Secion 3.5, Items 13, 15, 24, 33, 37, 55, 57, 59

13)
\[ y = \cos(a^3 + x^3) \]
\[ y' = -\sin(a^3 + x^3) \cdot 3x^2 \]
\[ y'' = -3x^2 \sin(a^3 + x^3) \]

15)
\[ y = e^{-mx} \]
\[ y' = e^{-mx}(-m) \]
\[ y'' = -me^{-mx} \]

24)
\[ s(t) = \left(\frac{t^3 + 1}{t^3 - 1}\right) \]
\[ s'(t) = \frac{1}{4} \left(\frac{t^3 + 1}{t^3 - 1}\right)^{\frac{3}{4}} \left(3t^2(t^3 - 1) - (t^3 + 1)(3t^2)\right) \]
\[ \frac{1}{(t^3 - 1)^2} \]
\[ \frac{1}{2} \left(\frac{t^3 + 1}{t^3 - 1}\right)^{\frac{3}{4}} \left(-3t^2\right) \]
\[ \frac{1}{2} \left(\frac{t^3 + 1}{t^3 - 1}\right)^{\frac{3}{4}} \frac{3t^2(t^3 - 1) - (t^3 + 1)(3t^2)}{(t^3 - 1)^2} \]

33)
\[ y = \left(1 + \cos^2 x\right)^6 \Rightarrow y' = 6(1 + \cos^2 x)^5 2\cos x(-\sin x) = -12\cos x\sin x(1 + \cos^2 x)^5 \]

37)
\[ y = e^{x\cos x} \Rightarrow y' = e^{x\cos x}(\cos x - x\sin x) \]

55)
(a) \( h(x) = f(g(x)) \Rightarrow h'(x) = f'(g(x)) \cdot g'(x), \) so \( h'(1) = f'(g(1)) \cdot g'(1) = f'(2) \cdot 6 = 5 \cdot 6 = 30 \)
(b) \( H(x) = g(f(x)) \Rightarrow H'(x) = g'(f(x)) \cdot f'(x), \) so \( H'(1) = g'(f(1)) \cdot f'(1) = g'(3) \cdot 4 = 9 \cdot 4 = 36 \)
(a) \( u(x) = f(g(x)) \Rightarrow u'(x) = f'(g(x))g'(x) \). So \( u'(1) = f'(g(1))g'(1) = f'(3)g'(1) \)
\[
= \left( -\frac{1}{4} \right)(-3) = \frac{3}{4}
\]
(b) \( v(x) = g(f(x)) \Rightarrow v'(x) = g'(f(x))f'(x) \). So, \( v'(1) = g'(f(1))f'(1) = g'(2)f'(1) \), which does not exist since \( g'(2) \) does not exist.
\[
(c) w(x) = g(g(x)) \Rightarrow w'(x) = g'(g(x))g'(x) \). So, \( w'(1) = g'(g(1))g'(1) = g'(1) = \left( \frac{2}{3} \right)(-3) = -2
\]

59)
\( h(x) = f(g(x)) \Rightarrow h'(x) = f'(g(x))g'(x) \). So \( h'(0.5) = f'(g(0.5))g'(0.5) = f'(0.1)g'(0.5) \).
We can estimate the derivative by taking the average of two secant slopes.
\[
f'(0.1) : m_1 = \frac{14.8 - 12.6}{0.1 - 0} = 22, \quad m_2 = \frac{18.4 - 14.8}{0.2 - 0.1} = 36. \quad \text{So}, \quad f'(0.1) \approx \frac{m_1 + m_2}{2} = 29.
\]
\[
g'(0.5) : m_1 = \frac{0.10 - 0.17}{0.5 - 0.4} = -0.7, \quad m_2 = \frac{0.05 - 0.10}{0.6 - 0.5} = -0.5. \quad \text{So}, \quad g'(0.5) \approx \frac{m_1 + m_2}{2} = -0.6.
\]
Hence, \( h'(0.5) \approx 29(-0.6) = -17.4 \)