 , 1 ) \] \text{ENTER}.

7. \[ \text{MATRX} \rightarrow \text{ALPHA} \cos \{ \times \text{row} + (\text{row}) \} \] \[ \text{[ANS]} , 3 , 1 ) \] \text{ENTER}.

Now the matrix is close to reduced row echelon form; the entries on the main diagonal are very close to 1, and the other entries in the first three columns are close to zero. Press or hold the \( \rightarrow \) until you can see the fourth column of the matrix. The value in the first row is \( x = 4.666666666 \), the value in the second row is \( y = -6.443631037 \), and the value in the third row is \( z = -24.07467057 \).

You can try to convert these decimals to fractional form by typing in \[ \text{[2nd]} \rightarrow \text{Frac} \] \text{ENTER}. In this case, it doesn’t help — the TI-83 only tries fractions with denominators less than or equal to 100 — but it may in others.

**Inverting a Matrix**

Inverting a matrix can be done on the TI without as much work; it is built-in to the calculator. Here we will invert the matrix

\[
\begin{bmatrix}
1 & 1 & -1 \\
2 & 1 & 1 \\
1 & 0 & 1
\end{bmatrix}
\]

1. **Enter the matrix into the calculator.**

This can be done as in the previous section; put it into matrix \( B \), then press \[ \text{[2nd]} \rightarrow \text{MODE} \rightarrow \text{QUIT} \] to get back into “calculation mode.”

2. **Calculate the inverse.**

Type in: \[ \text{MATRX} 1 \rightarrow \text{[B]} \rightarrow \text{x}^{-1} \] \text{ENTER}. You get

\[
\begin{bmatrix}
1 & -1 & 2 \\
-1 & 2 & -3 \\
-1 & 1 & -1
\end{bmatrix}
\]

Other matrix computations are possible. For instance, to find \( B^{10} \), type \[ \text{MATRX} 1 \rightarrow \text{[B]} \rightarrow ^1 \] \[ \text{[B]} \rightarrow 10 \] \text{ENTER}, the answer is

\[
\begin{bmatrix}
1897 & 1432 & -351 \\
3945 & 2978 & -730 \\
1432 & 1081 & -265
\end{bmatrix}
\]

\(^4\) You will have a \(-3.32E-10\) in your matrix. This is close enough to zero that we can count it as zero. Note that the CASIO calculators can work with exact (fractional) values here.
Matrix Operations on a TI-85 Graphing Calculator

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(This paper is based on a talk given in Spring Semester 2004.) The use of a graphing calculator can be useful and convenient, especially when reducing a matrix that has entries with many decimal places. The inverse of a matrix can also be found easily. One of the homework assignments for MAT 119 is to reduce a matrix with a graphing calculator.

Introduction

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A brief word on notation: When a box is drawn around a symbol, that means to press that key on the TI. Hence \( \times \) means to push the multiplication button. If two key presses are used, they will be to refer to functions printed in different colored keys on the calculator. The keypresses will be given, along with the operation to be performed, the latter in slanted type and in brackets. For instance, \( \text{2nd ln}[e^x] \) means to press \( \text{2nd} \), then \( \ln \), and this accesses the function \( e^x \); this is in yellow ink on the TI-85. Also, \( \text{ALPHA MATH}[A] \) puts the calculator in alphabetic mode and enters the letter A in the display; these letters are printed in blue ink on the TI-85.

Solving A System of Linear Equations

Suppose you want to solve the system:

\[
\begin{align*}
25x + 61y - 12z &= 10 \\
18x - 12y + 7z &= -9 \\
3x + 4y - z &= 12
\end{align*}
\]

Here’s how to solve it.

1. Enter the augmented matrix into the calculator.

Press \( \text{2nd 7}\) [MATRX] to put the calculator in MATRIX mode.

You will see five “words” above the keys \( \text{F1} \) through \( \text{F5} \): NAMES, EDIT, MATH, OPS, CPLX. To enter a matrix, press the key under EDIT (\( \text{F2} \)). You are now asked for the name of the matrix, which can be any single letter. The cursor is in alphabetic mode, so all you have to do is press \( \text{LOG}\) [A] [ENTER].

You are now prompted to enter the dimensions of the matrix \( A \); enter the number of rows \( 3 \) [ENTER] and the number of columns \( 4 \) [ENTER]. The TI-85 can handle matrices with up to 99 rows and/or columns, but cannot handle a matrix with both due to limited memory.

NOTE. At any time, if you realize you made a mistake, you can press \( \text{EXIT} \) and then start over.

Now you need to enter the coefficients and numbers on the right hand side. The cursor is placed in the first row, first column of the matrix. If you type in a number (or an expression) and hit \( \text{ENTER} \), it will put that number into the matrix and move to the next position, to the right (if you’re not at the right-most column) or down to the first column of the next row. After entering a number in the last row and last column, the cursor stays there.
So we need to put in the coefficients. The keystrokes for this are $2 \hspace{0.5cm} 5 \hspace{0.5cm} \text{ENTER} \hspace{0.5cm} 6 \hspace{0.5cm} 1 \hspace{0.5cm} \text{ENTER} \hspace{0.5cm} (-) \hspace{0.5cm} 1 \hspace{0.5cm} \text{ENTER} \hspace{0.5cm} 6 \hspace{0.5cm} 1 \hspace{0.5cm} \text{ENTER} \hspace{0.5cm} (-) \hspace{0.5cm} 1 \hspace{0.5cm} \text{ENTER}$ etc.\(^1\)

2. Perform the row operations.

Now you need to get back to the calculation mode. Press $\text{EXIT} \hspace{0.5cm} \text{2nd} \hspace{0.5cm} 7 \hspace{0.5cm} \text{[MATRX]} \hspace{0.5cm} \text{F4} \hspace{0.5cm} \text{[OPS]}$. You are ready to perform row operations on your matrix $A$.

The first thing we will do is to swap the first and third rows of $A$.\(^2\) Press the $\text{MORE}$ key to get the following “words” above the keys $\text{F1}$ through $\text{F5}$: $\text{aug, rSwap, rAdd, multR, and mRAdd}$. To swap two rows, press $\text{F2}$ $[\text{rSwap}()$. Now you need to type the rest in: the name of the matrix (press $\text{ALPHA}$ $\text{LOG}$ $[A]$), and the two rows to be swapped (press $1$, $3$) $\text{ENTER}$. This gives you the matrix

$$
\begin{bmatrix}
3 & 4 & -1 & 12 \\
18 & -12 & 7 & -9 \\
25 & 61 & -12 & 10
\end{bmatrix}
$$

If you need to see columns further to the right of the screen, press the $\rightarrow$ key.

NOTE: Doing row operations on $A$ will not change the value of $A$ on a TI-85, so we will need to keep track of the new matrix. We will do this by using the ANS (\text{2nd} (\text{−})) command. It is also a good idea to copy onto paper the matrix we get at this point, in case we have to go back to it.

Now we want to divide the first row by 3, so that there is a 1 in the upper left corner. We are still have the options $\text{aug, rSwap, rAdd, multR, and mRAdd}$, so we can just press $\text{F4}$ $[\text{multR}()$. We want to multiply the first row of ANS by $\frac{1}{3}$, so we type in: $1 \div 3$, $\text{2nd}$ (\text{−}) $\text{[ANS]}$, $1$ $\text{ENTER}$. The matrix we get is

$$
\begin{bmatrix}
1 & 1 \cdot 333333333 & -333333333 & 4 \\
18 & -12 & 7 & -9 \\
25 & 61 & -12 & 10
\end{bmatrix}
$$

(Don’t worry about the decimal approximation; we’ll fix it at the very end.)

Now you need to subtract 18 times Row 1 from Row 2, to turn the entry in the 2nd row, 1st column into a 0. Enter: $\text{F5}$ $\text{mRAdd()}$ $[\text{−}]$ $\text{1}$ $\text{8}$ $\text{2nd}$ (\text{−}) $\text{[ANS]}$, $\text{1}$ $\text{3}$ $\text{ENTER}$. The new matrix is

$$
\begin{bmatrix}
1 & 1.333333333 & -333333333 & 4 \\
0 & -36 & 13 & -81 \\
25 & 61 & -12 & 10
\end{bmatrix}
$$

Now, you perform these same types of row operations over and over. The key sequences are below.

1. $\text{F5}$ $\text{mRAdd()}$ $[\text{−}]$ $\text{5}$ $\text{2nd}$ (\text{−}) $\text{[ANS]}$, $\text{1}$ $\text{3}$ $\text{ENTER}$ (subtract 25 times row 1 from row 3).
2. $\text{F4}$ $\text{multR()}$ $[\text{−}]$ $\text{1}$ $\div$ $\text{3}$ $\text{2nd}$ (\text{−}) $\text{[ANS]}$, $\text{2}$ $\text{ENTER}$ (divide row 2 by $-36$).
3. $\text{F5}$ $\text{mRAdd()}$ $[\text{−}]$ $\text{7}$ $\text{2nd}$ (\text{−}) $\text{[ANS]}$, $\text{2}$ $\text{3}$ $\text{ENTER}$ (subtract 27.66666667 times row 2 from row 3).\(^3\)
4. If you press $\rightarrow$ repeatedly, you will find that the entry in the 3rd row and 3rd column is approximately $6.32407407409$. This is what you want to divide row 3 by: $\text{F4}$ $\text{multR()}$ $\text{1}$ $\div$ $\text{6}$ $\text{3}$ $\text{2}$ $\text{4}$ $\text{0}$ $\text{7}$ $\text{4}$ $\text{0}$ $\text{9}$ $\text{2nd}$ (\text{−}) $\text{[ANS]}$, $\text{3}$ $\text{ENTER}$.

\(^1\) And so on for the other rows. Note that you use the unary minus to enter negative numbers.
\(^2\) You don’t have to do this to solve the system; it’s mainly to show how to do all three types of row operations on the calculator.
\(^3\) You will have a $-3.2\text{E}-11$ in your matrix. This is close enough to zero that we can count it as zero. Note that the CASIO calculators can work with exact (fractional) values here.
5. The matrix is now in row echelon form. To get it into reduced row echelon form, continue with: 
\[ \text{F5} \ \text{mRAdd}( \begin{bmatrix} 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \end{bmatrix}, 2\text{nd} \ (-) \ \text{[ANS]} \begin{bmatrix} 3 & 3 & 1 \end{bmatrix} ) \ \text{ENTER} . \]

6. \[ \text{F5} \ \text{mRAdd}( \begin{bmatrix} 3 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}, 2\text{nd} \ (-) \ \text{[ANS]} \begin{bmatrix} 3 & 1 \end{bmatrix} ) \ \text{ENTER} . \]

7. \[ \text{F5} \ \text{mRAdd}( \begin{bmatrix} 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \end{bmatrix}, 2\text{nd} \ (-) \ \text{[ANS]} \begin{bmatrix} 2 & 1 \end{bmatrix} ) \ \text{ENTER} . \]

Now the matrix is close to reduced row echelon form; the entries on the main diagonal are very close to 1, and the other entries in the first three columns are close to zero. Press or hold the \(-\) until you can see the fourth column of the matrix. The value in the first row is \(x = 4.5661786066\), the value in the second row is \(y = -6.44363103925\), and the value in the third row is \(z = -24.074670571\).

You can try to convert these decimals to fractional form by typing in \[ 2\text{nd} \ (-) \ \text{[ANS]} 2\text{nd} \times \]
\[ \text{[MATH]} \ \text{F5} \ \text{MISC} \ \text{MORE} \ \text{F1} \ (-\text{Frac}) \ \text{ENTER} . \] In this case, this only gives you an exact value for \(z\), namely \(-16443/683\). The TI-85 only tries fractions with denominators less than or equal to 1000, so you may get exact values in other situations. Note that roundoff errors kept us from getting exact answers for \(x\) \((3119/683)\) and \(y\) \((-4401/683)\).

### Inverting a Matrix

Inverting a matrix can be done on the TI without as much work; it is built-in to the calculator. Here we will invert the matrix \[
\begin{bmatrix}
1 & 1 & -1 \\
2 & 1 & 1 \\
1 & 0 & 1
\end{bmatrix}
\]

1. **Enter the matrix into the calculator.**

This can be done as in the previous section; put it into matrix \(B\), then press \text{EXIT} to get back into “calculation mode.”

2. **Calculate the inverse.**

Type in: \[ \text{ALPHA} \ \text{SIN} \ [B] \ 2\text{nd} \ \text{EE} \ [-1] \ \text{ENTER} . \] You get \[
\begin{bmatrix}
1 & -1 & 2 \\
-1 & 2 & -3 \\
-1 & 1 & -1
\end{bmatrix}, \] after a few seconds have gone by.

Other matrix computations are possible. For instance, to find \(B^{10}\), type \[ \text{ALPHA} \ \text{SIN} \ [B] 10 \ \text{ENTER} ; \] the answer is \[
\begin{bmatrix}
1897 & 1432 & -351 \\
3945 & 2978 & -730 \\
1432 & 1081 & -265
\end{bmatrix}. \]