(1) (2pts) Suppose 
\[ S = \left\{ \begin{bmatrix} 0 \\ -3 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ -4 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ -4 \\ 8 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ -1 \\ 2 \end{bmatrix} \right\}. \]

Suppose \( W \) is the subspace of \( \mathbb{R}^4 \) spanned by vectors in \( S \). Find a subset of \( S \) that forms a basis for \( W \). This is the same type of problem with 4.4 #14 of your HW4.

(2) (2pts) Find a basis \( T \) for \( \mathbb{R}^4 \) that contains the vectors in 
\[ \left\{ \begin{bmatrix} 2 \\ 4 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -4 \\ 4 \\ -2 \end{bmatrix} \right\}. \]

This is the same type of problem with 4.4 #18 of your HW4. You can also solve this problem by finding a basis for the orthogonal complement of the subspace spanned by the two given vectors. The two vectors you find and the two given vectors combined together form a basis for \( \mathbb{R}^4 \).

(3) (4pts) Suppose 
\[ S = \left\{ \begin{bmatrix} 3 \\ 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -2 \\ -5 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -18 \\ -10 \\ -15 \end{bmatrix} \right\}. \]

Suppose \( W \) is the subspace of \( \mathbb{R}^4 \) spanned by vectors in \( S \). Apply the Gram-Schmidt algorithm to transform \( S \) into an orthonormal basis for \( W \). (2pts) Next use the orthonormal basis to find the orthogonal projection \( \overline{p} \) of \( \overline{b} = (0, 0, 0, 1) \) into \( W \). (2pts)

(4) (2pts) Suppose 
\[ \overline{v}_1 = \begin{bmatrix} 0.5 \\ -0.5 \\ 0.5 \\ -0.5 \end{bmatrix}, \overline{v}_2 = \begin{bmatrix} -0.5 \\ -0.5 \\ 0.5 \\ 0.5 \end{bmatrix}, \overline{v}_3 = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}. \]

Find a vector \( \overline{v}_4 \) such that \( \overline{v}_1, \overline{v}_2, \overline{v}_3, \overline{v}_4 \) form an orthonormal basis for \( \mathbb{R}^4 \). Since \( \overline{v}_1, \overline{v}_2, \overline{v}_3 \) are unit vectors orthogonal to each other, you just need to find another unit vector orthogonal to \( \overline{v}_1, \overline{v}_2, \overline{v}_3 \). Recall what you did for WeBWorK Chapter-5-part-1 will help.