1. This is the graph of a function $g(x)$:

![Graph of $g(x)$](image)

**a. [5 pts]** Find the intervals where $g(x)$ is increasing and the intervals where $g(x)$ is decreasing.

**Solution:** The graph goes up and to the right from $(-4, -2)$ to $(-2, 0)$, down and to the right from $(-2, 0)$ to $(0, -2)$, and up and to the right from $(0, -2)$ to $(1, 1)$. Since the intervals are only the $x$ values, $g(x)$ is increasing on the intervals $(-4, -2)$ ($-4 < x < -2$) and $(0, 1)$ ($0 < x < 1$), and decreasing on the interval $(-2, 0)$ ($-2 < x < 0$).

Grading: −1 point if the $y$ values were included.

**b. [10 pts]** Sketch the graph of $-2g(-x) - 4$.

**Solution:** Since we don’t have a formula for $g(x)$, we have to use transformations to sketch the graph of $-2g(-x) - 4$. First, we look to see how we get from $g(x)$ to $-2g(-x) - 4$:

\[
g(x) \xrightarrow{\text{Replace } x \text{ with } -x} g(-x) \xrightarrow{\text{Multiply by } -1} -g(-x) \xrightarrow{\text{Multiply by } 2} -2g(-x) \xrightarrow{\text{Subtract 4}} -2g(-x) - 4
\]

Now find out how the graph changes at each step:

\[
g(x) \xrightarrow{\text{Replace } x \text{ with } -x} g(-x) \xrightarrow{\text{Reflect } y\text{-axis}} g(-x) \xrightarrow{\text{Multiply by } -1} -g(-x) \xrightarrow{\text{Reflect } x\text{-axis}} -g(-x) \xrightarrow{\text{Multiply by } 2} 2g(-x) \xrightarrow{\text{Stretch by 2}} 2g(-x) \xrightarrow{\text{Subtract 4}} 2g(-x) - 4 \xrightarrow{\text{Move down 4}} -2g(-x) - 4
\]

These are the transformations, in order, which we need to do. The graphs are on the next page.
\[ y = g(x) \]

\[ (−2, 0) \quad (1, 1) \]

\[ (−4, −2) \quad (0, −2) \]

\[ y = g(−x) \]

\[ (−1, 1) \quad (2, 0) \]

\[ (0, −2) \quad (4, −2) \]

\[ y = −g(−x) \]

\[ (0, 2) \quad (4, 2) \]

\[ (−1, −1) \quad (2, 0) \]

\[ y = −2g(−x) \]

\[ (0, 4) \quad (4, 4) \]

\[ (−1, −2) \quad (2, 0) \]

\[ y = −2g(−x) − 4 \quad \text{final answer} \]

\[ (0, 0) \quad (4, 0) \]

\[ (2, −4) \quad (−1, −6) \]
2. Let \( h(x) = \sqrt{2x + 3} \).

a. [5 pts] Find \( h(3) \).

\[
Answer: \quad h(3) = \sqrt{2(3) + 3} = \sqrt{9} = 3.
\]

b. [5 pts] What is the domain of \( h(x) \)?

Solution: The domain is all values of \( x \) which you can put into the formula for \( h(x) \). Since there is a radical (\( \sqrt{\_} \)) sign, the expression under it must be at least 0. So \( 2x + 3 \geq 0 \), which means that \( x \geq -\frac{3}{2} \). This is the domain, written as \( \left[ -\frac{3}{2}, +\infty \right) \) in interval notation.

Grading: +3 points for the inequality \( 2x + 3 \geq 0 \), +2 points for solving for \( x \). Grading for common mistakes: −3 points for \( x \geq -1 \).

c. [5 pts] Is \( h(x) \) even, odd, both, or neither?

Solution: To find out whether \( h(x) \) is even, odd, both, or neither, we need to calculate \( h(-x) \) and simplify it:

\[
h(-x) = \sqrt{2(-x) + 3} = \sqrt{-2x + 3}.
\]

This is neither \( h(x) = \sqrt{2x + 3} \), nor is it \( -h(x) = -\sqrt{2x + 3} \). So \( h(x) \) is neither odd or even.

Grading: +3 points for evaluating \( h(-x) \), +2 points for whether \( h(x) \) was even or odd. Grading for common mistakes: +2 points (total) for a right answer with no work.

d. [5 pts] What is the average rate of change of \( h(x) \) from \( x = 3 \) to \( x = 11 \)?

Solution: The average rate of change formula for \( h(x) \) from \( x = a \) to \( x = b \) is \( \frac{h(b) - h(a)}{b - a} \), which is

\[
\frac{h(11) - h(3)}{11 - 3} = \frac{\sqrt{2(11) + 3} - \sqrt{2(3) + 3}}{8} = \frac{5 - 3}{8} = \frac{1}{4}.
\]

Grading: +3 points for the rate of change formula, +2 points for evaluating it.

e. [5 pts] Write \( h(x) \) as the composition of two functions \( f(x) \) and \( g(x) \), neither of which is \( h(x) \).

(Make sure \( (f \circ g)(x) = h(x) \).)

Solution: There are several ways to do this. Two pairs of functions that work are \( f(x) = \sqrt{x} \) and \( g(x) = 2x + 3 \); and \( f(x) = \sqrt{x + 3} \) and \( g(x) = 2x \).

Grading for common mistakes: −1 point if \( h(x) = (g \circ f)(x) \) instead of \( h(x) = (f \circ g)(x) \); −2 points for something close; +3 points (total) for \( f(\sqrt{x}) \) and \( g(2x + 3) \).
3. The value of \( R \) varies inversely with \( z \) and directly with \( w^3 \).
   
   a. [5 pts] Write down an equation which expresses the relationship between \( R \), \( z \), and \( w \).

   \[
   R = k \cdot \frac{1}{z} \cdot w^3 = \frac{kw^3}{z}.
   \]

   b. [5 pts] In the equation above, what is the value of the constant if \( R = 30 \) when \( z = 2 \) and \( w = 3 \)?

   Solution: We need to solve for \( k \). Substituting values for \( R \), \( z \) and \( w \), we get the equation

   \[
   30 = k \cdot \frac{1}{2} \cdot 3^3 = \frac{27}{2} k,
   \]

   so \( k = \frac{2}{27} \cdot 30 = \frac{20}{9} = 2.222.\)

   c. [5 pts] What is the value of \( R \) when \( z = 3 \) and \( w = 4 \)?

   Solution: Using the value of \( k \) from part (b.),

   \[
   R = k \cdot \frac{1}{3} \cdot 4^3 = \frac{20}{9} \cdot \frac{1}{3} \cdot 4^3 = \frac{1280}{27} = 47.407.
   \]

4. [10 pts] Is the graph below the graph of a function \( f(x) \)? Justify your answer.

   Solution: The picture was redrawn to make it clearer. This graph does not represent a function, because a vertical line can be drawn which crosses the graph in two points.

   Grading for common mistakes: +7 points (total) for Yes, and the “Vertical Line Test” or for the Horizontal Line Test.
5. [10 pts] Sketch the graph of the function \( f(x) = \begin{cases} x^2, & x < -1; \\ |x - 1|, & x \geq -1. \end{cases} \) Be sure to label any important points.

**Solution:** Below are the graphs of \( y = x^2 \) and \( y = |x - 1| \). The second one comes from the graph of \( y = |x| \) after shifting to the right by 1 unit.

We only want the part of the graph of \( y = x^2 \) when \( x < -1 \), which is to the left of the dashed line. Also, the only part of the graph \( y = |x - 1| \) we draw should be where \( x \geq -1 \). If we redraw these two pieces, we get the graph below.
6. Let $f(x) = 2x + 1$ and $g(x) = \lceil x + 0.5 \rceil$. Calculate the following:

a. [5 pts] $(f + g)(x)$

Solution: $(f + g)(x) = f(x) + g(x) = 2x + 1 + \lceil x + 0.5 \rceil$.

Grading for common mistakes: −3 points if $\lceil \rceil$ was replaced with parentheses ( ) or brackets [ ].

b. [10 pts] $(f \circ g)(2.8)$

Solution: $(f \circ g)(2.8) = f(g(2.8)) = f(\lceil (2.8) + 0.5 \rceil) = f(\lceil 3.3 \rceil) = f(3) = 2(3) + 1 = 7$.

Grading: +5 points for writing the composition as $f(g(2.8))$, +5 points for evaluating the rest. Grading for common mistakes: −3 points for $f(x) \cdot f(x)$.

Fun fact: $g(x)$ is a function which simulates rounding to the nearest integer (rounding up if the number ends in .5).

c. [10 pts] $(f \circ f)(x)$

Solution: $(f \circ f)(x) = f(f(x)) = f(2x + 1) = 2(2x + 1) + 1 = 4x + 3$.

Grading for common mistakes: −8 points for $f(x) \cdot f(x)$; −5 points for $2x(2x + 1) + 1$.