This article is the fourth in a series of reports on findings from the MAA’s NSF-supported studies of college calculus. For further details, see the MAA Notes volume Insights and Recommendations from the MAA National Study of College Calculus (Bressoud, Mesa, and Rasmussen, 2015) or visit the website at maa.org/cspcc.

Learning from Calc I Finals

Marilyn P. Carlson and Michael A. Tallman

The final exam in Calculus I provides only one view of the types of problems students encounter. Nevertheless, the exam’s content reflects aspects of mathematical content the instructor values, thus shaping students’ images of the mathematics discipline and what it means to be mathematically proficient. It was surprising, therefore, to analyze 150 final exams and find that they generally require low levels of cognitive engagement, seldom contain problems stated in a real-world context, infrequently elicit explanation or justification, and rarely afford students the opportunity to demonstrate or apply their understanding of one of the course’s central ideas.

The overarching goal of the MAA initiative and NSF-funded project Characteristics of Successful Programs in College Calculus (CSPCC) was to identify the attributes of exemplary college calculus programs. In the first phase of the National Calculus Study, the MAA invited institutions of higher education across the nation to submit their Calculus I final exams and respond to a battery of surveys that assessed the views and
experiences of calculus coordinators, instructors, and students. The intent was to characterize and describe the content focus of the calculus courses from which the survey data was collected.

As part of the data collected, we had access to 253 Calculus I final exams administered during the 2010-2011 academic year at participating U.S. postsecondary institutions, including community colleges, liberal arts schools, public universities, and Ivy League schools. We analyzed a random subsample of 150 exams to get a sense of what college calculus instructors assess and how they are assessing it. These 150 exams collectively contained 3,735 items.

Only 14.72 percent required students to demonstrate an understanding of a calculus concept or procedure.

In this article, we present only our most salient findings and encourage the interested reader to consult our recently published manuscript in the International Journal of Research in Undergraduate Mathematics Education for a complete discussion of our methods and findings.

Setting Up a Framework

After reviewing a vast amount of literature on analyzing mathematics textbooks and assessments for ideas and direction, we developed our Exam Characterization Framework. We used it to classify individual exam items along three dimensions: (1) cognitive demand, (2) representation, and (3) format. We used a modification of the six intellectual behaviors of Benjamin Bloom’s familiar taxonomy to characterize the cognitive demand of exam items. This meant that we categorized the intellectual behavior required by items in the sample by applying one of the following labels to each item: remember, understand, apply understanding, analyze, evaluate, and create.

We found it relatively straightforward to specify the representation of problem statements on exams. Such representations included, but were not limited to, applied/modeling (items presented in a physical or contextual situation), symbolic (items that convey information in the form of mathematical symbols), tabular (items that convey information in the form of a table), and graphical (items that convey information in the form of a graph).

To characterize the format of the exam items, we determined whether they prompted students to select the correct answer among a number of options (multiple choice), to write down the one correct answer (short answer), to provide any one of a number of appropriate answers (broad open-ended), or to present their solution to a novel problem posed in a contextual setting (word problem). We were also interested in whether items prompted students to explain their thinking or justify their solution or problem-solving approach.

Concept Scores Low

Our analysis of the cognitive demand required by the 3,735 items in our randomly selected subsample final exams revealed that only 14.72 percent required students to demonstrate an understanding of a calculus concept or procedure. An example of such an item described a function \( r \) that defines the relationship between two quantities and asked students to interpret the meaning of the derivative at a point, the average rate of change of the function over an interval of the function’s domain, and the definite integral of the function for a specified interval.

In contrast, 85.21 percent of the items required students to recall or apply a rehearsed procedure or retrieve rote knowledge from memory. These items ranged from asking students to take the derivative or integrate a given function to questions that provide specific instructions for applying some method to answer a question, with no prompts to explain the meaning of what they were directed to compute.

An example of such an item prompted students to employ a procedure using the conclusion of the mean value theorem without requiring them to demonstrate an understanding of the theorem. Moreover, the item does not require students to justify why the statement of the mean value theorem is true, nor does it require students to interpret the solution’s meaning.

Show that the function \( f(x) = x^2 \) satisfies the hypotheses of the mean value theorem on the interval \([0, 4]\) and find a solution \( c \) to the equation

\[
 f'(c) = \frac{f(b) - f(a)}{b - a}
 \]

on this interval.
their items requiring students to demonstrate or apply their understanding.

It is also worth mentioning that the content of the Calculus I final exams did not align with the instructors’ perceptions of them when it came to how much students were asked to explain their thinking. Of instructors who submitted exams in our sample, 68.18 percent selected either 4, 5, or 6 on the question “How frequently did you require students to explain their thinking on exams?” (Choices ranged from 1 [not at all] to 6 [very often].)

A considerable amount of attention has been devoted throughout the past 25 years to the conceptual teaching of calculus. Educators have emphasized making connections among multiple representations, and recent curricular and instructional initiatives have been set up to improve students’ ability to apply their mathematical knowledge in contextual situations. In light of these efforts, it is noteworthy that

- only 9.6 percent of the exam items required students to present or interpret their solution in a mathematical representation that differed from that of the task statement;
- 38.67 percent of the exams had less than 5 percent of their items classified as word problems;
- only 22 percent had more than 10 percent of their items stated in the context of a real-world situation;
- 18 percent of the exams contained no real-world-based problems;
- 3 percent prompted students to explain their thinking; and
- 1.23 percent asked students to provide a justification for their answer or problem-solving approach.

When final exams are predominantly based on measures of low-level cognitive behaviors such as recalling memorized facts and procedures, students are likely to develop perceptions about mathematical proficiency that do not entail conceptual understanding and its creative application to solve novel problems.

**From Finals to Beginnings**

We invite the reader to consider what changes are needed in our nation’s Calculus I classes to produce students who know when and how to use both concepts and procedures when confronting a novel problem.

In David Bressoud’s February and March Launchings blog articles, he elaborated on the phenomenon of an instructor knowing clearly what he wanted his students to learn, but not attending to the meanings his students were constructing. In such a case, he didn’t understand what his students actually learned. This misalignment might be addressed by constructing assessments that reveal the meanings that students are developing.

A growing body of mathematics education literature has characterized productive meanings and ways of thinking for precalculus that are essential for learning calculus. Unfortunately, there is little evidence that this knowledge is having a widespread impact on instructors’ goals for student learning or instructor’s knowledge of how to foster the development of mathematical meaning of students.

Efforts to make undergraduate calculus instruction more meaningful must begin with a clear image and description of the meanings we desire students to have. These characterizations provide a target for curricular and instructional improvements, and they serve as a foundation for designing exams that focus on and assess the development of these meanings in students. The products and approaches for supporting and effecting student learning in mathematics might work better if curriculum developers adopted principles similar to those used by design engineers: they use research to guide their design then evaluate their products with measures that generate useful formative data to adapt their products.

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