SECONDARY PRECALCULUS PROFESSIONAL LEARNING COMMUNITIES:

A STRUCTURE FOR TEACHER DEVELOPMENT

by

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ABSTRACT

The purpose of this study was to investigate the creation and implementation of a professional learning community (PLC) and to characterize its influence on promoting growth in one of six secondary mathematics teachers in the PLC. The specific aspects of professional development this study explored were the participant’s conceptual knowledge, pedagogical content knowledge, beliefs about the learning and teaching of mathematics, and professed teaching practices. The primary aspects of teacher knowledge, beliefs, and practices that have been identified in the literature as influential in teacher professional growth led to the development of a PLC framework for analyzing the data.

Weekly PLC sessions, individual teacher interviews, and teacher email reflections were the primary sources of qualitative data. An instructional observational tool was used to document aspects of the teachers’ classroom practices, a concept assessment instrument provided information about the teachers’ conceptual knowledge, and a views assessment instrument aided in characterizing the teachers’ beliefs. The discourse of the emerging PLC was documented and aspects of the PLC that appeared to promote professional growth in six teachers were identified.

A PLC cycle of investigation provided a structure to promote meaningful discourse and instructional modification for the PLC teachers. Results from the case study of one of the six teachers revealed that PLC activities to investigate student thinking promoted growth in the teacher’s conceptual and pedagogical content knowledge. Analysis of the data further revealed that a participant’s beliefs about the learning and teaching of mathematics, as well as individual teacher behavioral traits can
either support or be considerable barriers to teachers’ willingness to make changes in their classroom practices.
Dedicated to my wife, Janell, and my sons, Caleb, Jordan, and Seth who were supportive, encouraging, and understanding throughout.
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>LIST OF TABLES</th>
<th>xiii</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF FIGURES</td>
<td>xiv</td>
</tr>
<tr>
<td>CHAPTER</td>
<td></td>
</tr>
<tr>
<td>1 INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>Statement of Research Questions</td>
<td>5</td>
</tr>
<tr>
<td>Motivation for the Study</td>
<td>6</td>
</tr>
<tr>
<td>Theoretical Perspective</td>
<td>8</td>
</tr>
<tr>
<td>Constructivism</td>
<td>8</td>
</tr>
<tr>
<td>Reform Instruction</td>
<td>9</td>
</tr>
<tr>
<td>Theoretical Framework</td>
<td>10</td>
</tr>
<tr>
<td>Overview of the Study</td>
<td>10</td>
</tr>
<tr>
<td>2 REVIEW OF RELATED LITERATURE</td>
<td>14</td>
</tr>
<tr>
<td>Current Status of Mathematics Instruction</td>
<td>14</td>
</tr>
<tr>
<td>Teacher Change Complexities</td>
<td>16</td>
</tr>
<tr>
<td>Professional Development Models for Teachers</td>
<td>18</td>
</tr>
<tr>
<td>Learning Communities for Teachers</td>
<td>20</td>
</tr>
<tr>
<td>Considerations of the Professional Learning Community</td>
<td>24</td>
</tr>
<tr>
<td>Focus of the Professional Learning Community</td>
<td>30</td>
</tr>
<tr>
<td>Conceptual Knowledge Regarding the Function Concept</td>
<td>32</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge</td>
<td>37</td>
</tr>
<tr>
<td>Beliefs About Learning and Teaching Mathematics</td>
<td>40</td>
</tr>
<tr>
<td>Conclusion</td>
<td>45</td>
</tr>
<tr>
<td>3 THEORETICAL FRAMEWORK</td>
<td>49</td>
</tr>
<tr>
<td>Introduction</td>
<td>49</td>
</tr>
<tr>
<td>Participant Learning</td>
<td>51</td>
</tr>
<tr>
<td>Conceptual Knowledge</td>
<td>52</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge</td>
<td>54</td>
</tr>
<tr>
<td>Beliefs about Learning and Teaching Mathematics</td>
<td>58</td>
</tr>
<tr>
<td>Conclusion</td>
<td>62</td>
</tr>
<tr>
<td>4 METHODOLOGY</td>
<td>63</td>
</tr>
<tr>
<td>Method of Inquiry</td>
<td>63</td>
</tr>
</tbody>
</table>
CHAPTER | Page
---|---
Setting | 64
School | 64
School Mathematics Curriculum and the Precalculus Program | 65
Professional Learning Community Participants | 66
Preliminary Communication | 66
Research Team | 67
Participating Teachers | 68
Method of Data Collection | 71
Weekly Meeting Observations and Reflections | 72
Individual Teacher Interviews | 73
Professional Learning Community Artifacts | 74
Teacher Assessment | 74
Classroom Observation | 77
Method of Data Analysis | 78
Phase 1 | 78
Phase 2 | 79
Phase 3 | 81
Phase 4 | 82
Phase 5 | 83
Phase 6 | 84
Phase 7 | 85
5 THE EMERGENT PROFESSIONAL LEARNING COMMUNITY | 86
Introduction | 86
Role of the Facilitators | 87
Professional Learning Community Structure | 91
Participant Expectations | 92
Investigative Topics | 92
Defining Professionalism | 93
Beliefs about Professional Development | 97
Professional Learning Community Participant Goals | 114
Norms of Collaboration | 121
Emergence of an Investigative Structure | 126
Initial Structure | 126
The Professional Learning Community Cycle of Investigation | 137
Session 1: Conceptual Knowledge | 138
Session 2: Pedagogical Content Knowledge | 140
Session 3: Intervention | 142
Session 4: Implementation | 147
Session 5: Assessment | 149
Session 6: Reflection | 151
Conclusion | 154
# RESULTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>156</td>
</tr>
<tr>
<td>The Case of Jeanne: An Experienced Secondary Mathematics Teacher</td>
<td>158</td>
</tr>
<tr>
<td>Characterization of Jeanne</td>
<td>159</td>
</tr>
<tr>
<td>Conceptual Knowledge</td>
<td>159</td>
</tr>
<tr>
<td>Classroom Practices</td>
<td>160</td>
</tr>
<tr>
<td>Beliefs about the Learning and Teaching of Mathematics</td>
<td>162</td>
</tr>
<tr>
<td>Summary of Beliefs</td>
<td>177</td>
</tr>
<tr>
<td>Behavioral Traits that Affect Pedagogical Decisions</td>
<td>180</td>
</tr>
<tr>
<td>Initial Profile of Jeanne</td>
<td>181</td>
</tr>
<tr>
<td>Professional Learning Community Experience</td>
<td>182</td>
</tr>
<tr>
<td>Investigating Student Thinking</td>
<td>183</td>
</tr>
<tr>
<td>PLC Session 16</td>
<td>183</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge and Beliefs</td>
<td>183</td>
</tr>
<tr>
<td>Alignment of Curriculum</td>
<td>187</td>
</tr>
<tr>
<td>Research, Beliefs, and Conceptual Knowledge</td>
<td>189</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge and Conceptual Knowledge</td>
<td>195</td>
</tr>
<tr>
<td>Discussion</td>
<td>199</td>
</tr>
<tr>
<td>Conceptual Knowledge</td>
<td>201</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge</td>
<td>202</td>
</tr>
<tr>
<td>Beliefs about the Learning and Teaching of Mathematics</td>
<td>203</td>
</tr>
<tr>
<td>PLC Session 17</td>
<td>203</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge and Research</td>
<td>204</td>
</tr>
<tr>
<td>Research to Practice</td>
<td>208</td>
</tr>
<tr>
<td>Discussion</td>
<td>215</td>
</tr>
<tr>
<td>Conceptual Knowledge</td>
<td>217</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge</td>
<td>218</td>
</tr>
<tr>
<td>Beliefs about the Learning and Teaching of Mathematics</td>
<td>219</td>
</tr>
<tr>
<td>Classroom Intervention</td>
<td>219</td>
</tr>
<tr>
<td>PLC Session 18</td>
<td>219</td>
</tr>
<tr>
<td>Activity Development and Beliefs</td>
<td>220</td>
</tr>
<tr>
<td>Discussion</td>
<td>227</td>
</tr>
<tr>
<td>Conceptual Knowledge</td>
<td>229</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge</td>
<td>229</td>
</tr>
<tr>
<td>Beliefs about the Learning and Teaching of Mathematics</td>
<td>229</td>
</tr>
<tr>
<td>PLC Session 19</td>
<td>230</td>
</tr>
<tr>
<td>Activity Development and Pedagogical Content</td>
<td>230</td>
</tr>
<tr>
<td>Knowledge</td>
<td>245</td>
</tr>
</tbody>
</table>
# Research Methodology

This section details the methods used to conduct the research and analyze the data. It includes descriptions of the data collection tools, procedures for data analysis, and the criteria for data reliability and validity. The methods section is crucial for ensuring that the research is conducted in a rigorous and transparent manner, which is essential for the credibility of the research findings. It also provides a basis for future studies that may be conducted in similar contexts or with similar research questions.
### CHAPTER

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Behavioral Traits</td>
<td>306</td>
</tr>
<tr>
<td>Question 5 - Professed Teaching Practices</td>
<td>308</td>
</tr>
<tr>
<td>Professional Learning Community Theoretical Framework</td>
<td>312</td>
</tr>
<tr>
<td>Conceptual Knowledge</td>
<td>313</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge</td>
<td>314</td>
</tr>
<tr>
<td>Beliefs About the Learning and Teaching of Mathematics</td>
<td>315</td>
</tr>
<tr>
<td>Emergent Framework for Analyzing Participants in a PLC</td>
<td>316</td>
</tr>
<tr>
<td>Limitations of the Study</td>
<td>320</td>
</tr>
<tr>
<td>Recommendations for Future Research</td>
<td>323</td>
</tr>
<tr>
<td>Concluding Remarks</td>
<td>328</td>
</tr>
</tbody>
</table>

### REFERENCES

330

### APPENDIX

<p>| A  | Factors that Influence Secondary Mathematics Teachers’ Instructional Decisions | 355  |
| B  | Ten Necessary Qualities for Building Community                              | 358  |
| C  | Professional Learning Community: Theoretical Framework                      | 360  |
| D  | Views About Mathematics Survey (VAMS) Taxonomy                              | 365  |
| E  | Participant Pre-interview Protocol                                          | 367  |
| F  | Nature of Mathematics Survey for Teachers                                   | 369  |
| G  | Nature of Mathematics Education Survey for Teachers                        | 371  |
| H  | Participant Post-interview Protocol                                         | 373  |
| I  | Reformed Teacher Observation Protocol (RTOP)                                | 389  |
| J  | Sample Coding Form                                                         | 393  |
| K  | PLC Sample Agenda                                                          | 395  |
| L  | Participant Consent Form                                                   | 397  |</p>
<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Theoretical Framework: Conceptual Knowledge</td>
<td>54</td>
</tr>
<tr>
<td>2</td>
<td>Theoretical Framework: Pedagogical Content Knowledge</td>
<td>57</td>
</tr>
<tr>
<td>3</td>
<td>Theoretical Framework: Beliefs About Learning and Teaching Mathematics</td>
<td>61</td>
</tr>
<tr>
<td>4</td>
<td>Precalculus Concept Assessment (PCA) Taxonomy</td>
<td>76</td>
</tr>
<tr>
<td>5</td>
<td>Precalculus Professional Learning Community Framework</td>
<td>80</td>
</tr>
<tr>
<td>6</td>
<td>First Semester Planned Calendar of Events</td>
<td>128</td>
</tr>
<tr>
<td>7</td>
<td>Second Semester Calendar of Events</td>
<td>136</td>
</tr>
<tr>
<td>8</td>
<td>PLC Selected Topics for Investigation as Represented in the PCA Taxonomy</td>
<td>146</td>
</tr>
<tr>
<td>9</td>
<td>Summary of Jeanne’s Beliefs About Learning and Teaching Mathematics</td>
<td>178</td>
</tr>
<tr>
<td>10</td>
<td>Jeanne’s Behavioral Traits</td>
<td>181</td>
</tr>
<tr>
<td>11</td>
<td>Findings from Session 16</td>
<td>199</td>
</tr>
<tr>
<td>12</td>
<td>Findings from Session 17</td>
<td>215</td>
</tr>
<tr>
<td>13</td>
<td>Findings from Session 18</td>
<td>227</td>
</tr>
<tr>
<td>14</td>
<td>Findings from Session 19</td>
<td>245</td>
</tr>
<tr>
<td>15</td>
<td>Findings from Session 20</td>
<td>258</td>
</tr>
<tr>
<td>16</td>
<td>Reported Pedagogical Content Knowledge Growth</td>
<td>267</td>
</tr>
<tr>
<td>17</td>
<td>Reported Beliefs About Learning and Teaching Mathematics</td>
<td>273</td>
</tr>
<tr>
<td>18</td>
<td>Framework for Analyzing Participants in a PLC</td>
<td>317</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Professional learning community seating arrangement</td>
<td>87</td>
</tr>
<tr>
<td>2.</td>
<td>The professional learning community cycle of investigation</td>
<td>138</td>
</tr>
<tr>
<td>3.</td>
<td>Conceptually-based fraction problem</td>
<td>207</td>
</tr>
<tr>
<td>4.</td>
<td>Observer/facilitator’s drawing</td>
<td>212</td>
</tr>
<tr>
<td>5.</td>
<td>Facilitator/researcher’s drawing</td>
<td>213</td>
</tr>
<tr>
<td>6.</td>
<td>Part 1 of the activity developed by Jeanne and Diane</td>
<td>232</td>
</tr>
<tr>
<td>7.</td>
<td>Parts 2, 3, and 4 of the activity developed by Jeanne and Dianne</td>
<td>234</td>
</tr>
<tr>
<td>8.</td>
<td>PLC-developed activity for evaluating sine and cosine conceptually</td>
<td>243</td>
</tr>
</tbody>
</table>
CHAPTER 1: INTRODUCTION

As early as 1983 in *A Nation at Risk* (NCEE, 1983) and then again in 1999 in the *Third International Mathematics and Science Study* (TIMSS) (IAEEA, 1999), it was reported that U.S. students were not having as much success in mathematics as their international counterparts. Suggestions have been made that the improvement of mathematics education for all students requires effective mathematics teaching in all classrooms (NCTM, 2000). One response has involved educational researchers focusing classroom-based investigations on teacher improvement as a way of remedying the lack of mathematical understanding and proficiency in American students (Ma, 1999; Stigler & Hiebert, 1999; Boaler, 2002; NCTM, 2000; NCTAF, 2003). Recent research findings of such studies reveal that many American mathematics educators have a shallow understanding of the mathematics they teach (Carlson, M., Cox, T., Engelke, N., Lancaster, V., Oehrtman, M., & Zandieh, M., 2003; Ma, 1999; Stigler & Hiebert, 1999; Ball 2005; Dubinsky & Harel, 1992; Norman, 1992) and do not have adequate knowledge of how to teach mathematical concepts effectively to their students (Ma, 1999; Stigler & Hiebert, 1999; RAND, 2003; Ball, 2005; Sowder, 2005; Thompson, 1984).

It has frequently been observed that mathematics teaching in the United States lacks an interaction between the study of the mathematics to be taught and the study of how to teach it (Stevenson & Stigler, 1992; Ma, 1999; NCMST, 2000; CBMS, 2001; Lewis, 2002). American lesson planning typically consists of teachers working through in isolation the examples they plan to use and making sure they can do the assigned
problems without considering how to help students make mathematical connections (Ma, 1999; Cooney, 1999; Mason & Spence, 1999; CBMS, 2001). It is also widely thought by mathematics education researchers that teachers’ beliefs influence their instructional decisions whether they are consciously held views or not (Ernest, 1991; Cooney, 1999; Cooney, Wilson, Albright, & Chauvot, 1998; Schoenfeld, 1998; Thompson, 1984). Consequently, to focus teacher improvement efforts solely on content knowledge and methodology yet ignore such affects as values, goals, beliefs, and attitudes, or only to treat them as “more cognitions”, seems shortsighted (Howell & Bradley, 2001). This aspect, too, reveals a difficulty for American mathematics teachers as many have been shown to hold views about the learning and teaching of mathematics that do not align with teaching for student understanding (Ernest, 1994; Stigler & Hiebert, 1999; Cooney, 1999; Carlson et al., 2003; Thompson, 1992).

Teaching mathematics well is a complex endeavor, and there are no easy recipes for helping all students learn or for helping all teachers become effective. Nevertheless, much is known about effective mathematics teaching, and this knowledge should guide professional judgment and activity. Research and education communities have been called upon to get involved and to become instruments of change in classrooms by helping to identify the knowledge that teachers need to develop students’ mathematical proficiency (RAND, 2003; Sowder, 2005). To provide the necessary content knowledge and the capacity to use that knowledge effectively in practice, the researchers recommend that a significant program of research and development be aimed at building resources for improved learning and teaching. Two high priority focus areas laid out for such plans are:
Many past efforts to improve classroom instruction have not had the anticipated results for a number of reasons: the complexity of the task, misplaced focus and ineffective strategies, lack of clarity on the intended results, failure to persist, and lack of understanding of the change process (DuFour & Eaker, 1998). External mandates by outside agencies and organizations continue to fall short also because they fail to empower teachers as change agents in the classroom and fail to recognize the cultural dimension of teaching. Teaching is primarily learned through prolonged, informal participation, not by formal study or a prescribed list of steps to follow (Lortie, 1975; Stigler & Hiebert, 1999). This means that short-term political initiatives or workshops, and even rigorous certification programs, are not sufficient methods to sustain ongoing improvements.

There is growing evidence that the best hope for significant school improvement is by grouping teachers into collaborative partnerships called professional learning communities (DuFour & Eaker, 1998; Gamoran, Secada, & Marrett, 2000). Many teachers need to grow in the profession of teaching by seeking the support and advice of colleagues and have continued access to new developments in their fields in order to improve their instruction (Cibulka, 2000). Gamoran et al. (2000) point to two fundamental changes needed for promoting teacher improvement: (1) long term professional development that engages teachers in learning communities, and (2)
partnerships between these teacher learning communities and outside agents such as university-based researchers. Research groups are seen as an invaluable link that can provide the support necessary to help with the development and sustenance of teacher learning communities.

Research has shown that the change process needs to be driven by knowledge of how mathematics is learned and by the effectiveness of one’s own classroom in promoting mathematical thinking and understanding in students (CBMS, 2001; Sowder, 2005; Ball, 2005). Teachers need experiences to assist them in acquiring mathematical knowledge that allows them to teach mathematics as a coherent and interconnected body of knowledge with itself and other disciplines (CBMS, 2001). One approach for organizing effective teacher learning communities is to have the teachers explore challenging content and problem solving in teams (LaChance and Confrey, 2003).

Additionally, researchers have recommended that teachers be provided opportunities to develop the habits-of-mind of a mathematical thinker so that they are able to model the processes involved in learning and doing mathematics for their students (CBMS, 2001; Carlson, 1999).

Building upon the previous work related to the learning community structure, the definition of the precalculus professional learning community (PLC) I have developed for this study will be:

A collaboration of six secondary teachers who meet weekly for sixty minutes with the assistance of a trained facilitator for the shared purpose of deepening their conceptual and pedagogical content knowledge, and understanding the process by
which precalculus students acquire understandings and reasoning abilities for the function concept in order to inform and improve their practice.

Statement of the Research Questions

The authors of *Principles and Standards* (NCTM, 2000) stress that effective teaching requires continuing efforts to learn and improve. These efforts include learning about mathematics and pedagogy, benefiting from interactions with students and colleagues, challenging deeply held beliefs, and engaging in ongoing professional development and self-reflection. Although much support exists in the literature for implementing professional learning communities for teacher improvement, only very recently has the development process, structure, and functioning of PLCs for specific disciplines such as mathematics or specific content areas within secondary mathematics been investigated (Cobb, McClain, & Lamberg, 2003). Therefore, in this study I outline a structure for developing and implementing a professional learning community that focuses participating teachers on mathematical content and reflection on student thinking. Furthermore, through a case study I document one participating teacher’s development in conceptual knowledge, pedagogical content knowledge, and beliefs about learning and teaching mathematics in order to demonstrate the effect the PLC had in drawing out these important aspects of teaching. My main research questions for this study follow.

- What professional learning community attributes are associated with the development of conceptual knowledge and pedagogical content knowledge relative to the function concept in secondary mathematics teachers?
• What professional learning community support tools are associated with improvements in the facilitation of secondary teachers’ reflection on students’ thinking and reasoning relative to the function concept?

• What conceptual knowledge and pedagogical content knowledge are exhibited by a secondary mathematics teacher as she participates in a professional learning community?

• What beliefs about the learning and teaching of mathematics are exhibited by a secondary mathematics teacher as she participates in a professional learning community?

• What self-reported classroom practices does a secondary mathematics teacher display as she participates in a professional learning community?

Motivation for the Study

I began my career as an educator teaching secondary mathematics for fourteen years. While teaching at that level and particularly now as a community college instructor, I have become interested in helping secondary mathematics teachers grow professionally. Over the years, I have become increasingly convinced that dramatic change needs to occur in classroom instruction in order to facilitate student understanding. Research has shown that many students do not learn the mathematics they are taught very deeply (Carlson, 1997; Carlson, Buskirk, & Halloun, 1999) and are generally taught using methods that emphasize the memorization of rules and mimicking procedures demonstrated by the teacher (Stigler & Hiebert, 1999; Ma, 1999). Reflecting on my own experiences both as a student and an instructor of mathematics, I see how
much more my conceptual knowledge has grown by being intimately engaged in mathematics rather than being held on the sidelines. In other words, I have learned far more by being the “driver” as opposed to being an observing “passenger”.

Through this research project, I develop and examine strategies and techniques that may help my high school colleagues improve their instruction so that secondary students have more beneficial educational experiences in mathematics. Not until my community college career began did I have the time to step outside my own world of teaching to lend a hand to others seeking change and growth. My goals for change coincide with the current reform efforts in mathematics education, namely the *Principles and Standards* laid out by the National Council of Teachers of Mathematics (2000). My passion is to use my years of teaching experience and knowledge of research as a bridge between the hands-on world of secondary school practitioners and the research culture of learning and progress.

In this study, my strategy for affecting the teachers’ practice was to center the discussions in a professional learning community on precalculus teachers’ and students’ understanding of the function concept. Through this process the researcher thought that the participating teachers’ level of motivation for change would rise and, as a result, their conceptual knowledge and pedagogical content knowledge would be built up. In addition, my goal as PLC facilitator was to activate change by challenging the beliefs each teacher holds about the learning and teaching of mathematics through confronting them with students’ conceptions of mathematics.
Theoretical Perspective

Constructivism

Central to this research is my conceptualization of knowledge and coming to know; this conceptualization establishes the purpose of my investigation and structures what I pay attention to in the data. I understand knowledge development as a set of social and cognitive processes. Underlying the social aspects of this perspective is a view of learners as participants in particular social groups or communities. The professional learning community is a specific example of this that can be analyzed from this perspective. From an interactionist perspective (Voigt, 1995), meanings are interactively constituted by the members of the community.

Cognitively, I view knowledge development from a constructivist perspective. I hold that knowledge is not passively received either through the senses or by way of communication, but is rather actively built up by the individual (von Glaserfeld, 1991). In this study, the two facilitators’ task was to structure a learning environment (the professional learning community) in which teachers were able to actively construct mathematical and pedagogical concepts. Constructivism includes a philosophical position that human beings construct their experiential realities from perceptions and experiences that are themselves afforded and constrained by prior conceptions. A conception is an explanatory mechanism used to account for observations of a learner. I consider the work of Piaget (1964), particularly his notions of the assimilation and accommodation of concepts into one’s own cognitive structures, foundational. From a constructivist perspective, learning is explained as a transformation and reorganization of the learner’s
conceptions. A learner’s current state of knowing (conceptions) influences learning in two ways. First, as in Piaget’s (1964) notion of assimilation, the learner’s conceptions define the learner’s experiential reality; what a learner perceives and the interpretations that he makes are structured by his current knowledge. Second, as in Piaget’s (1964) notion of accommodation, the current state of knowing is transformed. Thus, the possibilities for learning are afforded and constrained by the current state of knowing.

I use constructivism to make sense of the mathematical and pedagogical learning of the participants of the professional learning community in a variety of ways. First, the perspective of constructivism orients me to focus on understanding teachers’ conceptions. That is, I attempt to articulate and describe explanatory mechanisms for the teachers’ professional growth. Second, constructivism structures what I notice about teachers’ perspectives. Finally, my constructivist perspective contributes to defining the nature of the conclusions drawn from this work.

Reform Instruction

Principles and Standards (NCTM, 2000) outline the need for reform to take place in mathematics instruction and curriculum design. The authors point out that while some mathematics teachers are emphasizing thinking and problem-solving, many students still experience mathematics that is dominated by memorization and drill, without any meaningful context. In “reform” classrooms, students use technology to model and explore ideas, are challenged to find ways to solve problems based on what they know and understand, and have opportunities to link mathematics to real-world problems. My belief is that these “reform” instructional techniques and curriculum materials that
support this type of instruction are beneficial to mathematics students, and, therefore, a goal in this research project was to encourage the teachers in the professional learning community to grow in their desire and ability to implement such methods.

Theoretical Framework

This study is best portrayed as a qualitative analysis of a professional learning community and one of the PLC participants. Drawing on the research literature regarding general professional learning communities as well as the knowledge required of teachers of mathematics and the beliefs they hold, the PLC Theoretical Framework, described in Chapter 3, was constructed and used to organize and study the participants’ PLC experience. The analysis of the data was done to determine the role that the PLC played in drawing out and supporting participating teacher growth in three areas: conceptual knowledge, pedagogical content knowledge, and beliefs about learning and teaching mathematics.

Overview of the Study

This study investigates the initial development and implementation of a professional learning community at the secondary level and the PLC’s affect on one teacher’s conceptual knowledge, pedagogical content knowledge, and beliefs about the learning and teaching of mathematics. I discuss the role that the PLC played in the professional growth of a participating teacher and what PLC support tools were associated with improvements in teacher reflection. I report on the changes in the teacher’s professed classroom practices and beliefs toward the learning and teaching of mathematics as a result of experiencing the professional learning community discussions
and activities. Audiotaped data were collected in the professional learning community for 32 weeks of the 2003 - 2004 academic year. Additional data were collected from individual teacher interviews before and after the PLC was implemented and through periodic teacher email reflections. The transcribed data were analyzed using the PLC Theoretical Framework in an attempt to provide insight into the research questions.

The results of the study reveal that teachers require multiple opportunities, over time, to grow in content knowledge, pedagogical content knowledge, and beliefs about learning and teaching mathematics. In analyzing these data, I recognize specific struggles that a participant experiences in her efforts to make change in the context of a professional learning community. As a result, I recommend adjustments in the PLC described in this study for future iterations that may help participants to overcome these barriers.

The results of the study further reveal that the cycle of investigation (as defined in Chapter 5) served as an effective organizing structure for the professional learning community. The cycle of investigation provided a logical and sequential foundation that was based on the body of research and the needs of the participants in this study. The cycle of investigation highlighted the important and necessary aspects of teaching for student understanding while accommodating the learning community environment. Through this structure, the facilitators were successful in advancing the chosen participant’s conceptual understanding and pedagogical content knowledge as well as challenging her beliefs about learning and teaching. Growth was shown in all three aspects albeit in gradual and small increments.
The study also revealed the challenges faced when attempting to change teacher classroom practices. Despite the improvements made in the teacher’s conceptual knowledge and pedagogical content knowledge, the degree of change noted in classroom practice was minimal. As seen in the case study of the one participating teacher, there was some development in teacher questioning, considering student thinking when planning an intervention, and making individual practice public. The case study demonstrates how a participant’s beliefs about the learning and teaching of mathematics can be a considerable barrier to teacher growth.

As a result, I recommend that future iterations of PLCs–

- focus on developing an improved understanding of the function concept,
- engage the PLC participants in activities that investigate and analyze student thinking,
- explore ways to encourage participants to incorporate suggested changes into their classroom practices, and
- attend more closely to the individual participant’s behavioral traits as an added factor to be considered when attempting to grow teachers professionally.

The background literature that helped to support this study is presented in Chapter 2. The development of the PLC Theoretical Framework is described in Chapter 3, and the research methodology is presented in Chapter 4. The structure of the professional learning community that evolved throughout the study is depicted in Chapter 5 and the results of the study are reported in Chapter 6. Chapter 7 contains conclusions supported
by the results, limitations of the study, and recommendations for future PLC iterations and research.
CHAPTER 2: REVIEW OF RELATED LITERATURE

In this chapter, I review the literature that is most relevant to the study. I present a review of the literature regarding the current status of mathematics instruction, the complexities of teacher change, as well as some current professional development models. More directly, I include the literature related to learning communities for teachers that leads to and supports the definition of the precalculus professional learning community (PLC) that I use in this study. Finally, I specify aspects of effective PLCs and areas targeted for growth including teachers’ conceptual knowledge, pedagogical content knowledge, and beliefs about learning and teaching mathematics.

Current Status of Mathematics Instruction

Researchers believe that children need enthusiastic teachers who have a solid grounding in their disciplines and the professional training to teach their subjects well (NCMST, 2000). To achieve this end, decades of reforms and billions of dollars have been invested to bring about improvements in teaching, but few of these efforts have succeeded in altering instructional practice or increasing student achievement (Cohen & Hill, 1998; Elmore, 2000; Gallimore & Stigler, 2003). Over the last half-century teaching methods in mathematics classes have remained virtually unchanged (Callahan, 1962; Cuban, 1983; Tyack & Cuban, 1995; Wilms, 2003). Videotape studies of classes in the United States revealed that the basic teaching style in American mathematics classrooms remains essentially the same as two generations ago. The approach generally used begins with the teacher reviewing material or going over homework questions from the previous lesson followed by the teacher demonstrating how to solve example problems using a
new concept/skill. Often working in isolation, students are given time to practice problems of low-level procedures that imitate those demonstrated by the teachers. The lesson then culminates with an assignment of homework as another cycle begins again with the following lesson (Olson, 1999; Stigler & Hiebert, 1999; Wilms, 2003).

The literature is quite extensive in calling for changes in this deeply entrenched teaching style as a way to improve student achievement in mathematics (Ma, 1999; Stigler & Hiebert, 1999; NCTM, 1991, 2000; NCMST, 2000; AMATYC, 2003; Boaler, 2002; CBMS, 2001; RAND, 2003). In *Principles and Standards for School Mathematics*, the National Council of Teachers of Mathematics (2000) paints a vivid picture of what the desired classroom should be and how the class should operate to enhance student understanding of mathematics.

Imagine a classroom where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodation for those who need it. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. Technology is an essential component of the environment. Students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make
progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it (page 3).

This vision for mathematics education is highly ambitious and idealized. Despite the concerted efforts of many classroom teachers, administrators, teacher-leaders, curriculum developers, teacher educators, mathematicians, and policymakers, this portrayal of mathematics learning and teaching is not the reality in the vast majority of classrooms, schools, and districts (NCTM, 2000).

**Teacher Change Complexities**

The desired goal of many teacher improvement programs is to move mathematics educators more towards the aspiration that has been laid out (NCTM, 2000; AMATYC, 2003; RAND, 2003). Recent research has shown, however, that there is a vast array of powerful factors that affect today’s secondary mathematics teacher, making change a difficult undertaking (Ernest, 1994; Schoenfeld, 1998; Sowder, 2000; Spillane, 1999; Nelson, 1998; Gregg, 1995; Pugach, 1992). A research team from Arizona State University (Adamson, Banks, Burtch, Cox, Judson, Lawson, & Turley, 2003) has compiled many of the factors (Appendix A) that have been shown to influence secondary mathematics teachers’ pedagogical decisions. The study revealed that *social factors* such
as the availability of a supportive network of colleagues, the accessibility of professional
development opportunities, the existence and make-up of standardized tests, and adequate
preparation and instructional time are all such influences. Personal factors include
teacher’s beliefs about the learning and teaching of mathematics, goals and personal
characteristics, and knowledge of the mathematical content and how to teach it.

In addition to the many variables at play, the challenge of teacher change is
confounded by the fact that what is known about how to implement and sustain
successful change in teachers leads to conflicting results. DuFour and Eaker (1998) have
outlined some of the explanations for the failure of teacher change initiatives that include
change moving too fast versus change moving too slowly; change lacking strong
leadership versus change relying too heavily on leadership; change addressing too much
too soon versus change that is too small and not aggressive enough; change that is top-
down without participant buy-in versus change that is bottom-up without administrative
support; change celebrated too soon versus change not recognized and celebrated; and
unwillingness to change versus the embracing of every change that comes along.
Research and practice offer one inescapable, insightful conclusion: change is difficult.
Teacher change is a complex and formidable task that is certain to be accompanied by
dissonance and disequilibrium.

To help chart a course for the facilitators of change, the Concerns-Based
Adoption Model (CBAM) (Hall & Hord, 2001) lays out a foundation for the change
process by providing a set of principles that represent some of the predictable patterns of
change in educational settings. The principles described in the CBAM include:
• change is a process, not an event;
• planned interventions are the actions and events that are key to the success of the change process;
• appropriate interventions reduce the challenges of change;
• a horizontal perspective with administration is best (not top-down or bottom-up);
• facilitating change is a team effort;
• the context of the school influences the process of change;
• there are significant differences in the development and implementation of an innovation;
• innovations come in different sizes;
• change takes time and persistence.

Any effort, plan, or strategy for teacher development would be well-advised to take into account these important principles and to use them as guideposts in the creation and implementation of such (Hord, 2004).

Professional Development Models for Teachers

The common approach to executing change is through professional development activities and programs. Undoubtedly the most common type of professional development for the teaching profession is the workshop (Whittington, 2002). A workshop is a structured approach to professional development that occurs outside the teacher's own classroom. A workshop generally involves a leader with special expertise and participants who attend sessions at scheduled times – often after school, on the
weekend, or during the summer (Loucks-Horsley, Hewson, Love, & Stiles, 1998). Institutes, courses, and conferences are other typical forms of professional development that share many of the features of workshops. Although these formats are quite common, they have been criticized as being ineffective tools for increasing teachers’ knowledge base, for lacking meaningful learning experiences, and for not incorporating techniques that help to motivate teachers to transfer what they learn into the classroom (Loucks-Horsley et al., 1998; Whittington, 2002; DuFour & Eaker, 1998).

According to Ball and Cohen (1999), most professional development funding is spent on sessions and workshops that are often intellectually superficial, disconnected from deep issues of curriculum and learning, fragmented, and noncumulative. Teachers are thought to need updating rather than being given opportunities for serious and sustained learning about curriculum, content, students, and teaching (Ball & Cohen, 1999). In many school districts a “one size fits all” mentality limits the types of professional assistance teachers receive (Lord, 1994). Some frame the issue even more dramatically – Miles (1995) called current professional development a “joke” – “pedagogically naïve, a demeaning exercise that often leaves its participants more cynical and no more knowledgeable, skilled, or committed than before” (p. vii). Others (Hargreaves, 1995; Middleton, Sawada, Judson, Bloom, & Turley, 2002) have been milder in stating their criticism of current professional development efforts, but, nonetheless, have found them to be ineffective in terms of current reform and accountability demands.
The belief that many conventional programs are ineffective and wasteful has led to growing interest in other more innovative models for professional development, such as mentoring (Friedman & Phillips, 2002), action research teams (Ball, 2000), teacher networks (Pennell & Firestone, 1996), inquiry groups (Hammerman, 1997), vertical teaming (Blankstein & Cocozzella, 2004), study groups (Murphy, 1999; Mitchell, 1989), and lesson study (Ma, 1999; Lewis, 2000, 2002). Professional development models such as these differ from the more common and traditional methods in several respects. For instance, they often take place during the process of classroom instruction or during regularly scheduled teacher planning time. By locating opportunities for professional development within a teacher's work day and at his or her own school site, professional development has been shown to be more effective in making connections with classroom teaching (Garet, Porter, Desimone, Birman, & Suk Yoon, 2001). We are also starting to see that these novel formats for growth have more direct influence on changing classroom teaching practices (Darling-Hammond, 1995, 1996; Hargreaves & Fullan, 1992; Stiles, Loucks-Horsley, & Hewson, 1996; DuFour & Eaker, 1998) and result in incremental teacher change over time (Garet et al., 2001). Furthermore, by design they are more responsive to individual teacher’s needs and goals (Darling-Hammond, 1997) and are more aligned with how teachers learn (Ball, 1996).

**Learning Communities for Teachers**

Reflection and analysis are often individual activities, but they can be greatly enhanced by teaming with an experienced and respected colleague, a new teacher, or a group of teachers (Lave & Wenger, 1991). Collaborating with colleagues regularly to
observe, analyze, and discuss teaching and students' thinking is a powerful, yet neglected, form of professional development in American schools (Stigler & Hiebert, 1999). Interest is rising in what is seen as a promising model of professional development designed for groups of teachers from the same school, department, or grade level. There are many terms used in education attributed to a group of educators who come together on a regular basis to explore different aspects of learning and teaching. Some of the terms that carry this notion in the literature on professional development include teacher inquiry groups (Hammerman, 1997), professional study groups (Mitchell, 1989), learning circles (Collay, Dunlap, Enloe, & Gagnon, 1998), communities of practice (Lave & Wenger, 1991; Wenger, 1998), teacher support groups (Rich, 1992), teacher professional groups (Avalos, 1998), faculty learning communities (Cox, 2003), and professional learning communities (DuFour & Eaker, 1998; Hall & Hord, 2001).

Nomenclature notwithstanding, there is an astonishing concurrence regarding the effectiveness of this type of approach and that learning communities currently offer the best hope for stimulating significant teacher improvement and increased student learning (Senge, 1990; Drucker, 1992; Fullan, 1993; Joyce & Showers, 1995; Newmann & Wehlage, 1995; Darling-Hammond, 1996; DuFour & Eaker, 1998; Hall & Hord, 2001; Cox, 2003; Hord, 2004). The question: “Why Learning Communities? Why Now?”, is addressed by Cross (1998) and three reasons are given: (1) philosophical - because they fit into a changing philosophy of knowledge, (2) research based - because they fit with what research tells us about learning, and (3) pragmatic - because they work.
Regarding the particular discipline of mathematics, Anderson, Ashmann, Secada, and Williams (2003) note that a strong social network of professional colleagues constitutes a crucial resource for teachers as they attempt to cope with how to manage the tensions and ambiguities that come to the fore when attempting to teach mathematics. The researchers state that one goal of standards-based instruction and, therefore, many mathematical learning communities is to “teach for understanding”. This commonly involves focusing on student thinking, engaging students in powerful mathematical ideas and practices, and attempting to develop opportunities in which all students can come to know significant mathematical ideas. Collaboration among colleagues offers a forum for developing plans that respond to the uncertainties of teaching for understanding in the challenging world of mathematics.

It seems unlikely that even the most dedicated teachers would be able to acquire and sustain the resources and the commitment necessary for teaching for understanding by themselves. They need the support and advice of colleagues, and they need continuing access to new developments in their fields (Gamoran, Secada, & Marrett, 2000). Gamoran et al. (2000) claim that two fundamental changes needed to support teaching for understanding are: (1) long term professional development that engages teachers in learning communities, and (2) partnerships between these teacher learning communities and outside agents such as university-based researchers. In concert with the calls for involvement by institutions of higher education, outside agents are seen as an invaluable link that can provide the support necessary to help with the development and sustenance of teacher learning communities (NCMST, 2000; AAC & U, 2002).
Thinking pragmatically, it is important to follow up on current efforts which focus on the need to support the development of teacher learning communities that can both nurture and sustain growth (Franke & Kazemi, 2001; Nelson & Hammerman, 1996; Wilson & Berne, 1999). Franke, Carpenter, Levi, and Fennema (2001) refer to this as *generativity* and means to not only maintain new practices over time, but also to modify and adapt practices continually in response to new learning and reflection that occurs as a result of a persistent focus on student thinking. Franke et al. (2001) argue that both maintaining practice and generativity can be observed at the level of individual teacher, but a crucial difference between the two is that generativity occurs in the context of collaborative inquiry rather than in isolation. In other words, these generative practices occur and become normative in the context of participating in a teacher learning community.

From a theoretical standpoint, what remains constant in the research literature throughout the discussion of learning communities for teachers is the social situatedness of teachers’ learning within the activities (Franke & Kazemi, 2001; Grossman, Wineburg, & Woolworth, 2000; Lehrer & Schauble, 1998; Rosebery & Warren, 1998; Stein, Silver, & Smith, 1998; Warren & Rosebery, 1995). Collective participation in the same activity can provide a forum for debate and improving understanding, which increases teachers' capacity to grow (Ball, 1996). Secada and Adajian (1997) assert that mathematics teachers’ learning communities provide an important context for understanding the nature of teachers’ practices, change, and learning. Teachers who are from the same school, department, or grade are likely to share similar types of students and common curriculum
materials, course offerings, and assessment requirements. By engaging in joint professional development, they are able to integrate what they learn from each other with aspects of their instructional and institutional context (Ball, 1996).

**Considerations of the Professional Learning Community**

Rarely has research given school practitioners such a consistent message and clear sense of direction. However, even if educators are persuaded that forming a learning community offers the preeminent strategy for teacher improvement, difficult questions remain regarding creation, implementation, and assessment strategies. The best way to initiate consideration of these questions is to describe some key characteristics of a learning community and the conduct, as well as the habits-of-mind, of the people who work within it. A clear vision of what learning communities look like and how people operate within them offers insight into the process of forming a powerful alliance that is oriented towards and committed to ongoing change that benefits students.

Professional learning communities (PLCs), the focus of this study, incorporate the important aspects of the change process by involving a collaboration of teachers who focus on the examination of actual classroom data to understand student thinking. These data are then used by the teachers as the basis for an attempt to implement the discussed changes directly into their own instruction for the enrichment of their students’ learning (Hall & Hord, 2001; Hord, 2004). Each word of the phrase **professional learning community** has been chosen purposefully to help codify its components. DuFour and Eaker (1998) define a professional learning community as:
• A **professional** is someone with expertise in a specialized field, an individual who has not only pursued advanced training to enter the field, but who is also expected to remain current in its evolving knowledge base.

• The notion of **learning** suggests ongoing action and perpetual curiosity.

• In a professional learning **community**, educators create an environment that fosters mutual cooperation, emotional support, and personal growth as they work together to achieve what they cannot accomplish alone.

DuFour and Eaker (1998) suggest further that research both inside and outside education has arrived at the same conclusion: PLCs require educators to function as a collaborative team characterized by shared mission, vision, and values; collective inquiry; supportive and shared leadership; an orientation toward action and experimentation; commitment to continuous improvement; and a focus on results. Hord (2004) includes that the PLC should be characterized by “shared practice”. This involves the review of a teacher’s classroom behavior by colleagues and includes feedback and assistance activity to support individual and community improvement. In a related notion, Wenger (1998) described a **community of practice** as having three dimensions: (1) mutual engagement, (2) a joint enterprise, and (3) a shared repertoire. He emphasizes that a community of practice, in contrast to community, is an important distinction in which the interrelations that form the community structure arise out of collaboration and a shared enterprise or mission and engagement in practice and not some idealized view of community.

Additionally, Woolcock (1997) suggests four necessary conditions for fostering sustainable growth in a learning community: integration, linkage, organizational integrity,
and synergy. Integration refers to trust, mutual expectations, shared values, and the potential for establishing norms within a community. To break out of isolation, the community needs linkage to the wider environment, in the form of social interaction and relationships, that allow members to attract material and human resources (e.g., funding, equipment, expertise) so the community can continue to thrive (Gamoran, Anderson, & Williams, 2003). Organizational integrity refers to the coherence, competence, and capacity of institutions to manage a process of change (Woolcock, 1997). The fourth condition for sustainability is the degree of synergy between efforts of the community and those of the organizations in its larger environment. A focus on synergy recognizes that not only are individual actors embedded in a context, but organizations are embedded in an environment of organizations that are reliant upon one another.

Further guidance in building successful learning communities for teachers is provided by Cox (2004). With over twenty-five years of experience in the development of learning communities for teachers across a variety of disciplines, he has presented “Ten Necessary Qualities for Building Community” (Appendix B) that have been shown to be effective in creating and implementing faculty learning communities. Cox (2004) intentionally points out the importance of the sense of community. Among other factors for emphasis, he highlights:

- Safety and Trust – In order for participants to connect with each other, there must be a sense of safety and trust. This is especially true as participants reveal weaknesses in their teaching or ignorance of teaching processes or literature.
• Respect – In order to coalesce as a learning community, members need to feel as though they are valued and respected as professionals.

• Responsiveness – For an effective learning community, members must respond to each other, and the leader(s) must respond quickly to the other participants.

• Collaboration – The importance of consultation and group discussion on individual members' needs and on achieving learning outcomes hinges on the group’s ability to work with and respond to each other.

• Enjoyment – Learning community activities must include social opportunities to lighten up, bond, and should take place in invigorating environments.

Early on in the creation of a learning community discussions involve sharing concerns and frailties with the other members that participants are not accustomed to. Hence, the elements of safety and trust in the community are of utmost importance and a factor that demands constant monitoring (Cox, 2004).

Although the notion of a professional learning community often conjures images of a culture of consensus, shared values, and social cohesion, caution is recommended here. Although the overwhelming literature on learning communities emphasizes the effective conditions under which they should operate, Achinstein (2002) points out that in practice, however, when teachers collaborate, they most often run headlong into deep and strongly-held beliefs and practices. The researcher reported that in growth advocates’ optimism about caring and supportive communities, they often underplay the role of diversity, dissent, and disagreement in community life, leaving practitioners ill-prepared
and conceptions of collaboration underexplored. Building from case studies of two urban, public middle schools, Achinstein (2002) found that when teachers enact collaborative reforms in the name of community, certain unavoidable hurdles present themselves.

Three major concepts were found that cast new light on understanding learning communities for teachers: conflict, border politics, and ideology (Achinstein, 2002). First, communities are often born in conflict because they demand substantial change in school and individual norms and practices, challenge existing norms of privacy, independence, and professional autonomy, and may question existing boundaries between cultures and power groups at school sites (Hargreaves, 1994; Johnson, 1990; Lieberman & Miller, 1990; Little, 1990; Talbert, 1993). They remain in conflict as their valued norms of consensus and critical reflection, of unity and discord, are oftentimes incompatible. As many have argued, critical reflection is as essential as collaboration to strong communities (Dewey, 1916; Gardner, 1991; Lieberman & McLaughlin, 1992). Critical reflection, however, involves challenging the taken-for-granted assumptions of teaching and schooling practices and imagining alternatives for the purposes of changing conditions (Louden, 1992; Tabachnic & Zeichner, 1991).

The second dimension of collaborative teacher efforts left underexamined by past research is the process involved in defining community borders – negotiating which people and ideas belong. Borders identify the extensiveness and inclusiveness of the community. Thinking of learning communities projects notions of belonging, connectedness, and caring relationships (Noddings, 1992; Sergiovanni, 1994). For teachers, the metaphor of community may be particularly powerful in countering their
experiences of isolation; yet, those same movements to define a sense of community construct walls and borders that designate outsider status as well (Krainer, 2003). As Noddings (1992) explains, "we tend to draw circles around groups to which we belong and often define those outside our circles in disturbing ways" (p. 117). Putnam's (2000) distinction between bridging (inclusive) and bonding (exclusive) is relevant here. Communities may simultaneously construct insider and outsider status. As they reinforce shared identities, they also are distinguishing members from nonmembers and making it difficult for others to enter the collegial environment.

The third and final underexamined dimension of teacher collaboration is how communities are shaped by ideology. Ideological stances represent "educational perspectives and commitments of teachers" (Ball, 1987, p.281) and are a central concern of the political makeup of the community. For teachers, ideology determines the framework of shared values about the philosophy of education, schooling, and students. Ideology as a political process refers to the management of meaning – how individuals and communities make sense of their work and ultimately take action (Ball, 1987). Although past researchers and advocates of teacher communities identify the importance of having shared values and commitments, they ignore the content or ideological substance of such values (Westheimer, 1996, 1999). For example, Sergiovanni (1994) expressed the importance of communities bonding around shared values or philosophy, while finding the subject matter of this focus and clarity may well be secondary.

Despite obstacles and challenges, the literature is quite extensive in suggesting that the most promising strategy for sustained, substantive teacher improvement is
developing the ability of school personnel to function as effective PLCs. Therefore, building upon the previous work related to the learning community structure, the definition of the precalculus *Professional Learning Community* for this study is:

A collaboration of six secondary teachers who meet weekly for sixty minutes with the assistance of a trained facilitator for the shared purpose of deepening their conceptual and pedagogical content knowledge, and understanding the process by which precalculus students acquire understandings and reasoning abilities for the function concept in order to inform and improve their practice.

**Focus of the Professional Learning Community**

Ultimately, the success in creating a professional learning community depends upon the competence and commitment to professionalism of the teachers participating (DuFour & Eaker, 1998). Even the most well-conceived improvement programs fall flat if teachers lack the skills to implement them. Teachers are the ones that must bring the principles of the PLC to life. Situated in the classroom, teachers are essential to any meaningful reform effort and are in the best position to have a positive impact on the lives of children (Shulman, 1996).

Despite the critical role teachers play in providing effective instruction, they are commonly regarded as functionaries in the culture of education rather than as professionals (DuFour & Eaker, 1998). A profession has structures in place to ensure the competence of its members. Candidates for the profession must graduate from an accredited school, and then only those who can demonstrate that they have satisfactory content knowledge and proficient skills of practice are allowed to continue in the
profession. Consequently, to be a professional educator of mathematics, teachers need an extensive knowledge base, technical teaching skills, and epistemological stances that emphasize learning rather than teaching (Elmore, 2002; Sykes, 1999). To meet these needs, professional learning communities must strive to provide opportunities for growth on the part of teachers and motivate them to develop the knowledge, skills, and dispositions required to teach for understanding not simply to disseminate information (NCTM, 2000). Related outcomes of participating in a PLC must include changing teachers’ understanding of how students learn mathematics, the nature of mathematics, mathematical knowledge, and teaching mathematics well (Sowder, 2005). Ideally, this new understanding will result in mathematics teachers equipped to choose appropriate curricula, plan instruction, and organize their classrooms to promote and support learning for all students they teach.

In order for this to occur, Ball and Cohen (1999) suggested that teachers need to become serious learners of practice rather than learners of strategies and activities. For teachers to become such learners entails their coming to understand well the mathematics they teach and what it means to reason mathematically. They need to learn to attend to their students in insightful ways; a skill that “requires expertise beyond what one gathers from one’s own experience” (pp. 8 – 9). They need to “develop and expand their ideas about learning, [which would require that] longstanding beliefs and assumptions about learning would need to be examined” (p. 9).

In the following section I address individually the needs of teachers that serve as the goals for the participants in the PLC. These goals are to: (a) develop a sound
conceptual understanding of the function concept, (b) deepen pedagogical content knowledge, and (c) examine and challenge beliefs and assumptions about learning mathematics. Based on the research, it is important to note that growth in the PLC is expected to be characterized by continual, gradual, and incremental improvement (Stigler & Hiebert, 1999) as this type of change comes in small steps, not dramatic leaps (Tyack & Cuban, 1995).

*Conceptual Knowledge Regarding the Function Concept*

In recent decades, psychological and educational research on the learning of complex subjects such as mathematics has solidly established the important role of conceptual understanding in the knowledge and activity of persons who are proficient. Being proficient in a complex domain such as mathematics entails the ability to use knowledge flexibly – applying what is learned in one setting appropriately in another. One of the most robust findings of research is that conceptual understanding is an important component of proficiency, along with factual knowledge and procedural facility (Bransford, Brown, & Cocking, 1999). The alliance of factual knowledge, procedural proficiency, and conceptual understanding makes all three components usable in powerful ways. Students who memorize facts or procedures without understanding often are not sure when or how to use what they know and such learning is often quite fragile (Bransford, Brown, & Cocking, 1999). Learning with understanding also makes subsequent learning easier. Mathematics makes more sense and is easier to remember and to apply when students connect new knowledge to existing knowledge in meaningful ways (Schoenfeld, 1998). Well-connected, conceptually grounded ideas are more readily
accessed for use in new situations (Skemp, 1976). Conceptual understanding is an
essential component of the knowledge needed to deal with novel problems and settings.
Moreover, as judgments change about the facts or procedures that are essential in an
increasingly technological world, conceptual understanding becomes even more
important. Change is a ubiquitous feature of contemporary life, so learning with
understanding is essential to enable students to use what they learn to solve the new kinds
of problems they will inevitably face in the future.

Research shows that teachers must have a solid understanding of their content to
teach the subject well (Ball, 2005; Sowder, 2005). Ma (1999) revealed that shallow
subject knowledge restricts teachers’ capacity to promote conceptual learning among
students. How well teachers know mathematics is central to their capacity to use
instructional materials wisely, to assess students' progress, and to make sound judgments
about presentation, emphasis, and sequencing (Ball, 2003). Teachers need to understand
the big ideas of mathematics and be able to represent mathematics as a coherent and
connected activity (Ma, 1999; Cooney, 1999; Mason & Spence, 1999). Teachers also
need to understand the different representations of a concept, the relative strengths and
weaknesses of each, and how they are related to one another (Wilson, Shulman, and
Richert, 1987; NCTM, 2000). At the 2003 Department of Education Secretary’s Summit
on Mathematics, Ball, in her presentation, said that the qualities of mathematics
knowledge needed for effective instruction included being “respectful of the integrity of
the discipline, able to be extended and opened up for learners – unpacked, justified and
reasoned, connected within and across domains, building on earlier ideas and anticipating
more advanced topics, and organized psychologically as well as logically.” Teacher
decisions and their actions in the classroom—all of which affect how well their students
learn mathematics—should be based on this knowledge.

A specific mathematical concept that is of importance at the secondary
precalculus level and is investigated in this study is that of the function concept. The
notion of function has long been central to undergraduate mathematics (Breidenbach,
Dubinsky, Hawks, & Nichols, 1992; Selden & Selden, 1992; Thompson, 1994b). The
function concept has been shown to be essential for success in calculus; important ideas
such as limit, derivative, and accumulation each require a strong function understanding
(Carlson, 1998; Carlson, Jacobs, Coe, Larsen, Hsu, 2002; Kaput, 1994; Thompson,
1994a; Zandieh, 2000). Defining, understanding, and translating between functions in
diverse representations are standards that have received emphasis (NCTM, 2000). In
*Everybody Counts* (NRC, 1989), the authors have advocated that the principal goal of
undergraduate mathematics should be to develop function sense in students.

Despite the central role of the concept and repeated calls to strengthen function
learning and teaching for students, a primary challenge is that novice and experienced
teachers exhibit a weak understanding of the function concept (Norman, 1992; Carlson,
Coe, Cox, & Larson, 2002; Carlson et al., 2002a). Mathematics teachers at all levels have
been found to know rules and procedures but lack knowledge of reasoning skills and
concepts (Wilson, Floden, & Ferrini-Mundy, 2001). When investigating secondary
precalculus teachers’ understanding of functions at the beginning of a graduate course,
Carlson et al. (2002b) identified weaknesses in teachers’ ability to construct and interpret
graphs of dynamic function situations. The study showed that some of the teachers did not understand what was involved in inverting a function or composing two functions, and that some had difficulty conceiving of a function as an entity that accepts input and produces an output. Norman (1992) found that the preservice secondary teachers he studied exhibited weaknesses in their conceptualizations of function as well. The researchers noted among other things that they preferred graphical representations of functions over symbolic and numeric ones, exhibited difficulty devising and identifying physical situations that entail functional relationships, and had not built strong connections between the formal function definition and the different representations and real world applications.

Furthermore, Carlson’s investigations (Carlson et al., 2002b, 2003) of teachers’ function understandings suggest that they do not possess the deep and flexible knowledge of function they need to provide conceptual learning and teaching for their students. Several teachers revealed that their previous weak understanding of function had left them powerless to take anything but a procedural approach to teaching functions to their students (Carlson et al., 2002a, 2002b, 2003). This information regarding the lack of teacher function knowledge supports other general findings that too few classroom teachers have the type of mathematical knowledge necessary for teaching (Borko & Putnam, 1995; Ma, 1999; RAND, 2003; Ball, 2005).

Given the significance of teachers’ understanding of content and its affect on instruction, questions have been raised about how to insure and maintain high-quality conversations in professional development programs based on deep understanding of
mathematical concepts (Ma, 1999; Sowder, 2005). Teachers cannot be expected to know or do what they have not had opportunities to learn for themselves. Improving the mathematics learning of every child depends on making central the learning opportunities for teachers. The current efforts to focus school mathematics instruction on students developing conceptual understanding of mathematics must go hand-in-hand with teachers developing this knowledge of mathematics also.

Lewis (2002) notes that participating in professional development alone does not ensure enhanced content knowledge. Professional development must provide opportunities for teachers to learn more mathematics, even when the primary lens is on student thinking, curriculum, or classroom events (Sowder, 2005; NCTM, 2000). Expanding conceptual knowledge for teachers calls for a shift from content as the sole focus to “content in relation to student learning and context” (Elmore, 2001, p. 11). This requires the careful development of well-designed professional learning communities that make a deliberate and sustained attempt to identify the conceptual knowledge needed for teaching mathematics and to understand its specific uses in teaching. This also suggests that conceptual understanding for teachers should not be an assumed benefit of a professional learning community; rather it must be a goal that should be actively targeted in the planning and implementation of PLCs.

**Pedagogical Content Knowledge**

Early efforts to study the relationship between teachers’ mathematical knowledge and student achievement used characteristics of teachers and their educational backgrounds, such as courses taken, degrees attained, or certification status as measures
of ability (Begle, 1979). Despite disappointing findings – little relationship between these measures and student achievement – interest has persisted in the connections between teacher knowledge and student achievement. By the mid-1980s, scholars began to see the problem differently, reframing the question of how teachers’ content knowledge might contribute to student learning by focusing on how such knowledge might be used in teaching (Shulman, 1986; Wilson et al., 1987; Ball, 1990; Ma, 1999). This change of perspective represented a different way to think about teacher knowledge. To start, this view suggested teacher knowledge is highly particularized, rooted in the specific problems of teaching mathematics content. Previous research studies suggest that student learning might result not only from teachers’ content knowledge, but also from the interplay between teachers’ knowledge of students, their learning, and strategies for improving that learning (Ball, 1990; Ball & Bass, 2000; Ball, Lubienski, & Mewborn, 2001; Ma, 1999).

The analytic distinction between teachers’ content knowledge and teachers’ knowledge of pedagogy began to fade, in large part due to Shulman’s (1986, 1987) work. Shulman argued that a distinctive form of teachers’ professional knowledge that he called pedagogical content knowledge exists, and that this form of knowledge builds upon, but is different from, teachers’ subject matter knowledge or knowledge of general principles of pedagogy. In Shulman’s view, pedagogical content knowledge is a form of practical knowledge that is used by teachers to guide their actions in highly contextualized classroom settings. This form of practical knowledge entails, among other things the: (a) knowledge of how to structure and represent academic content for direct teaching to
students, (b) knowledge of the common conceptions and difficulties that students encounter when learning particular content, and (c) knowledge of the specific teaching strategies that can be used to address students’ learning needs in particular classroom circumstances. Pedagogical content knowledge builds on the other forms of professional knowledge, and is therefore a critical—and perhaps even the paramount—element in the knowledge base of teaching (Shulman, 1987).

As a consequence, professional development for teachers of mathematics should not be simply for the purpose of producing individuals who know more mathematics. The goal is to improve students' learning. Teachers' opportunities to learn must equip them with the mathematical knowledge and skill that enables them to teach mathematics effectively (Ball, 2003). In other words, being able to do calculations or solve problems oneself is insufficient for being able to respond well to a student who doesn’t understand. Even understanding the procedure in the formal terms that one might learn in a mathematics course does not equip one to explain mathematical concepts in ways that are both mathematically valid and accessible to secondary school students. The capacity to do this is a form of mathematical work that has been often overlooked in the current discussions of improving teaching quality (Sowder, 2005).

What is the “mathematical work” that secondary mathematics teachers need to be able to do to teach for understanding in their students? Ball’s (2003) description of the central components of the knowledge for teaching is helpful to those developing professional development opportunities for mathematics teachers. She outlines that teachers need to be proficient at–
• designing mathematically accurate explanations that are comprehensible and useful for students,

• using mathematically appropriate and comprehensible definitions,

• representing ideas carefully, mapping between a physical or graphical model, the symbolic notation, and the operation or process,

• interpreting and making mathematical and pedagogical judgments about students' questions, solutions, problems, and insights (both predictable and unusual),

• responding productively to students' mathematical questions and curiosities,

• making judgments about the mathematical quality of instructional materials and modifying them as necessary,

• posing good mathematical questions and problems that are productive for students' learning, and

• assessing students' mathematics learning and taking informed next steps.

Relating the research to this study, it has been shown that the primary focus of a professional learning community needs to be on student learning rather than on teaching (DuFour & Eaker, 1998; Louis, Kruse, & Marks, 1996). The difference is much more than semantics. The reorientation of teacher focus in this way represents a fundamental shift in the teacher-student relationship. This new relationship would not allow for the familiar teacher comment, “I taught it but they just didn’t learn it.” Teachers in a PLC need to recognize that teaching has not occurred until learning has occurred, and need to act accordingly as a community. Their emphasis, therefore, must not simply be on
covering the required material but rather on engaging students in the consideration of
essential content in ways that help them develop a deep understanding of that content
(Ball, 2003).

Beliefs about Learning and Teaching Mathematics

Mathematics education researchers widely hold that teachers’ beliefs (Ernest, 1991; Thompson, 1984; Cooney et al., 1998; Schoenfeld, 1998; Cooney 1999) largely influence how they teach their courses. Schoenfeld (1998) has argued that what a teacher believes about the subject matter, about learning, and about the students plays a critical role in shaping what the teacher does, both in terms of planning and in terms of what happens “on the fly” as instruction takes place. For this reason, the beliefs that teachers hold about the learning and teaching of mathematics must be a strong consideration while constructing and implementing a professional learning community.

A synthesis of the research by Thompson (1992) suggests that teachers’ beliefs about mathematics are often limited and dualistic in the sense of having a right/wrong answer with mostly single procedures to arrive at the correct answer. The conception of mathematics teaching that follows from this view is one in which concepts and procedures are presented in a clear way with students required to practice identifying concepts and to perform procedures (Thompson, 1992). An alternative account of the meaning and nature of mathematics emerges from the analysis of mathematical knowledge based on the practice of mathematicians. Carlson (1999) asserts in her research that mathematicians depict mathematics as a kind of mental activity, a social construction involving conjectures and proofs. For many mathematicians, the results of
mathematics are subject to change, and validity must be judged in relation to a social and cultural setting. The conception of mathematics teaching that can be gleaned from this view is one in which students engage in activities that grow out of problem situations, requiring reasoning and creative thinking, gathering and applying information, discovering, inventing, and communicating ideas, and testing those ideas through critical reflection and argumentation (Ernest, 1994; Thompson, 1992). This view of mathematics teaching contrasts with the alternative view in which the mastery of concepts and procedures is the ultimate goal of instruction. However, this perspective does not necessarily deny the value and place of skills and procedures in the mathematics curriculum.

The typical American lesson has been shown to be driven by a singular belief that school mathematics is a set of procedures (Ernest, 1991; Thompson, 1992; Stigler & Hiebert, 1999). Although teachers might understand that other things must be added to these procedures to get the complete definition of mathematics, many behave as if mathematics is a subject whose use for students is as a set of procedures for solving clearly laid-out problems (Stigler & Hiebert, 1999; Schoenfeld, 1998). In a study by Stigler and Hiebert (1999), American mathematics teachers were asked what “main thing” they wanted students to learn from the lesson they had taught. Sixty-one percent of the teachers surveyed specified skills they wanted their students to acquire, including the ability to perform a procedure or solve a particular kind of problem.

Cooney (1999) found that prospective secondary mathematics teachers view mathematics as existing outside the human domain. The results show teachers align
“doing mathematics” with “getting the right answers”. One teacher expressed the view that math consisted of “certain undeniable truths or laws” and that “math could be discovered but not created because it already exists”. Many teachers were shown to not have entered their teacher education programs with a solid grounding in school mathematics or in how mathematics can be applied to real world situations. Some, however, embraced a pluralistic view of mathematics and acknowledged the importance of intuition, curiosity, and reasoning.

Not only do instructors’ views about the nature of mathematics seem to have an impact on the chosen teaching style or methods they implement but so do their views about the nature of what constitutes good mathematics teaching (Ernest, 1991; Thompson, 1984). How do teachers believe that mathematics should be taught? What do mathematics educators believe is the best way for students to learn mathematics? Cooney et al. (1998) found that a consistent theme in a study with secondary preservice teachers was their equating of good teaching with good telling; meaning that teachers should explain mathematics step-by-step so that students are not confused. Not surprisingly, this view of the teaching of mathematics is, more or less, consistent with the way the preservice teachers experienced learning mathematics themselves (Cooney, 1998). The frequent experiences they had with mathematics teaching in the “telling mode” counter current reform efforts in mathematics education. As one future teacher in another study saw it, the responsibility of the teacher is to present clear and concise step-by-step instructions to solving problems; the responsibility of the student is to listen carefully, take good notes, and be sure to do the assigned work (Cooney, 1999).
Further supporting evidence for this widely-held belief is when Cooney (1999) asked the same preservice teachers what they thought constituted an excellent mathematics professor. The collaborated response was one who has the following characteristics: clarity, a good pace of instruction, and is available for answering questions. Cooney suggests that these are, in fact, characteristics of good instructors but none are the substance from which reform is derived but rather they lend themselves to an environment of telling. Cooney (1999) concludes: “Teaching in the schools would be much simpler if teachers were asked only to deliver instruction based on these three principles. Perhaps it might be easier, but reform it is not” (p. 169).

The preservice teachers in Cooney’s (1999) study mentioned another important characteristic of a good mathematics instructor as one who cared. For some, caring meant enabling students to master basic skills and to be comfortable in the classroom setting. This consisted of making tests a matter of mimicking what had been previously done in class so the students didn’t get uncomfortable trying to solve problems to which they hadn’t previously been exposed. Stigler and Hiebert (1999) found that teachers they studied appeared to feel responsible for shaping mathematical tasks into pieces that were manageable for most students, providing all the information needed to complete the task and assigning plenty of practice. To the teachers, providing sufficient information meant, in many cases, demonstrating how to complete a task just like those assigned for practice. Teachers acted as if confusion and frustration were signs of weakness or incompetence on their part. When teachers noticed confusion, they quickly assisted students by providing whatever information it took to get the students back on track.
In what seems to be a conflicting finding, many experienced mathematics teachers believe that learning mathematical terms and practicing skills, as many of them emphasize in their courses, is not very exciting. As Stigler and Hiebert (1999) analyzed the instruction of many American teachers they saw them trying to “jazz up” the lesson and increase students’ interest in nonmathematical ways such as by the teacher being entertaining, by interrupting the lesson to talk about what they felt was other more interesting things, or by setting the mathematics problem in a real-life or intriguing context. The teachers acted as if student interest would be generated only by diversions outside of mathematics. Contrastingly, Japanese teachers studied seemed less concerned about motivating topics in nonmathematical ways. They acted as if mathematics is inherently interesting, and students will be interested in exploring it by developing new methods for solving unique problems (Stigler & Hiebert, 1999).

The goal of teaching open-ended problem solving is often seen by American teachers as risky business because it invites the unpredictable and may lead to losing “control” of the class which is what many teachers seem to covet (Cooney, 1999). Cooney (1999) goes on to say that:

Teacher education is consequently in the unenviable position of needing to help teachers unravel their notions about teaching and rebuild them in a rational way….The orientation toward telling with clarity and the overwhelming propensity to be a caring teacher puts at risk ideas that may appear to contradict these characteristics. How can it be, for example, that caring (emphasis added) can be translated into causing students to experience stress (emphasis added) in
solving problems? Clarity is a very seductive characteristic in teaching for it satisfies both teachers and students – not to mention parents and administrators….This is not to say that clarity is the enemy of good teaching, but it is to suggest that clarity without rationale can lead to a static form of learning (p. 170).

Finally, Cooney (1994) has illustrated through a case study that it is not easy for an experienced and good teacher – certified so by students, parents, and administrators – to admit that other approaches toward learning have merit. To admit such often requires a shift away from a teacher-centered classroom in which the teacher feels good, is comfortable, and has had “success”. The paradigm shift that occurred in this single teacher required dismantling much of what she had previously valued and believed about the learning and teaching of mathematics by confronting her with what her students were actually learning. We can see that before change can occur, teachers’ core beliefs may need to be challenged if they run contrary to what is required to teach mathematics for understanding (Sowder, 2005).

Conclusion

Despite numerous attempts to reform mathematics education, there is little evidence to suggest that mathematics teachers have become significantly more effective in meeting the challenges that confront them. Past efforts to improve instruction by becoming more in line with what professional organizations have called for have not had the anticipated results (Sowder, 2005). Change facilitators need not despair. There is growing evidence that the best hope for significant instructional improvement is by
forming isolated individual teachers into well-functioning professional learning communities.

According to the National Staff Development Council (2003), the most powerful forms of staff development occur in ongoing teams that meet on a regular basis for the purposes of learning, joint lesson planning, and problem solving. These teams, or professional learning communities, require teachers to operate in collaborative teams characterized by shared mission, vision, and values; supportive and shared leadership; collective inquiry; an orientation toward action and willingness to experiment; commitment to continuous improvement; and a focus on results (DuFour & Eaker, 1998). A professional learning community needs to be built around important components requiring teacher challenge and growth including conceptual knowledge, pedagogical content knowledge, and beliefs about the learning and teaching of mathematics.

The goal of this study is to describe how the forming of a PLC at the secondary mathematics level can be used as an instrument of change in the process of moving teachers along the continuum of professional growth. Effective mathematics teaching requires a serious commitment to the development of students' understanding of mathematics (NCTM, 2000). Because students learn by connecting new ideas to prior knowledge, teachers must understand what their students already know. If teachers know how to ask questions and plan lessons that reveal students' prior knowledge, they can then design experiences and lessons that respond to, and build on, this knowledge (NCTM, 2000). I speculate that teachers will be motivated to make changes by having the PLC focus on their students’ understanding of mathematical concepts. I believe that teachers
will discover that many students don’t understand very deeply the mathematics that has been taught to them. This study investigates the role that teacher reflection about student understanding and classroom practice in a PLC plays in teacher change. The intention is to learn how to provide a PLC that supports teachers’ efforts to improve through the investigation of student thinking about mathematics.

I expect that, through engagement in a PLC where teachers collaborate with their colleagues and are supported by mathematics educators, the participants will begin to deepen their conceptual knowledge, improve in pedagogical content knowledge, and align their teaching practices more closely with what has been called for in the literature. I envision a supportive PLC environment in which the teachers participate in the arena of ideas, admit weaknesses, challenge each others’ belief systems, and are nurtured by one another. In addition, I believe that as the PLC is being implemented it will be particularly powerful and relevant because of it being located in the participants’ own campus environment, focusing teachers on their own students’ thinking, using their own curriculum, and involving their own colleagues.

Another purpose of this dissertation is to describe the structural components of the PLC in which teachers participated and to illustrate how it was established. In addition, I provide examples of how focusing on student learning aided teacher growth, and I describe the PLC tools that were used to encourage the growth process. Specific examples of a teacher’s growth in conceptual knowledge, pedagogical content knowledge, and beliefs are provided in PLC weekly meetings, teacher self-reflections, and individual interviews. Finally, a participant’s changing beliefs toward mathematics
learning and teaching are portrayed and reasons, reported by the teacher, for those changes are discussed.
CHAPTER 3: THEORETICAL FRAMEWORK

Introduction

A synthesis of the body of literature support that many mathematics educators have a shallow understanding of the function concept (Carlson et al., 2003; Dubinsky & Harel, 1992; Norman, 1992), do not have adequate knowledge of how to teach mathematical concepts to their students (Ma, 1999; Stigler & Hiebert, 1999; RAND, 2003; Thompson, 1984), and hold beliefs that are not supportive of teaching for student understanding of concepts and principles (Thompson, 1992; Ernest, 1994; Sowder, 2005). As has been widely cited as an effective model for teacher professional development (DuFour & Eaker, 1998; Franke & Kazemi, 2001; Grossman et al., 2000; Lehrer & Schauble, 1998; Rosebery & Warren, 1998; Stein et al., 1998; Warren & Rosebery, 1995), a professional learning community served as the social environment and construct in which the participating teachers’ development was considered in this research.

A thorough review of the literature revealed there was no suitable theoretical framework for investigating a professional learning community for secondary precalculus teachers. There are broad frameworks for inter-disciplinary faculty learning communities at the collegiate level (Cox, 2004) and professional learning communities for general disciplines (Hord, 2004) but not specifically for mathematics. Although it has been shown that there are pedagogical practices that are aspects of effective instruction common to all disciplines (Shulman, 1996; Marzano, Pickering, & Pollock, 2001; Shulman, 1986, 1987), there are certainly special considerations that must be taken into account in the learning and teaching of mathematics. The types of special knowledge that
teachers of mathematics must possess (Ball, 2005) and the beliefs of mathematics
teachers that affect their choice of instructional techniques (Ernest, 1994; Thompson,
1992) are both such considerations.

In this dissertation, I depict the growth of a participating PLC teacher’s
conceptual knowledge, pedagogical content knowledge, and beliefs about the learning
and teaching of mathematics as well as the structural PLC components that were most
effective in facilitating such growth. To illuminate this, I focus on how participating in
the PLC facilitated a teacher’s efforts to grow professionally. Therefore, it became
necessary to create a framework by which teacher efforts to grow could be analyzed. I
create this framework by drawing on data collected from this study as well as a wealth of
research literature concerning professional learning communities (DuFour & Eaker,
1998; Hord, 2004; Bryson, 2005; Cox, 2004; NCTM, 2000; Franke et al., 2001),
conceptual knowledge (Shulman, 1987; NCTM, 2000; Ball, 2005; Santos-Trigo, 1998;
Ma, 1999; Sowder, 2005; CBMS, 2001; Wilson et al., 1987; Cooney, 1999; Mason &
Spence, 1999; Gravemeijer & Doorman, 1999; Adamson, 2005; Simon & Tzur, 2004),
pedagogical content knowledge (Silverman, 2005; Shulman, 1987; NCTM, 2000; NRC,
2005; Ball, 2003), and beliefs about learning and teaching mathematics (Carlson, 1997;
Sowder, 2005; NRC, 2005; Dewey, 1933; von Glaserfeld, 1991; Boaler, 1993;
Gravemeijer & Doorman, 1999; Adamson, 2005; Santos-Trigo, 1998; Cooney, 1993;
Lawson et al., 2002; Ernest, 1991; NCTM, 2000). Recall that a precalculus \textit{professional
learning community} for this study is defined as:
A collaboration of six secondary teachers who meet weekly for sixty minutes with the assistance of a trained facilitator for the shared purpose of deepening their conceptual and pedagogical content knowledge, and understanding the process by which precalculus students acquire understandings and reasoning abilities for the function concept in order to inform and improve their practice.

An all-encompassing theoretical framework necessary to study the numerous aspects of a precalculus professional learning community is extensive and beyond the scope of this study. Nevertheless, an initial PLC Theoretical Framework created by the researcher for this purpose is provided in Appendix C that details each aspect of a PLC—professional, learning, and community—and will be adapted and enhanced as future iterations of PLCs are implemented. I reveal in this chapter the portions of the full PLC Theoretical Framework that are relevant for this study—specifically, those which deal with participants’ learning. Further information of the framework development process is described in the data analysis (Phase 3) found in Chapter 4.

Participant Learning

Participant learning is the central core of what the professional learning community sets out to accomplish. There are three major aspects of participant learning that are important to this study of PLCs: conceptual knowledge, pedagogical content knowledge, and beliefs about learning and teaching mathematics. A brief portrayal of each of these aspects of the PLC Theoretical Framework follows.
As has been shown in the literature review of this dissertation, teacher conceptual knowledge is at the forefront of the knowledge base for teachers of mathematics along with factual knowledge and procedural facility (Bransford et al., 1999; Ball, 2005; Sowder, 2005; NCTM, 2000). Those who simply memorize facts or procedures without understanding often are not sure when or how to use what they know, and such learning is often quite fragile (Bransford et al., 1999). Furthermore, such a narrow focus on procedural competence and memorization of facts does not lead one to gain the ability to provide reasoned responses or to justify thought processes when problem solving (NCTM, 2000). Researchers have shown that mathematics makes more sense and is easier to remember and to apply when students connect new knowledge to existing knowledge in meaningful ways (Schoenfeld & Arcavi, 1988; Adamson, 2005; Ball, 2005; Ma, 1999). In addition, well-connected, conceptually grounded ideas are more readily accessed for use in new situations (Skemp, 1976) and are an essential component of the knowledge needed to deal with novel problems and settings (NCTM, 2000).

Others (Ma, 1999; Stigler & Hiebert, 1999) have revealed that shallow subject knowledge restricts teachers’ capacity to promote conceptual learning among students. How well teachers know mathematics conceptually is central to their capacity to use instructional materials wisely, to assess students' progress, and to make sound judgments about presentation, emphasis, and sequencing (Ball, 2003). Teachers need to understand the big ideas of mathematics and be able to represent mathematics as a coherent and connected activity (Ma, 1999; Cooney, 1999; Mason & Spence, 1999). Finally, teachers
need to understand the different representations of a concept, the relative strengths and weaknesses of each, and how they are related to one another in order to facilitate the same understandings in their students (Wilson et al., 1987; NCTM, 2000).

Given the significance of teachers’ understanding of mathematical concepts and its affect on instruction, providing professional growth opportunities that focus on developing teachers’ conceptual knowledge are of utmost importance. Teachers cannot be expected to know or do what they have not had opportunities to learn for themselves. The current efforts to focus school mathematics instruction on students’ developing conceptual understanding of mathematics must also go hand-in-hand with teachers’ developing this knowledge of mathematics. Lewis (2002) cautions, however, that participating in professional development alone does not ensure enhanced content knowledge for teachers. Professional development must purposefully provide opportunities for teachers to learn more mathematics, even when the primary lens is on student thinking or curriculum (Sowder, 2005; NCTM, 2000).

For the present study, expanding conceptual knowledge required the careful development of a well-designed professional learning community that made a deliberate and sustained attempt to identify the conceptual knowledge needed for teaching mathematics and on understanding its specific uses in teaching. Consequently, a theoretical framework and definition of learning is adopted and displayed in Table 1 and was used to analyze the data and document a teacher’s conceptual knowledge growth in Chapter 6. This framework incorporates the many characteristics of conceptual knowledge as outlined in the research literature described above.
Table 1

Theoretical Framework: Conceptual Knowledge

<table>
<thead>
<tr>
<th>Learning</th>
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<tbody>
<tr>
<td>Ongoing action, perpetual curiosity, examination of data, and a continuous effort to deepen one’s knowledge base and refine instructional practice with the goal of enhancing student understanding (DuFour &amp; Eaker, 1998; Hord, 2004; Cox, 2004; NCTM, 2000)</td>
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</table>

L1: Conceptual Knowledge

Conceptual knowledge is characterized by having the ability to—

L1.1 recognize the central and peripheral ideas of mathematics and their relative importance (Schifter, 1999; Sowder, 2005; Shulman, 1987),

L1.2 provide reasoned responses and justify thought processes when problem solving (NCTM, 2000),

L1.3 build a concept upon earlier ideas, formulate connections within and across domains, and create further extensions (Ball, 2005; Santos-Trigo, 1998; Ma, 1999; Sowder, 2005),

L1.4 understand the different representations of a concept, the relative strengths and weaknesses of each, and how they are related to one another (NCTM, 2000; CBMS, 2001; Wilson, Shulman, & Richert, 1987; Ma, 1999; Cooney, 1999; Mason & Spence, 1999),

L1.5 articulate contextual meaning (NCTM, 2000; Gravemeijer & Doorman, 1999) and exhibit sense-making regarding a concept (Adamson, 2005; Simon & Tzur, 2004), and

L1.6 use knowledge flexibly by appropriately applying or adapting what is learned in one setting in another (NCTM, 2000).

Pedagogical Content Knowledge

One of the significant problems facing mathematics education is how to promote students’ development of new mathematical concepts. To meet this challenge, mathematics educators need an understanding of learning processes and the role of
mathematical tasks in the learning process. Mathematics educators need to design and use mathematical tasks to promote mathematical conceptual learning. Recent studies have reported that many teachers do not have a deep understanding of this kind of pedagogical content knowledge (Shulman, 1986; Stevenson & Stigler, 1992; Cooney et al., 1998; Ma, 1999; Lewis, 2002). Pedagogical content knowledge is a special aspect of practical teacher knowledge that entails understanding the intricacies of teaching a subject and ways of representing and formulating the subject that make it comprehensible to others. This includes “knowing” such things as the common conceptions students have, how to structure and represent concepts for teaching to students, how to use effective analogies, metaphors, or examples, and having knowledge of specific teaching strategies that can be implemented for the enhancement of student learning (Shulman, 1986, 1987).

As a consequence of this lack of knowledge in many teachers, the professional learning community is not designed simply for the purpose of producing individuals who know more mathematics. The goal is to improve students' learning which dictates that teachers' opportunities to learn must equip them with the mathematical knowledge and skill that enables them to teach mathematics more effectively (Ball, 2003). In other words, being able to do the calculations oneself or even knowing mathematical concepts deeply is insufficient for being able to respond well to a student who doesn't understand. Teachers need to have knowledge of mathematics that enables them to make the subject accessible to all students.

Relating the research to this study has established that the primary focus of the professional learning community needs to be on student learning rather than on teaching
(DuFour & Eaker, 1998; Louis et al., 1996). Teachers in the PLC need to recognize that teaching has not occurred until learning has occurred, and need to act accordingly as a community. Their emphasis, therefore, must not simply be on covering the required material but rather on engaging students in the consideration of essential content in ways that help them develop a deep understanding of that content (Ball, 2003). Heeding the call for developing a professional growth model that enhances pedagogical content knowledge in teachers, a portion of the PLC Theoretical Framework was developed to address this and is displayed in Table 2. The framework was subsequently used to characterize changes in a PLC participant’s pedagogical content knowledge as described in Chapter 6.
**Table 2**

*Theoretical Framework: Pedagogical Content Knowledge*

<table>
<thead>
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<th>Learning</th>
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<tbody>
<tr>
<td><strong>Ongoing action, perpetual curiosity, examination of data, and a continuous effort to deepen one’s knowledge base and refine instructional practice with the goal of enhancing student understanding</strong> (DuFour &amp; Eaker, 1998; Hord, 2004; Cox, 2004; NCTM, 2000)</td>
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<td><strong>L2: Pedagogical Content Knowledge</strong></td>
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<tr>
<td>Pedagogical content knowledge is characterized by having the ability to–</td>
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<tr>
<td>L2.1 imagine what it looks like for someone else to have an understanding of a concept (Silverman, 2005),</td>
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<tr>
<td>L2.2 articulate/reveal common conceptions students have regarding a particular concept (Shulman, 1987; NCTM, 2000; NRC, 2005),</td>
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<tr>
<td>L2.3 plan a roadmap for how others might acquire understanding of a concept by building upon their existing knowledge base (Silverman, 2005; Ball, 2003; NRC, 2005),</td>
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<tr>
<td>L2.4 use powerful and effective analogies, metaphors, representations, models, verbal descriptions, and examples in facilitating and enhancing the learning experience of others without taking over the process of thinking for them and thus eliminating the challenge (Shulman, 1987; NCTM, 2000),</td>
<td></td>
</tr>
<tr>
<td>L2.5 utilize both real-world experiences of students and purely mathematical contexts that are intriguing, challenging, and invite speculation and hard work by those with varied levels of expertise (NCTM, 2000), and</td>
<td></td>
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<tr>
<td>L2.6 accommodate and integrate students’ culture and language into the learning process and equitably provide effective learning experiences for all (NCTM, 2000).</td>
<td></td>
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</tbody>
</table>
Beliefs about Learning and Teaching Mathematics

Schoenfeld (1998) has argued that what a teacher decides to do in the classroom and why is dependent on the teacher’s knowledge, goals, and beliefs. The teacher’s beliefs about what is important about the subject matter, about learning, and about the students play a critical role in shaping what the teacher does, both in terms of planning and in terms of what happens “on the fly” as instruction takes place. Ernest (1994a) holds that a mathematics teacher’s mental contents or “schemas” includes knowledge of mathematics but also beliefs concerning mathematics and its learning and teaching. Knowledge of content and pedagogy is important, but the knowledge alone is not enough to account for the differences between mathematics teachers. Ernest (1994a) states that two teachers can have a similar knowledge base, but while one teaches mathematics with a problem solving orientation, the other has a more didactic approach which can be partly attributed to their individual beliefs. For these reasons, the beliefs that teachers hold about the learning and teaching of mathematics must be a strong consideration while constructing and implementing the professional learning community.

Carlson (1997) studied the beliefs about knowing and learning mathematics and developed a taxonomy to portray the range of beliefs that individuals can have. The Views about Mathematics Survey (VAMS) Taxonomy (Appendix D) is divided into two broad dimensions: epistemological and pedagogical. The epistemological dimension pertains to the structure and validity of mathematical knowledge. The pedagogical dimension pertains to the methods of mathematics including the role of reflective critical thinking and the personal relevance of mathematics. The VAMS Taxonomy is presented
in the form of contrasting views in each of six aspects (Structure, Methodology, Validity, Learnability, Critical Thinking, and Personal Relevance). For this study, the pedagogical dimension is included in the PLC Theoretical Framework and incorporates only the Learnability, Critical Thinking, and Personal Relevance aspects of the VAMS Taxonomy because they tie to the learning of mathematics directly and the goals of this study.

The researcher came to believe during data analysis (see Chapter 4) and further study of the body of research that the PLC Theoretical Framework must also include Instructional Philosophy as the fourth aspect of participants’ beliefs about learning and teaching mathematics. Beliefs regarding the learning of mathematics and its teaching form the basis of one’s philosophy of mathematics although some teacher's views may not have been elaborated into fully articulated or even consciously held philosophies (Ernest, 1994a; Schoenfeld, 1989). The Instructional Philosophy aspect of participating teachers’ beliefs describes the teacher's conception of the type and range of teaching roles, actions and classroom activities associated with the teaching of mathematics.

The inclusion of Instructional Philosophy is an important addition to the framework because it is possible for teachers to practice classroom teaching techniques that are not in alignment with their professed teaching philosophy regarding how students learn. Ernest (1994a) discovered that a potential cause for this is the teacher’s level of consciousness regarding his/her own beliefs about each aspect of his/her teaching philosophy regarding important issues. Some mathematics educators are not conscious of what they believe or where they stand on mathematical matters so they are not as capable of making informed decisions about how to align their beliefs and teaching practices
(Schoenfeld, 1989; Sosniak, Ethington, & Varelas, 1991). Others may be aware of their beliefs but do not take the time to reflect in order to see if their practice and beliefs coincide. To von Glasersfeld (1991), reflection is the ability of an individual to “step out of the stream of directed experience, to re-present a chunk of it, and to look at it as though it were direct experience, while remaining aware of the fact that it is not” (p. 47). Cooney (1999) suggests that an effort to improve instruction begins with the teacher reflecting on what mathematics means to him/her and how he/she envisions its teaching. “What seems critical is that teachers see something problematic about the doing, teaching, and learning of math” (p. 175).

As the research suggests, the importance of participants’ beliefs necessitates that a professional learning community confront and challenge the beliefs teachers hold about learning and teaching mathematics. The PLC Theoretical Framework that characterizes the belief structure is presented in the form of contrasting views in Table 3 by continuing the format of the VAMS Taxonomy and includes different aspects of the beliefs laid out in the literature.
Table 3

Theoretical Framework: Beliefs About Learning and Teaching Mathematics

| **Learning** |  
|---|---|
| **Ongoing action, perpetual curiosity, examination of data, and a continuous effort to deepen one’s knowledge base and refine instructional practice with the goal of enhancing student understanding** (DuFour & Eaker, 1998; Hord, 2004; Cox, 2004; NCTM, 2000) |  
| **L3: Beliefs about Learning and Teaching Mathematics** |  
| **L3.1 Learnability:** (Carlson, 1997) |  
| a. Mathematics is learnable by anyone willing to make the effort rather than by a few isolated people. |  
| b. Achievement depends more on persistent effort than on the influence of the teacher or textbook. |  
| **L3.2 Critical Thinking:** (Carlson, 1997) |  
| For meaningful understanding of mathematics, one needs to-- |  
| a. concentrate more on the systematic use of general thought processes rather than on memorizing isolated facts and algorithms, |  
| b. examine situations in many ways, and not feel intimidated by committing mistakes rather than follow a single approach from an authoritative source, |  
| c. look for discrepancies in one’s own knowledge instead of just accumulating new information, and |  
| d. reconstruct new knowledge in one’s own way instead of memorizing it as given. |  
| **L3.3 Personal Relevance:** (Carlson, 1997) |  
| a. Mathematics and related technology are relevant to everyone’s life rather than being of exclusive concern of mathematicians. |  
| b. Mathematics should be studied more for personal benefit than for just fulfilling curriculum requirements. |  
| **L3.4 Instructional Philosophy:** |  
| For effective mathematics instruction to occur, a teacher needs to— |  
| a. focus primarily on student understanding of principle concepts and ideas instead of on procedures and quick computation (Sowder, 2005; NRC, 2005), |  
| b. build upon students’ prior knowledge and personal experience rather than presenting material without regard to individual meaning and sensemaking (Dewey, 1933; von Glaserfeld, 1990; Boaler, 1993; Gravemeijer & Doorman, 1999; Adamson, 2005), |  
| c. purposefully provide tasks that offer diverse and appropriate challenges rather than striving to alleviate the struggle and ambiguity of doing mathematics (Santos-Trigo, 1998; Cooney, 1993), |  
| d. design instruction that requires active participation by students and encourages curiosity, creativity, investigation, collaboration, and healthy questioning rather than only expecting that students listen intently, take good notes, follow directions and examples, behave themselves, and do the assigned work (Lawson et al., 2002; Ernest, 1991; Cooney, 1993), |  
| e. require students to communicate their thinking with others in a variety of ways rather than simply providing to the teacher correct written solutions to problems that they individually arrive at (NCTM, 2000; NRC, 2005), and |  
| f. facilitate students’ self-monitoring by teaching them to critically reflect, estimate, and assess their own understanding rather than looking only to an authoritative source for verification (NRC, 2005). |  

Conclusion

The purpose of this chapter was to provide an appropriate lens by which the data presented in Chapter 6 can be analyzed. Utilizing this framework, the professional growth of a PLC participant is described with respect to her conceptual knowledge, pedagogical content knowledge, and beliefs about learning and teaching mathematics.
CHAPTER 4: METHODOLOGY

In this chapter, the methodology used in this dissertation study is described. The reader is provided with a sense of how this study was undertaken as well as the rationale for certain decisions regarding its design and implementation. A full description of the method of inquiry, the setting and participants, and the methods of data collection and analysis are presented. In short, this chapter explains the principles and techniques that were used to create and implement a professional learning community and to study the effects the PLC experience had on a participating secondary precalculus teacher.

Method of Inquiry

This study provides a detailed case study of one PLC teacher’s growth in the areas of conceptual knowledge, pedagogical content knowledge, and beliefs about the learning and teaching of mathematics specifically with regard to aspects of the function concept. The dissertation focuses on the function concept due to its importance in precalculus and calculus (NCTM, 2000; Zandieh, 2000; Carlson et al., 2002) and the desires of the study’s participating precalculus teachers. The teachers chose to investigate six aspects of the function concept: transformations, rate of change, logarithmic functions, trigonometric functions, composition of functions, and rational functions. Rather than casting a broad research net over many facets of the function concept, the feature that is given prominence in this written report is evaluating trigonometric functions measured in radians. The study follows the trigonometric function cycle of investigation of the PLC (outlined in Chapter 5) and includes the case study of one participating PLC teacher (described in Chapter 6).
Setting

This section provides a portrayal of the school setting as well as the mathematics curriculum used at this school, including a brief outline of the precalculus program.

School

The study was conducted at a secondary school of grades 9-12 located in a large city in the southwestern United States with just over 2,500 students and nineteen mathematics instructors. The school has been in existence since 1993. The community in which this school is situated is generally regarded as one of the most affluent suburbs within its greater metropolitan area and is experiencing tremendous population growth. In the academic year 2002-2003, 52% of the school’s teachers had a Master’s degree with the largest percentage (42%) having only three or fewer years of teaching experience. This school was chosen for the study because some teachers have previously participated in summer professional development workshops at the supporting research institution, there are multiple teachers who teach at the precalculus level, and the teachers have shown interest in ongoing professional improvement. In the year the study was conducted (2003-2004), the school reported an attendance rate of 97% versus 94% for the state where the school is located and a 0% dropout rate compared to 8.5% across all state high schools. In the same academic year, the school scored the highest on the state standardized examination in mathematics.
School Mathematics Curriculum and the Precalculus Program

The course offerings in mathematics at the school are “integrated” in nature meaning that they incorporate different aspects of mathematics in one course (i.e. geometry, trigonometry, statistics, etc.) The titles of the courses are: Math Topics 1/2, Math Topics 3/4, Honors Math Topics 3/4, Math Topics 5/6, Honors Math Topics 5/6, Precalculus, Honors Precalculus, Calculus I, AP/Honors Calculus I, Calculus II, and AP/Honors Calculus II. Calculus III and Differential Equations are offered to the students through a partnership with a local community college. Graphing calculators, but not computer algebra systems, are used extensively throughout all mathematics courses above the Math Topics 1/2 course. The reason given by the department chair for not allowing any technology at the Math Topics 1/2 level was because the students can’t use calculators on the standardized state assessments that are required for graduation.

Textbooks by the John Wiley & Sons Company are employed in both the Precalculus and Calculus courses: Functions Modeling Change and Calculus respectively. The precalculus program at this school is divided into “regular” and “honors” tracks. Honors students are determined by a placement exam or teacher recommendation. Both groups use the same textbook with the difference between the two courses primarily being that the honors classes cover more material. There are ten sections of regular Precalculus at the school taught by four different teachers and three additional sections of Honors Precalculus taught by a single instructor. The precalculus teachers have constructed a common final examination that is revised after each school year to reflect the goals of the department. The school’s availability of conceptually-
focused curriculum, the desire of teachers to pursue ongoing professional development, and the philosophy of the mathematics department were important factors considered in the site selection process.

Professional Learning Community Participants

In this section, an explanation is provided of how the professional learning community was formed and how teacher participants were selected. A description is also presented of the research team that was in place to initiate, implement, and study the PLC. This section concludes with an illustration of the participating teachers.

Preliminary Communication

A meeting was held at the end of the spring semester prior to the study’s implementation between the researcher, an experienced mathematics education researcher from the supporting university, and the school’s mathematics department chair. The discussion centered on the motivation for the study and the requirements expected of participating teachers. The primary goals of the project were relayed as a way to help teachers enhance their students’ understanding of precalculus mathematics through an innovative professional development opportunity—professional learning community—for precalculus teachers. The supporting university committed to offer graduate credit to teachers for their participation. Additional incentives were made available for state teacher recertification and/or district salary advancement. After the meeting, the department chair recruited and selected the teachers for participation in the PLC. The professional learning community commenced in the fall semester of 2003 and continued in the subsequent spring semester of 2004.
Research Team

The researcher also served as the main facilitator for the PLC. I am a veteran mathematics teacher of nineteen years. Fourteen years were spent at the secondary level, and a total of seven years have been spent teaching community college students, either in an adjunct role or, as in the past five years, as a full-time teacher. I regularly teach Intermediate Algebra, College Algebra, Trigonometry, Precalculus, Calculus I, and Calculus II. Prior to this study, I participated in an extensive research project investigating the factors that influence secondary mathematics teachers’ pedagogical decision-making processes and the effectiveness of their chosen methods of instruction. As a way to prepare to be part of the research team before instituting the study and facilitating the PLC, I participated in the “New Faculty Learning Community Developers’ Institute” at California Polytechnic University – Pomona directed by Milt Cox from Ohio University. Through my doctoral studies, I have participated in various projects that focused on students’ conceptual understanding of function.

In addition to the facilitator/researcher, an observer/facilitator participated in the PLC who has recent experience as a leader in a formal “lesson study” program and has mathematics teaching experience at the junior high level. (More details on the two facilitators’ roles are found in Chapter 5.) Throughout the study the facilitators accessed support from two expert mathematics education researchers. These team members are faculty from the department of mathematics at the supporting university. Both have extensive backgrounds investigating student understanding of precalculus mathematics and have experience teaching and developing research-based curriculum for the course.
These individuals served as consultants but did not play an active role in the PLC meetings.

**Participating Teachers**

The participants included five female, secondary precalculus teachers (including the mathematics department chair) and one female who taught the course in the year following the study. Their (pseudo) names are Jeanne, Loretta, Cooper, Diana, Lorene, and Mary Beth. The six participants’ teaching experience ranged from five to sixteen years, and all had their Bachelor’s degree in Secondary Education and a major in mathematics. Five of the six teachers have Master’s degrees, and all have participated in additional professional development either through taking advanced coursework, teacher workshops, conferences, or district-led growth opportunities. A description of each of the six participating teachers follows.

Jeanne is the school’s department chair and was the first mathematics teacher hired when the school was opened in 1993. The principal hand-selected her out of another district school to establish the Mathematics Department. She has served continually as the department chair since the school opened. In her twelve-year tenure as chair, Jeanne has been the primary influence on the curriculum and personnel decisions of the school’s mathematics department. Jeanne is a self-described “disciple” of the reform movement. Her strong desire for teachers to facilitate student understanding of mathematics and her commitment to ongoing professional development have been set firmly in the philosophy of the department from its inception. Jeanne did not go into teaching immediately after graduating from college because she wanted to be a wife and
stay-at-home mother first. As a result, her teaching career did not begin until she was forty years of age. She had sixteen years of teaching experience at the time of the study and a Master’s degree. Jeanne is the teacher that was chosen for a case study in this research project because of her strong influence over the other teachers in the department and her deeply entrenched beliefs about the learning and teaching of mathematics. In addition, she provided valuable information, experience, and guidance in the implementation and facilitation of the PLC. (A more detailed characterization of Jeanne is provided in Chapter 6.)

At the time of the study, Loretta was a teacher with ten years of experience and was teaching Precalculus and Math Topics 5/6. She went into the teaching profession after five years of working in the medical field managing records. Loretta shared her reason for becoming a teacher: “I turned to teaching as an adult and came to teaching after I had already done other things. When I went back to school to finish up my degree, the math was just so much fun at that point. I chose to become a math teacher because as an adult it sang to me. I don’t know how else to describe it but that I saw it as being such a creative process, and it wasn’t just numbers and rules. To me it was creativity and how you could express things and show things. As an adult I felt very excited about that but as a student in high school I was not.” Loretta is eager to learn and progress as a teacher through professional development. Her genuine excitement to participate in the PLC was apparent from the outset of the study.

Cooper transferred to the school when it opened. She transferred from the same district secondary school as Jeanne. Cooper stated that she did so primarily because her
teaching philosophy is in alignment with Jeanne’s, and she wanted to go to a school
where she could teach “as she wanted to”. At the time of this study, Cooper had fourteen
years of teaching experience, a Master’s degree, and had taught at all different levels of
mathematics. She was teaching Honors Precalculus and Honors Calculus during the
study. From the pre-PLC interviews it was clear that Cooper had some very strongly-held
beliefs with regard to the development of teachers. Reflecting on her preservice training,
she felt that it was “a joke” because none of her training applied (except for
administrative tasks such as recording grades) to what she does in her job as a classroom
teacher. According to Cooper, “I seriously thought that these college professors who are
‘professionals’ must have sat around at happy hour and came up with what they teach and
assign (in preservice training) because it didn’t seem like there was any thought given,
and if there was thought, then it was all in their head because they haven’t been in the
day-to-day.” Cooper’s beliefs and headstrong personality traits were factors the
facilitators had to consistently deal with in the PLC. The manner in which Cooper
affected the development and implementation of the PLC is discussed in Chapter 5.

At the time of the study, Lorene was an experienced teacher of fifteen years and
also had a Master’s degree. She had taught Precalculus for ten years, and during the study
was teaching Precalculus and Calculus. Her stated reason for PLC participation was to
share instructional ideas with her colleagues and to earn more professional development
hours for salary advancement. She pointed out early that she had gone through some bad
professional development in the past and was curious to see if the PLC would be any
better.
Diana and Mary Beth were the other two participating PLC teachers. At the time of the study, Diana had five years of teaching experience and a Master’s degree. She was teaching Math Topics 5/6 and Precalculus for the first time. Her desire was to learn precalculus mathematical content better through the PLC experience and “to get some good ideas from the others who have taught it before”. Before the PLC, Mary Beth had taught for eight years but never Precalculus. She was the only teacher in the study without her Master’s degree. Her motivation for participation in the precalculus PLC was to learn from her experienced colleagues and to prepare for teaching the course for the first time in the following school year. She also liked that a professional development opportunity was available at her own school site with her fellow teachers and that participating in the PLC would help her earn a Master’s degree.

Method of Data Collection

Recent work done on professional development programs for teachers provided insight into the process of collecting and analyzing data for this study. Lachance and Confrey (2003) performed a professional development intervention that attempted to start a professional community among secondary education mathematics teachers through inservice work on mathematical problem solving and technology. Because their outcomes were highly dependent on the context and because the researchers anticipated that the participants would have many different perspectives on the intervention, a wide variety of sources (interviews, surveys, teaching portfolios, classroom observation, and videotape) were employed for data collection (Lachance & Confrey, 2003). Similarly, this research study utilizes a wide variety of data sources to draw a rich picture of Jeanne’s growth.
with respect to her conceptual knowledge, pedagogical content knowledge, and beliefs as well as to describe the emergence and effectiveness of the PLC structure and support tools.

Both qualitative and quantitative data were collected in this study. The study utilized primarily the qualitative data but the quantitative data served as supplemental information and was valuable in helping to characterize Jeanne’s classroom practices and to compare her conceptual knowledge to other precalculus teachers. The following data were collected:

- weekly meeting observations and reflections,
- individual teacher interviews,
- professional learning community artifacts, including developed, activities/problems, individual participant work, and agendas,
- teacher assessment, and
- classroom observation.

*Weekly Meeting Observations and Reflections*

Over the period of data collection, PLC discussions from thirty meetings were audiotaped and fully transcribed. Facilitator observational data from the weekly meetings were also compiled. This data consisted primarily of the fieldnotes that the members of the facilitation team created immediately after each session in a post-PLC debriefing meeting. Furthermore, weekly email correspondences between the two facilitators were gathered to more formally organize the data. This was done to corroborate and improve the accuracy of the real-time data collected as well as to promote concurrence or rejection
of any preliminary assertions made regarding occurrences in the PLC. Additional assistance and input were sought periodically concerning the observations that had been made from the two expert mathematics education researchers supporting this research study. This information was then used to plan and improve subsequent PLC sessions.

Occasionally, the fieldnotes were supplemented with other forms of data produced by the researchers and participating members of the PLC. Participant reflections, reactions, and email correspondences were found to be helpful in determining how well individual teachers believed the PLC was performing and in capturing individual participant’s growth. While requiring teachers to reflect, via email, on their learning, they were able to provide meaningful data that was used to improve the PLC experience. The corpus of observational data produced a summary of the activities, conversations, and learning that took place during the weekly meetings. The data captured the facilitators’ initial interpretations of the interactions between participants and regarding the effectiveness of the PLC structure and tools in garnering professional growth.

*Individual Teacher Interviews*

Clinical interviews were conducted with each teacher participating in the professional learning community and were audio recorded. Before the study commenced, two one-hour pre-PLC interviews (Appendix E) were conducted to establish baseline teacher data and included use of the Nature of Mathematics (NOM) (Appendix F) and the Nature of Mathematics Education (NOME) (Appendix G) surveys. The audio recordings were reviewed and transcribed. These data were used to help characterize the teachers’
initial beliefs about the learning and teaching of mathematics. The information was also used to document the different factors that have enhanced or hindered the PLC participants’ past professional development. Two one-hour post-PLC interviews (Appendix H) were conducted at the end of the study with each teacher to draw a clearer overall picture of the teachers’ PLC experiences. The teachers were asked to reflect over the year and to provide feedback to the researcher regarding their perceived professional development. To detail their growth, the exit interviews included questions to probe the teachers’ conceptual knowledge, pedagogical content knowledge, and beliefs.

_Professional Learning Community Artifacts_

Artifact collection included the PLC-developed materials and activities to determine if the tasks that were required of students were more conceptually-based and focused on enhancing student understanding than the materials that the participants had been using in their classrooms. Additional artifacts studied from the PLC included the weekly agendas. The agenda for each of the hour-long sessions was developed by the facilitators to provide direction for the meeting and used as a record keeping tool as the discussions took place. The record of the PLC transactions that each agenda provided was used to determine what the important elements and obstacles were in creating and implementing a PLC and to mark the evolution of the PLC structure and tools.

_Teacher Assessment_

A quantitative instrument used to assess teachers’ conceptual knowledge of the function concept and to allow for a comparison of Jeanne to a larger population of precalculus teachers was the Precalculus Concept Assessment (PCA) (Carlson et al.,
Carlson has conducted in-depth investigations of students’ and teachers’ understanding of the concept of function (Carlson, 1995, 1998, 2002). As a result of these studies, researchers (Carlson et al., 2002) have developed a framework to define the precalculus level understanding of the function concept. This team of researchers (Carlson et al., 2002) has constructed The Precalculus Concept Assessment (PCA) Taxonomy (Table 4). The PCA describes the major reasoning abilities and conceptual understandings of precalculus mathematics that have been reported in the research as foundational for understanding the concept of function (Breidenbach et al., 1992; Monk, 1992; Carlson, 1998). Included in the reasoning abilities is the process view of function, an important “way of thinking” that has previously been documented as central to gaining a mature function understanding (Breidenbach et al., 1992; Dubinsky & Harel, 1992; Sfard, 1992; Carlson, 1998).
Table 4

*Precalculus Concept Assessment (PCA) taxonomy*

<table>
<thead>
<tr>
<th>Reasoning Abilities</th>
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<tbody>
<tr>
<td>R1</td>
<td>Apply proportional reasoning</td>
</tr>
<tr>
<td>R2</td>
<td>View a function as a process that accepts input and produces output</td>
</tr>
<tr>
<td>R3</td>
<td>Apply covariational reasoning</td>
</tr>
<tr>
<td></td>
<td>• Coordinate two varying quantities while attending to how the quantities change in relation to each other</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Conceptual Abilities</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>Interpret function information (table, formula, graph)</td>
</tr>
<tr>
<td></td>
<td>C1P At points (t, f, g)</td>
</tr>
<tr>
<td></td>
<td>C1I On intervals of the domain (t, f, g)</td>
</tr>
<tr>
<td></td>
<td>C1D By attending to domain restrictions inherent in the function (t, f, g)</td>
</tr>
<tr>
<td>C2</td>
<td>Represent contextual function relationships using function notation (table, formula, graph)</td>
</tr>
<tr>
<td></td>
<td>C2V Identify, define and relate variable quantities as functional relationships (t, f, g)</td>
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<tr>
<td></td>
<td>C2C Use function composition to relate specified quantities (t, f, g)</td>
</tr>
<tr>
<td>C3</td>
<td>Perform function operations (table, formula, graph)</td>
</tr>
<tr>
<td></td>
<td>C3E Function evaluation (t, f, g)</td>
</tr>
<tr>
<td></td>
<td>C3A Function arithmetic (t, f, g)</td>
</tr>
<tr>
<td></td>
<td>C3C Function composition (t, f, g)</td>
</tr>
<tr>
<td></td>
<td>C3I Function inverse (t, f, g)</td>
</tr>
<tr>
<td></td>
<td>C3T Function translations (t, f, g)</td>
</tr>
<tr>
<td>C4</td>
<td>Understand how to reverse the function process (table, formula, graph)</td>
</tr>
<tr>
<td></td>
<td>C4E Solve equations that involve functional relationships (t, f, g)</td>
</tr>
<tr>
<td></td>
<td>C4IN Solve inequalities that involve functional relationships (t, f, g)</td>
</tr>
<tr>
<td></td>
<td>C4IF Understand the meaning of an inverse function (t, f, g)</td>
</tr>
<tr>
<td>C5</td>
<td>Interpret and represent function behaviors (table, formula, graph)</td>
</tr>
<tr>
<td></td>
<td>C5L Linear (t, f, g)</td>
</tr>
<tr>
<td></td>
<td>C5P Polynomial (t, f, g)</td>
</tr>
<tr>
<td></td>
<td>C5R Rational (t, f, g)</td>
</tr>
<tr>
<td></td>
<td>C5E Exponential (t, f, g)</td>
</tr>
<tr>
<td></td>
<td>C5L Logarithmic (t, f, g)</td>
</tr>
<tr>
<td></td>
<td>C5T Trigonometric (t, f, g)</td>
</tr>
<tr>
<td>C6</td>
<td>Interpret and represent rate of change information for a function (table, formula, graph)</td>
</tr>
<tr>
<td></td>
<td>C6C Interpret the relative growth of the input and output variables (t, f, g)</td>
</tr>
<tr>
<td></td>
<td>C6A Determine average rate-of-change (t, f, g)</td>
</tr>
<tr>
<td></td>
<td>C6I Interpret rate-of-change information on intervals of the domain (t, f, g)</td>
</tr>
<tr>
<td></td>
<td>C6E Understand multiplicative rate-of-change (exponential growth) (t, f, g)</td>
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</tbody>
</table>
The PCA taxonomy provided a foundation for the development of the Precalculus Concept Assessment (PCA) instrument, a multiple choice instrument of which one version was used for this project to assess the teachers’ conceptual knowledge of function. Each of the 125 items in the PCA item pool provides information about strengths and weaknesses relative to specific components of the PCA Taxonomy. Each item has five answer choices that include the correct answer and common incorrect responses that have emerged in past research, and from multiple cycles of interviews with students. Each PCA item has undergone multiple cycles of revision to assure that the: (1) wording of the question is clear and consistently interpreted according to the design intent, (2) selection of the correct answer consistently reflects the understanding assessed, and (3) selection of each of the incorrect answers represents a common incorrect response consistently provided by students. PCA version G, Form 1 with its twenty-five items as selected from the larger pool of 125 was chosen for this study because it had the most data available for comparative analysis.

Classroom Observation

Finally, each teacher was observed early in the study for the purpose of understanding the classroom setting that the participants worked in and to corroborate information gathered from the interviews, surveys, and weekly meetings. Two additional fifty-five minute classroom observations were conducted during the year-long study using the Reformed Teaching Observation Protocol (RTOP) instrument (Appendix I) while the participating teachers implemented PLC-developed activities and problems. The observations were done so the facilitators could use the data gathered to inform the
development of subsequent PLC sessions. The RTOP was created by the Evaluation Facilitation Group (EFG) of the Arizona Collaborative for Excellence in the Preparation of Teachers (ACEPT). The RTOP is an observational instrument designed to measure the level of “reformed” teaching taking place in mathematics classrooms (ACEPT, 2000). The protocol consists of twenty-five items divided into three subsets: Lesson Design and Implementation, Content, and Classroom Culture. The second and third subsets are each divided into two smaller groups of five items. The first subset was designed to capture what had become the ACEPT model for reformed teaching. It depicts a lesson that begins with recognition of students’ prior knowledge and preconceptions, that attempts to engage students as members of a learning community, that values a variety of solutions to problems, and that often takes its direction from ideas generated by the students. The second subset was directed at content and was divided into two parts. The first part assessed the quality of the content of the lesson, and the second part attempted to capture the ACEPT understanding of the process of inquiry. The final subset, consisting of ten items, was directed at the climate of the classroom. The researchers’ intention was to capture the full range of reformed teaching that occurred in the participants’ classrooms as defined by ACEPT with these twenty-five items.

Method of Data Analysis

The data analysis consisted of seven phases as revealed below.

Phase 1

In Phase 1, data analysis consisted of the weekly meetings held by the facilitator/researcher and the observer/facilitator following each PLC session. All of the
observational data collected was synthesized immediately after each weekly learning community session by the facilitators discussing each point of the agenda in a post-PLC debriefing meeting. Both of the facilitators’ reactions, thoughts, and suggestions were recorded and summarized. In addition, the observer/facilitator’s notes taken during the debriefing were compiled the following day and sent to the facilitator/researcher for additional input. The correspondences between the two facilitators were used to formally organize the data, promote concurrence or rejection of any preliminary assertions, and to enhance the subsequent PLC sessions. PLC participating teachers’ periodic reflections regarding their learning and the functioning of the community were incorporated into the planning of future PLC sessions as well. Bearing in mind that it is important to provide professional growth that teachers deem worthwhile and relevant, the participants’ input was highly valued by the facilitators and had a large impact in the PLC evolution process.

Phase 2

Cobb, Stephan, McClain, and Gravemeijer (2001) designed an interpretive framework for analyzing student mathematical activity and learning in a classroom context and noted its potential use as “an analytic tool for and means of supporting the development of professional learning communities” (p. 155). To accommodate the design of this study, the researcher modified the framework to better describe professional learning communities for precalculus teachers (see Table 5).
Table 5.

Precalculus Professional Learning Community Framework

<table>
<thead>
<tr>
<th>Social Perspective</th>
<th>Psychological Perspective</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLC social norms</td>
<td>Beliefs about the Learning and Teaching of Mathematics:</td>
</tr>
<tr>
<td></td>
<td>Beliefs about own role, others’ roles, and the general nature of mathematical activity in school</td>
</tr>
<tr>
<td>PLC sociomathematical norms</td>
<td>Conceptual Knowledge:</td>
</tr>
<tr>
<td></td>
<td>Mathematical interpretations and reasoning relative to a specific mathematical concept</td>
</tr>
<tr>
<td>PLC mathematical practices</td>
<td>Pedagogical Content Knowledge:</td>
</tr>
<tr>
<td></td>
<td>Relative to a specific mathematical concept</td>
</tr>
<tr>
<td>Community interaction</td>
<td></td>
</tr>
</tbody>
</table>

In Phase 2 of the data analysis, every participant response in all interviews and PLC meetings was coded into one of the components of the framework. This exhaustive technique enabled the researcher to gain a global and comprehensive view of the data. This process also helped the researcher discover the most notable sources of data for further study and analysis and resulted in the researcher more narrowly focusing on one set of five PLC sessions (16 – 20) and also on one individual participant (Jeanne) as a case study. However, as the coding process using this framework unfolded, the researcher and one of the expert mathematics education researchers concluded that it did not adequately serve the needs of this study. The two researchers noted that the PLC implemented and investigated in this study was more complex than what this framework would accommodate, and it became evident that a more productive instrument was needed. This realization led to the development of the PLC Theoretical Framework that subsequently comprised Phase 3 of the data analysis.
Phase 3

Phase 3 consisted of the researcher initially developing the PLC Theoretical Framework (Appendix C). To structure the framework, four areas for teacher growth were identified from the literature: conceptual knowledge (Carlson et al., 2002; Ball, 2005), pedagogical content knowledge (Ma, 1999; Shulman, 1989), beliefs about the learning and teaching of mathematics (Ernest, 1994), and community (Cox, 2004). For each of these four professional development areas, some of the desired outcomes for the PLC participants were identified from the body of literature and the knowledge of practice, while others emerged from this study’s early data analysis phases. Drawing from this body of literature, the researcher grouped the outcomes that were observed to be highly related to one another into clusters. This process resulted in new components being created, omitting components, rewording components, and consolidating components. The researcher consulted a team of five expert mathematics education researchers—the PLC Research Team—from the supporting university throughout the process for feedback and help in fine-tuning the framework. As a result, a total of approximately 120 initial PLC desired outcomes and beliefs were clustered into the four teacher growth areas and were consolidated into the current list of six desired outcomes for conceptual knowledge, six desired outcomes for pedagogical content knowledge, fourteen beliefs about the learning and teaching of mathematics, and ten characteristics of community. (Chapter 3 provided a more detailed description of the PLC Theoretical Framework and its components.)
**Phase 4**

The scope of data analysis narrowed in Phase 4. The researcher focused attention on the specific PLC sessions that contained the most informative data. This process led the researcher to look closely at PLC sessions 16 to 20 specifically with an eye toward understanding the broader themes and issues illuminated by the analyses in earlier phases and providing insight into the study’s research questions. The noted PLC sessions for further study were the first meetings of the second semester. Participant reflections at the conclusion of the first semester evaluating the effectiveness of the PLC and soliciting suggestions had provided the facilitators with insight and guided the development of the subsequent PLC sessions.

In this phase, the transcribed data for PLC sessions 16 – 20, the relevant interview excerpts for Jeanne, and the facilitators’ observations were analyzed by manually coding the qualitative data into the major components of the PLC Theoretical Framework and its subcomponents. The coding was done for each piece of the data by recording the date/session of occurrence, the person the data pertained to, the component, and any related comments made by the facilitators. (A sample coding form is included in Appendix J.) Professional growth instances explicitly cited by Jeanne or subjectively inferred by the researcher were so noted. Analysis included looking for emergent patterns in the data with respect to Jeanne’s conceptual knowledge, pedagogical content knowledge, beliefs about the learning and teaching of mathematics, and verbalized classroom practices. Phase 4 was when the researcher noticed that Jeanne’s behavioral traits (e.g. independence, impatience, etc.) were emerging as an important factor that
largely impacted her classroom decision-making processes. This facet had not been previously incorporated into the working model of the PLC Theoretical Framework but was added during this phase of analysis. Chapter 6 provides greater detail on this aspect of the emergent framework and its components are displayed in Table 10.

**Phase 5**

Phase 5 consisted of triangulation of the data coding. Two experts independently coded a sample of the data using the approach specified above. These experts, both of whom are teachers and researchers of mathematics, were experienced in using a similar approach to coding data. I refer to the expert coders as C1 (coder 1) and C2 (coder 2). For the purposes of establishing validity and reliability in the coding, both coders coded PLC session 15. This task was performed after C1 and C2 had become familiar with sample data from the preceding (14) and following (16) PLC sessions and the PLC Theoretical Framework. Before coding took place, a meeting was held to discuss the meaning of each of the components in the framework. Following this discussion, each coder, including the researcher, proceeded to code the data from PLC session 15.

There was not total agreement, initially, because in more than one instance each coder named a component that the others had not named. In other instances, however, there was complete agreement. When there was disagreement, the three coders discussed the differences and concluded that the discrepancies in coding were primarily comprised of possible multiple or secondary codings. For example, the statement by Loretta: “We need to get out the wicky sticks activity where they can cut out radians on the unit circle.” was coded by all three coders as the second (L2) professional growth area (Pedagogical
Content Knowledge) and the fourth (L2.4) component (Pedagogical content knowledge is characterized by having the ability to use powerful and effective analogies, metaphors, representations, models, verbal descriptions, and examples in facilitating and enhancing the learning experience of others without taking over the process of thinking for them and thus eliminating the challenge). However, C1 and the researcher also included coding this statement as the third (L3) professional growth area (Beliefs about Learning and Teaching Mathematics) and the first (L3.4a) component (For effective mathematics instruction to occur, a teacher needs to focus primarily on student understanding of principle concepts and ideas instead of on procedures and quick computation). C2 did not include the second code initially, but after listening to the rationale for including the additional label, he concurred.

Although it would have been desirable to repeat and refine this process with other PLC sessions, time constraints made this impossible. The primary benefit of the verification process just mentioned was increased confidence about the reliability and validity of the coding method for the study and also in the comprehensiveness of the PLC Theoretical Framework.

Phase 6

In Phase 6, the researcher took a final pass through the qualitative data regarding Jeanne after it had been manually coded into the PLC Theoretical Framework in Phase 4. For a second time, analysis was done to see what growth patterns emerged and how Jeanne was affected as the PLC progressed over time. In this phase of the analysis, assertions that had been made in earlier phases were challenged by looking for
contradictory evidence. Any prior claims that were deemed to not be sufficiently supported with evidence were disregarded or noted as possible areas for future research. Jeanne’s self-reported conflict between her behavioral traits and classroom practices was documented through this process. A more robust characterization of Jeanne’s growth in conceptual knowledge, pedagogical content knowledge, and beliefs about learning and teaching mathematics resulted from this analysis.

**Phase 7**

In the final phase, Phase 7, the findings from analyzing the qualitative data were verified and supplemented with information gleaned from the different quantitative data collected. The findings from the quantitative data supported the characterization of Jeanne’s conceptual understanding and classroom practices. More specifically, to further detail and substantiate the description of her conceptual understanding of the function concept the Precalculus Concept Assessment (PCA) test results were analyzed, and to illustrate Jeanne’s classroom teaching Reformed Teaching Observation Protocol (RTOP) scores were investigated.

Jeanne’s PCA score was used as comparative data with approximately 100 precalculus teachers who had previously taken the exam making it possible to discuss her conceptual knowledge in relationship to a wider population of teachers. Similarly, analyzing Jeanne’s RTOP scores as outlined in the RTOP manual (ACEPT, 2000) and comparing to the findings of Adamson et al. (2003) allowed the researcher to place her into a wider population of teachers with regard to the level of reform instruction taking place in her classroom.
CHAPTER 5: THE EMERGENT PROFESSIONAL LEARNING COMMUNITY

Introduction

According to the National Staff Development Council (2003), the most powerful forms of staff development occur in collaborative teams that meet on a regular basis for the purposes of learning, joint lesson planning, and problem solving. The literature is quite extensive in suggesting that the most promising strategy for sustained, substantive teacher improvement is developing the ability of teachers to function in teams known as professional learning communities (PLCs). In this study, a precalculus PLC was established with six participating teachers around important components requiring teacher challenge and growth including conceptual knowledge, pedagogical content knowledge, and beliefs about the learning and teaching of mathematics.

The PLC met weekly in one of the participating teachers’ classrooms on Tuesday afternoons from 2:30 – 3:30 p.m. Generally, the thirty-two sessions were held in the same room but occasionally had to be moved to a different location. As shown in figure 1, eight desks (six for participating teachers and two for facilitators) in the classroom were arranged in a semi-circle facing the front board to encourage participant participation, collaboration, and discourse. The two facilitators sat in desks adjoining one another to facilitate communication during the meetings. The other participants sat in the remaining seats in random fashion as they arrived for each session. An audio taping device was located on a desk located in the center of the semi-circular seating arrangement.
This section begins with a depiction of the facilitators’ roles in creating and implementing the PLC and is followed by a detailed accounting of the emergence of the structural components of the study’s professional learning community. This includes the participants’ initial expectations of the PLC, a description of the norm-setting process, and an explanation of the *cycles of investigation* that evolved during implementation and served as the major structural component for PLC meetings.

**Role of the Facilitators**

The professional learning community was primarily led by two facilitators: the observer/facilitator and the facilitator/researcher. Both had prior training and experience in leading groups of teachers in professional growth. The observer/facilitator also has recent experience as a leader in a formal “lesson study” program, has led professional growth workshops for secondary mathematics teachers, and has mathematics teaching
experience at the junior high level. During a typical PLC meeting, the observer/facilitator participated minimally in the group’s discussions and activities. Her tasks generally were to:

- take observational fieldnotes,
- manage the logistics of the group such as room changes or meeting dates or times and obtaining special equipment,
- manage the audio recording,
- distribute materials, and
- maintain recordkeeping for attendance and participation.

The observer/facilitator also provided input into the PLC discussions at appropriate times but tried to minimize active participation so that her focus could be on the observation of PLC interactions. The observer/facilitator participated fully in the post-PLC debriefing meetings with the facilitator/researcher as well as in the development of PLC agendas through ongoing cursory data analysis in order to be informed for subsequent PLC sessions.

The researcher of this dissertation study was the primary facilitator of the weekly PLC sessions. As a way to prepare for facilitating the PLC, I participated in the “New Faculty Learning Community Developers’ Institute” at California Polytechnic University – Pomona directed by Milt Cox from Ohio University. In addition, through my doctoral studies I have been grounded in the research on knowing and learning the concept of function and have experience analyzing and interpreting student thinking using concept frameworks for this topic. I am a veteran mathematics teacher of nineteen years.
years were spent at the secondary level, and a total of seven years have been spent teaching community college students, either in an adjunct role or, as in the past five years, as a full-time teacher. I regularly teach Intermediate Algebra, College Algebra, Trigonometry, Precalculus, Calculus I, and Calculus II. Prior to this study, I participated in an extensive research project investigating the factors that influence secondary mathematics teachers’ pedagogical decision-making processes and the effectiveness of their chosen methods of instruction.

The facilitator/researcher’s role in this study was to lead people to progress towards the components found in the PLC Theoretical Framework, through writing agendas, selecting examples and problems for study, and posing questions. As established in this study, the roles and responsibilities of the PLC primary facilitator were to:

- complete appropriate readings and training before the PLC began,
- meet with the school’s department chair to formulate the PLC, prepare the site, and to help select the teacher participants,
- plan all sessions with the assistance of the observer/facilitator weekly,
- prepare for the sessions (review materials, complete associated readings, plan to meet the needs of the group’s dynamics, and create the agenda),
- facilitate the sessions through questioning, probing participant thinking, and challenging their beliefs,
- compile teacher reflections and group processing forms,
- maintain copies of all materials used including those submitted by teachers,
- debrief with the observer/facilitator at the conclusion of each session,
• complete a weekly reflection log and conduct ongoing data analysis (see Phases 1 and 2),
• keep notes regarding suggested changes, ideas and/or concerns regarding the PLC tools and processes, and
• begin to nurture indigenous leadership.

One of the primary tools employed by the facilitators in the PLC was the weekly meeting agenda. Each week a draft of the agenda was crafted by the facilitator/researcher and sent to the observer/facilitator for additional input. Upon considering the suggestions made by the observer/facilitator, the facilitator/researcher finalized each agenda and made preparations for implementing the meeting including making arrangements to secure all necessary materials (e.g. research articles, mathematical activities, handouts). The agenda was instrumental in providing direction and focus to the meetings by encouraging the facilitators and participants to stay narrowly focused on what was set to be accomplished. Considering the brief one hour time available for each PLC meeting, a well-thought-out agenda was imperative for promoting efficient time usage. A sample agenda is included in Appendix K.

For additional support and counsel, the facilitators dialogued with and sought guidance from two expert mathematics education researchers. These team members are faculty from the department of mathematics at the supporting university. Both have extensive backgrounds investigating student understanding of precalculus mathematics and have experience teaching and developing research-based curriculum for the course. These individuals served as consultants but did not play an active role in the PLC
meetings. A goal of the research team is to build the PLC into a self-sustaining entity over time that continues to solicit outside support as needed. Consequently, building an amiable and effective working relationship between the participating teachers and the researchers is of utmost importance.

Professional Learning Community Structure

In this section, I discuss important structural aspects of the professional learning community emerging from early findings of the study. I describe the participants’ expectations of the PLC as well as the development of the norms of collaboration that were utilized to facilitate an effective working environment. I also share the central organizing structure that evolved during implementation—the PLC cycle of investigation—including the primary tools that were found to be effective in focusing the participating teachers on students’ mathematical thinking and on deepening their own conceptual knowledge.

This section provides insight into the first two research questions of this dissertation study:

- What professional learning community attributes are associated with the development of conceptual knowledge and pedagogical content knowledge relative to the function concept in secondary mathematics teachers?

- What professional learning community support tools are associated with improvements in the facilitation of secondary teachers’ reflection on students’ thinking and reasoning relative to the function concept?
Participant Expectations

From a constructivist perspective, the facilitators believed that to create an excellent professional learning community the participating individuals’ needs, beliefs, and desires had to be taken into account in the development process. As stated previously, an important feature and benefit of a PLC is that this professional growth model is situated on the participants’ campus in their own school culture and utilizing the teachers’ students and fellow colleagues. For these reasons, the facilitators crafted agendas for the first three PLC meetings that elicited participant input and focused on:

1. choosing the mathematical concepts for investigation and creating a projected calendar of actions,
2. establishing the definition and goals for the precalculus professional learning community, and
3. setting norms of collaboration.

The initial meetings provided opportunities for the facilitators to collect participant contact information, enumerate the benefits of teacher participation, discuss participant expectations and goals of the project, and to begin to establish the professional learning community.

Investigative Topics

To begin the process of determining the mathematical concepts the participants wanted to investigate, the facilitators engaged the teachers in the “Learning to Grow Activity”. In this exercise, two tasks were given to the participants. The first was to name a topic/concept in precalculus they wanted to learn more about or thought was
challenging for their students, and the second was to name a concept/topic that they felt particularly knowledgeable in or had confidence in their ability to teach. After individual participants had time to complete these tasks, each was asked to present her responses to the others. Three mathematical topics of investigation for each semester were chosen in the resulting collaborative discussion: transformations, rate of change, and logarithms in semester one followed by trigonometric functions, function composition, and rational functions in semester two. After choosing the topics to study, the teachers proceeded to look at their school’s calendar and the department’s plans for the school year with regard to what material they would need to cover. By mutual consensus, a Calendar of Activities (as described later in this chapter) was then developed to estimate when the chosen topics would be examined by fitting the six topics into an appropriate timeline.

*Defining Professionalism*

With a timeline in place, the facilitators directed the discussions towards forming a definition for the professional learning community. Desiring not to be perceived as ones “coming to the teachers’ rescue” or “living in the ivory tower”, the facilitators did not want to dictate a definition for the PLC to the participating teachers. It was learned, however, and reinforced by a variety of data sources that it would have been beneficial to clearly explain to the participants what was meant by a “professional learning community” to help them understand the vision. Confusion as to what was to be accomplished is evidenced in the participants by statements they made in pre-PLC interviews and occurrences in early PLC meetings as shown below.
In an effort to launch the PLC and begin communicating its purpose and objectives in the initial session, the facilitators reviewed with the participants a short research article by Hiebert, Gallimore, and Stigler (2002). The article describes the type of professional knowledge required for the teaching profession and outlines aspects of the “Japanese lesson study” professional development model currently receiving a great deal of attention. The question “What is a professional learning community?” was posed by the facilitators to spur discussion within the group. The plan was to have the collection of individual teachers construct a definition for the PLC and create together with the facilitators’ guidance the PLC vision. The facilitators believed this would result in a higher level of participant acceptance and understanding. However, the ensuing PLC discussion tended to center on the participants providing peripheral characteristics of professionalism that they had experienced as teachers instead of working to arrive at a definition. Consequently, a consensus was not realized for what it was that the assembled group was attempting to build in a professional learning community. This caused confusion and a lack of clarity throughout the early part of the study.

Facilitator/Researcher (F/R):

Considering the research that we have just reviewed, let’s start with professional. What does it mean to be a “professional”?

Diana: We don’t get the opportunity to be professional.

Loretta: There’s no forum for it.

The participants suggested that they felt professional at their school, but there was some disagreement about how much time they had to collaborate with their colleagues. Instead
of discussing what it meant to be a professional, the participants proceeded to talk about their professional development opportunities at the school.

Cooper: We’re not encouraged to go to conferences because then we’re out of the classroom and it costs too much money.

Loretta: In our department, though, we have many opportunities to do this (talk with colleagues).

Lorene: Yeah, I think we’re professional within our own community or department.

In the post-PLC debriefing, the two facilitators believed that the general feeling of professionalism and pride in their department was a reason that the teachers wanted to be at the school. In particular, Lorene makes an extra effort to teach at the school by driving a long distance despite living in close proximity to what is perceived as a good secondary school. She made it known that she takes pride in the mathematics department at the school.

The facilitator again tried to refocus the discussion and draw out of the participants their definition for what it means to be “professional”.

F/R: Okay, but what does being a “professional” mean to you?

Mary Beth: The person went to college.

Diana: They do their job seriously.

Loretta: It is someone who is learning and improving.

F/R: Is a TV technician a professional?
Jeanne: No—it is also based on education.

Lorene: It also has to do with if you present yourself in a professional manner.

Mary Beth: Is it how much money one makes?

Cooper: No, a plumber makes a lot of money but isn’t a professional. A professional is someone who goes to school to do a specific thing.

F/R: So, what’s the answer? After all, it is referred to as the “teaching profession”. Is a businessman a professional?

Diana: A CEO of a company is, but the manager at Fry’s isn’t. Is he?

The discussion regarding the meaning of “professional” ended by being unfocused and arrived at no general consensus by the group. This showed that the group was unsure of what it meant to be a professional but it was important for them to be perceived as “professional” by the public—whatever that meant.

Summarizing, the characteristics of being a “professional” that the teachers mentioned included having an education, making good money, serving an internship, providing a service to the community, and having a special license. For the purposes of this study it is important to note that the PLC participants did not include that a professional is one who stays current in their field and seeks input from other professionals considering that the PLC structure is designed to encourage ongoing professional growth and collaboration with colleagues.
Beliefs about Professional Development

To conclude the first PLC session the participants were asked to read the Hiebert, Gallimore, and Stigler (2002) article more closely and to prepare to discuss the article further in the subsequent meeting. It was also requested that they write a few paragraphs reacting to the article (e.g., their definition of “professional”, their comparison of practitioner knowledge and professional knowledge, their explanation of how to turn practitioner knowledge into professional knowledge). This request prompted some intriguing dialogue and led to some interesting findings regarding the participants’ beliefs about professional development and their thoughts about the relevance of mathematics education research. Reflecting on her personal experiences, Lorene shared a “horror” story regarding a mathematics education research class she had recently taken in which she felt that the instructor was out to “punish the secondary teachers in the class” by giving them an extremely heavy workload and “tried to see if they would drop the course”. She expressed her strong dislike for the course and the reading of research articles that did not directly apply to her classroom instruction.

Cooper noted that her early perception of a PLC was that the structure was based on the research article the participants were asked to read that singled out Japanese lesson study as an effective professional development model. To her, it was not “fair” to compare American and Japanese students because of their dissimilarity and to try to generalize teaching techniques to different classrooms even with comparable types of students from the same school is not a wise practice. Cooper also shared the difficulty she
has in working collaboratively with other teachers to develop teaching ideas and materials.

The way I see it is that they (the researchers) want to come up with the perfect lesson and everybody teach it the same way. I don’t know if that is something that you can do because each person has their own personality. My class and what I do greatly involves my personality, and it is really hard for me to do something that I work on in a group. I just have always been that way. That is kind of like church. I don’t go to church because it doesn’t do much for me to go to church. It doesn’t do anything more for me to go to church than what I do on my own because I am like that. Now I believe in church for other people, and I think it is wonderful, it is important, but for me it is not something that does something for me. I have my own ideas. I have my own relationship with God, and that is what works for me. In teaching, these other girls (referring to Loretta, Mary Beth, and Diana) will get together, and they will make packets, and they will all follow them, but I can’t do it. It is just something that I can’t do.

Cooper did not understand that the goal of lesson study and, likewise, the PLC is not to create a “perfect lesson” but rather to build and maintain a systematic procedure for looking at student thinking and, subsequently, developing classroom interventions based on the findings—to be more professional. Her viewpoint was very strong in this regard, however, and became a factor to deal with throughout the study.

Jeanne expressed similar convictions in an email reflection by stating how she maintained that each student, class, and teacher is so different that each teacher must
individually develop a lesson that “fits” them and their students. To Jeanne, collaborating with colleagues to prepare lesson plans is fine, but when actual instruction takes place in the classroom, the teacher must be able to react to the students. She also frequently expressed her desire to feel “comfortable” with what will transpire in her classroom.

So much of being successful in teaching mathematics is the person doing the teaching. You could have a beautiful lesson that works with your kids, and I could take that exact same lesson and go in there, and it just bombs because I can’t do it or because it’s not right for me or my kids. The lesson has to be something I am comfortable with. If you are teaching auto mans or robots then that is fine, but we are not. I don’t understand the idea of a collaborative research lesson, but maybe that is how it works in Japan where you have got all of these kids who are expected to pay attention because it is life and death for them…it pretty much is…and this teacher is going to teach it to them. That doesn’t work here. Boy, some days the lesson just moves, but the next class it is just awful. The teacher has to constantly monitor and adjust and not just follow a prescribed worksheet.

Cooper and Jeanne’s affinity toward and insistence upon rugged individualism supports what has been shown in the research literature regarding American teachers’ value of their independence and strong emphasis on student individualism (Stevenson & Stigler, 1992; Cox, 2001). American teachers are comfortable being in charge, like to work on their own, and cherish the authority they have over their schedule, curriculum, and classroom management (Stevenson & Stigler, 1992). These values can form cold
barriers to communication, cutting teachers off from their colleagues. This view poses a challenge when working with teachers in a collaborative environment such as a PLC.

Maintaining this position in a mid-year email reflection, Jeanne expressed her lack of interest in implementing the collaboratively developed PLC activities because of her discomfort with them and her philosophy of reacting to her individual students’ needs by “seeing” them in a teaching moment and attempting to remedy the situation.

I didn’t feel comfortable with the activities we developed, so I don’t think they helped me at all in the classroom. I tend to be a loner when it comes to teaching. That is, I look at the students and see where they are struggling, and then I try to find a way to help them. It is, therefore, difficult for me to make up an activity one month in advance of instruction.

Jeanne’s views in this regard are very deeply held as her post-PLC interview shows when she again forcefully states her belief that teachers must “see” the kids and react to them. Jeanne feels that if teachers don’t react to students and instead just try to cover the content, then they are not doing their job as educators even if they know mathematics very well themselves. By “seeing”, Jeanne appears in this transcript to be primarily referring to watching students for classroom management purposes rather than for focusing on student thinking. (Note that bold-faced type in a transcript denotes emphasis added by the participating teacher.)

We have had engineers teach here that just know the mathematics, and we have had one just “brief” the students. He would say here is how it works, and he would do all the work and show them the answer and tell them, “Okay, now you
go do the work.” He could **not** figure out how these kids couldn’t get it! He just struggled the whole time. That is one thing—you can know all you want, but you have to be able to see those kids. If you don’t see them…. Isn’t that right? How can you talk to a room full of people and not see they are there? Not see that one kid is sleeping and that one kid has a headset on, one person is talking...Some people are better at it than others, but if you don’t see those kids and what they are doing…. Lots of times I can tell if a student is screwing around, and I don’t even have to literally see them. If you can’t see them so that you can tell if they are misbehaving or if one of them…. Why are you scrunching your eyes? What did I do? Or they start laughing…. What did I do wrong up here?

Jeanne goes further with her explanation and begins to allude to focusing on students more for instructional purposes. She expresses her belief that a “good” mathematics instructor will consider students’ needs, and she links college mathematics educators in with those she perceives as poor teachers. To Jeanne, this is because of their goal to cover content by imparting knowledge and not focusing on students.

Some teachers just focus on covering the content. Maybe that is college, but I even think a good college or university teacher has to see the students, too. But I see a transition taking place where it is more about kids in the elementary level rather than content, then we transition to kind of content and students at the secondary level, and then when you get to the university or community college level, it’s all content. When I taught at the junior college, they could ask me before or after class for help—but don’t ask me at other times: “I am an adjunct
faculty and not making much money. I am doing this for the bucks and experience. It is your (the students’) responsibility to get it.”

Jeanne’s statements about the importance of teachers being able to adjust to students needs are supported by the research conducted by Mason and Spence (1999) who describe different forms of knowledge required of classroom teachers. The researchers state that *knowing to* act when the moment comes requires more than having accumulated *knowledge about* teaching. It requires relevant knowledge to come to the fore, so it can be acted upon. They claim that an extra degree of awareness is required on the part of a teacher: *knowing to* as well as *knowing how* to create suitable conditions and then to direct student attention effectively. Although Jeanne may hold that *knowing to* act in the moment is important, it doesn’t appear that she is aware that being able to do so requires more than simply reaction. Mason and Spence (1999) also make the case that *knowing to* act requires awareness, and that it is working on this awareness which provides the fulcrum for professional development. Active, practical knowledge—knowledge that enables teachers to act creatively rather than merely react to stimuli with trained or habituated behavior—involves *knowing to* act in the moment that requires having thought about the moment previously. They claim that it is impossible to be aware *in the moment* that you “do not *know to* act”, but you can become aware afterwards. Teachers recognize when students “don’t *know to* act”, because their own awareness extends beyond the awareness of their students, but they themselves often suffer not *knowing to* act while teaching.
Mason and Spence (1999) surmise that knowing to is developed through connections being established between past, present, and future, so that in the future, past experience informs practice in the moment. In order to inform practice, it is essential that something brings to mind a possible action in the moment, just in time when it is needed. The tricky bit is moving the moment of recognition of a possibility from the retrospective to the present or, better yet, in the anticipatory stage such as in teacher planning. To bring a possibility to mind in the moment requires either luck or intentional preparation. The researchers conclude that reflection on mathematical content knowledge and pedagogical content knowledge is the sort of preparation required for planned success. This relates to the PLC in that Jeanne needs to learn that even though it is fitting for her to act in the moment she needs to understand that those decisions can be greatly enhanced by anticipating and contemplating what students know and how they will react. The PLC is an appropriate and well-suited environment in which to do this in collaboration with the other mathematics educators.

Another obstacle to the PLC structure that emerged early in the study was a position held by most of the participating teachers. Jeanne’s statement represented that it wasn’t a good idea to look so intensely at only a few mathematical topics like they do in the Japanese lesson study model because American secondary school teachers have too many topics to cover and have many demands on their time. In response to an interview question, Jeanne expressed her feelings.

F/R: In one professional development model, colleagues study together a concept and how to teach it for an extended period of time with
multiple revisions of the lesson taking place after seeing how students perform. Do you believe that there is merit to studying extensively one math concept and how to teach it effectively? Explain.

Jeanne: Ha, ha, ha, ha! Come on. **One** math concept! How many concepts do we have to teach in a year? I mean give me a break. You are talking about that Japanese thing aren’t you? That is absolutely ludicrous! How can you spend all of that time on one concept?

Jeanne does not see the value in looking intently at a concept to find the nuances and challenges for students. This is in alignment with her deeply held belief that teachers need to react to students and to not purposefully plan ahead for their learning. Later in the PLC the participants requested increasing amounts of time be allocated in the PLC for each topic of study because they felt they were being restrained by the limits of time (see cycles of investigation presented later in this chapter for more information).

When the facilitators’ prompted the participants to share how their prior training had prepared them professionally, Cooper articulated her strong conviction that the education classes she took in college "were crap" and that what is done to prepare teachers doesn't work. She stressed that college professors should not be considered more professional simply for making their ideas public as was suggested in the Stigler et al. (2002) article because some professors’ ideas aren't really "true"—they are not based on real classroom experience. All but one of the others shared similar experiences but not in such graphic terms. Cooper continued her animated diatribe:
I don’t think there are many people that will tell you that they gained anything through a mathematics education class. They are stupid, they’re useless, and they’re ridiculous. I could have sworn having taken them that the professor sat around during happy hour thinking up stupid stuff for us to do; most of which students made up and just did it. It is busy work, useless, and it’s ridiculous. It is a waste of money and a waste of time when you could have used that time to really learn your subject and the ins and outs of the things that you are actually going to be teaching.

This showed that Cooper’s feelings of professionalism are largely influenced by how society views her. The participants as a whole did not feel that the general public values teachers in secondary and elementary schools as highly as college professors and were disappointed by this. They resented that in the field of education the label of “professional” seemed to be reserved by many for college faculty only. As a result, the PLC teachers’ sense of professional identity was heavily impacted by how they were perceived by others rather than by what they did as educators.

Loretta and Mary Beth were the two participants to share positive experiences regarding the professional development programs in which they had participated. Loretta explained that the motivation she has for accessing a variety of professional growth opportunities is to learn mathematical content more deeply and to become a better classroom teacher. In addition, she saw the benefit of professional development programs that include collaboration with other mathematics teachers and direct application.
Everything I have done I’ve tried to make valuable. I don’t want to waste my time and take some silly class. I’ve taken a lot of math classes just because I need to. The more mathematics I know; the better I am, and the better I can understand. I’ve done a lot of classes through local universities and even a few online which were interesting and kind of fun. The one online course I took was a correspondence course, but I missed the discussion. If you take those, you are in isolation. I’ve done workshops and Advanced Placement workshops to see where the kids need to be at the honors level. I’ve gone to the National Council of Teachers of Mathematics national and state conferences that have been good because I got ideas from other secondary mathematics teachers of what works in their classes.

The facilitator/researcher asked the participants to reflect further about the article they had read and, specifically, about the emphasis the authors placed on connecting practitioner knowledge and research knowledge. Cooper agreed that they should be integrated but stated that one hour per week in the PLC would not be enough for a researcher to really know what's going on in a class/school like theirs. She and Lorene were unsure about allowing individuals to come in from outside the classroom to discuss with them learning and teaching because they felt that most researchers don’t know what they experience as practitioners. They did establish that research knowledge was valuable if the research related directly to their students and teaching but noted that time was a barrier to them doing any formal research. For the purpose of implementing the PLC it is important to understand the participating teachers’ views regarding research and the
research community in order to provide an excellent experience that the teachers feel is applicable to them. The researcher/facilitator learned from these statements that the research incorporated into future sessions would need to be carefully chosen.

Despite their stated concerns and reservations about professional development and mathematics education research, the participants showed interest in the PLC model because it appeared to be relevant to them by accommodating the needs of their students and directly applicable to the classroom.

Cooper: How a teacher spends their time—if they have time out of the classroom—they want to think about their students, unlike professors, or they choose to be with their families, but time for research is not there.

Lorene: Time spent researching isn't an effective or efficient use of the little prep time we have. You can spend hours looking for one article and often it won't even help you teach what you need to teach tomorrow.

F/R: If research was readily available, in a format you could use, and reflected your needs—would that be useful?

Lorene: Yes, but a video library of best practices mentioned in the Stigler article may not be beneficial for me though. I think that may take too much time. What's beneficial is this group, because we are all using the same book, and working with the same kind of kids. A video of another class with a different book may not help me as
much.

At this point Cooper again expressed her disdain for college professors and their being considered “professional” when she, as a secondary school teacher, is not. She demonstrated her angst by sharing what she felt was the widely-held perception that what practitioners do isn’t “professional”. Furthermore in her pre-PLC interview, Cooper exhibited her lack of respect for college professors and stated that she considers them not to be good teachers who “just present information to students and don’t care if they get it”. She suggested that college professors are generally not concerned about student learning but, rather, are immersed in their own research interests. Cooper relayed more of her thinking by speaking out about an engineer the school had recently hired.

In the past we have had people like that come in, but they leave right away. Because for some reason engineers, I believe, think that they are God-like. They come in really thinking that teaching is easy. They don’t understand that having a relationship with the students is very important. We had one engineer come in and teach like his college professors had and never really put much into thinking through how to do it differently so he could reach his kids. He would always talk about how stupid his students were because they didn’t understand what he was saying. Just because you are an engineer doesn’t mean you are a teacher. I have a student in my calculus class who is very, very sharp. He can do it, but he can’t explain anything—those are two different things. That is like college professors presenting math lessons, they are bad at it because they are not teachers, and there
is a difference. It is not the knowing of the math that is hard, it’s the presenting it.

Knowing it and presenting it are two different things.

Cooper had made the case in her own unique way that both conceptual knowledge and pedagogical content knowledge are important components of being a successful teacher. The researcher notes that these two aspects are central elements of the PLC structure and this research study. Another important aspect revealed here is the prevalence of disrespect for college educators. The establishment of a PLC attempts to link the collegiate world of research with classroom practice. A researcher coming in from outside the school culture that the participants are in should note the participants’ feelings about college professors. Lack of mutual respect is a hindrance to the effort to establish an effective PLC.

Because of the participants’ interest, questions, and concerns regarding professional development and a desire to meet their needs, the facilitator/researcher engaged the PLC in talking much longer about the research article than was originally planned for by the two facilitators. The facilitator/researcher relayed that the authors of the article were not attempting to place more value or respect on research knowledge; rather, they were defining “professional” in one specific way. Practitioner knowledge did not fit the definition they espoused for professional knowledge but was another type of equally important knowledge. One of the purposes for having the participants read the selected article was to assert that researchers need to value practitioner knowledge, and practitioners need to value research knowledge. The facilitators proposed that researchers and practitioners communicate and work together to produce the best results for students
through the vehicle of the PLC. Despite the facilitators’ planned outcomes, the PLC participants held firmly to their belief that practitioner knowledge is not valued equally with research knowledge by those in mathematics education and that practitioners are, therefore, seen as subordinates.

Finally, the participants noted the lack of high-quality opportunities for them to grow professionally. The perception of the PLC participants was that secondary school mathematics teachers are mostly left unattended to in this regard, and that most professional growth opportunities are geared toward elementary teachers. The PLC teachers relayed their desire for professional development programs that incorporate the specialized knowledge that they knew they needed and were seeking. Cooper and Loretta represent what each of the participants communicated.

Cooper: I get so frustrated with workshops set at an elementary or middle school focus. Secondary teachers seem to always get overlooked.

Loretta: A lot of workshops don’t focus on mathematics specifically or if they do, it’s generally at the elementary level. Some of us went to one this summer for 6 days, 8 a.m. – 5 p.m., and not one word about math. There was a guy at the workshop that quickly said we have these computers that do the math for us. I tried to get to him after the workshop to set him straight, but he disappeared before I could get there.

Jeanne affirmed the same feelings by questioning the facilitators’ selection of the Stigler et al. (2002) article for the PLC.
I have a concern about the rationale for having us read this article. A quote from the reading says, “knowledge linked with practice must be grounded in context”. How is this article grounded in our context of teaching mathematics? I am tired of reading articles, attending seminars, etc. that show me how to teach using elementary grade materials. Why would I want to know how to teach a 1st grader about Billy Goats Gruff or how to add 4 pieces of gum to 7 pieces of gum? Where’s the context? Don’t tell me I should extrapolate the concepts to the high school level. I need help in understanding teenagers, how they think, what’s interfering with learning. To try and compare a 6-year-old and a 16-year-old is like comparing apples and bicycles. I would like to read an article that speaks to me about high school math content taught to high school students.

When the participants were asked their opinions about the research knowledge they had seen, the group again expressed its disappointment with the lack of accommodation for their desires and needs. They craved information that helps them in the classroom and focuses on them as the audience.

Lorene: Last year I took a class, and we had to read and do all of this research. I understand it is important for me to know what research is out there but none of the research that we looked at fit my world. Research has got to fit me and deal with what I am doing in my classroom, in my socio-economic, and my education level. We were talking about kids in 6th grade in inner city Chicago. That’s not going to help me.
Mary Beth: It is mostly written for other researchers as an audience. It is not written for me. If a researcher wants me to get something out of it, then they have to think about me and not just other researchers because that is really what it is written for—people who are spending their professional career researching and not necessarily teaching. They need to not use such specialized terminology. I don’t have the time to try and figure out what it is that they are saying.

Loretta: I like to read it and I think it is interesting, but the problem is a lot of times it seems like it isn’t appropriate for high school students. It is usually the younger levels K - 8 that I see research for. I would actually like to read research on high school kids. That is what would help me.

Cooper: I don’t have much of an opinion. I don’t think it is great or terrible. I have to say though that I want conclusions not just problem statement. I don’t put much stock in it because I think everything that is presented statistically can be skewed to say what the researcher wants to say.

Diana: It is boring, and I think they are saying the same thing over and over. Most of the stuff doesn’t pertain to what I am teaching. I read a couple that didn’t pertain to me because it was elementary math or something. I think that is what makes it boring. They may be
great papers if I was into the subject that they were talking about. It may not be boring then.

A transcript taken from one of the first sessions after Lorene had shared her attempt at creating a classroom intervention based on PLC research further illustrates the extent at which this belief is felt.

F/O: It (Lorene’s activity) is definitely great. It takes the research that illuminates the problem that students have, and then does something about it.

Lorene: To me that is the only valid reason to look at research. I don’t see value in simply thinking up specific terms for problems that I see every day.

Mary Beth: Yeah, show me how to fix it.

Cooper: That is why I have a problem with our “professionals”. I want the professional to solve a problem. Don’t just define it. To me it’s like people writing letters to the editor complaining about a problem. Don’t just define it, solve it! Solve the problem, you know! It’s like they (researchers) are the professionals, but we are the ones solving the real problems. Don’t university professors get paid for doing research?

F/O: They have to get funding in order to do the research.

Lorene: If there isn’t a solution piece in the research that is proposed, then they shouldn’t get any money.
F/R: That is what funding organizations are calling for now.

Cooper: That is the problem I have with a lot of the research is that I am not the audience. I feel that other researchers are the audience. Their goal is to impress not to express ideas. If I were the audience, they would be giving me solutions. They would be giving me suggestions. They would do something about our problems and use my language.

It became apparent from these results that came primarily from the first two PLC sessions that the professional learning community that was being developed at this school required taking the participants’ desires, wants, and needs into account by crafting a structure that was relevant to the teaching participants. Any research knowledge provided to the PLC participants would need to be applicable to them and to their classroom teaching as well as address high school teachers as the audience. Finally, the facilitators noted that the role that they played was critical in helping to first mend and then build bridges between the participants and the research community that desired to work with them. The participants made it abundantly clear that to present a professional development program that fails to do these things would quickly become a failure.

**Professional Learning Community Participant Goals**

Early data also revealed that the participants’ desires for their PLC experience were varied. Two major goals of the participants emerged in response to an early inquiry about their expectations for the learning community: (1) conceptual knowledge growth, and (2) creation of materials for classroom use. The two teachers seeking growth in their
conceptual knowledge base (Cooper and Jeanne) were the ones not excited to implement
the PLC-developed classroom materials, were most hesitant about having others observe
their instruction, and preferred to make classroom decisions in the moment. Alternatively,
the participants desiring to spend time with their colleagues discussing teaching strategies
and constructing instructional materials (Diana, Loretta, Mary Beth, and Lorene) were the
ones who were most enthusiastic and participatory when time was spent in the PLC
developing activities, were excited to try implementation, welcomed others into their
classrooms to observe and report back to the group, and preferred to have well-thought-
out lesson plans from which to work while teaching.

Jeanne, the department chair, believed strongly that the way to develop the
teachers in her department was to focus primarily on deepening their conceptual
knowledge rather than on learning pedagogy or in creating classroom teaching materials.
She stated that she hires only certified teachers, so “they should already know how to
teach”. Her concern is that they learn more mathematics.

I think a more effective way is to teach colleagues so that they understand the
math. I go back to the same thing. If you know the math inside and out, then you
don’t come in as a teacher and ask me, “The kids are wondering if you can have a
vertical asymptote and a zero at the same time.” Now if the kids have asked that,
then the teacher is coming to make sure that they were right. See, that scares me
and doesn’t make any sense. A teacher should not be asking that while they are
teaching the students. So what I would prefer to do is, don’t spend time on
discussing how to teach the lesson but instead learn the stuff. Maybe in the
Japanese lesson study model they think the teachers already know the math so they are focused on how to teach it.

Jeanne and Cooper expressed their lack of interest in implementing the activities that were developed cooperatively early in the PLC by explaining how they value their freedom as a teacher to teach as they want. Jeanne stated that she would use the materials occasionally but did not want to feel obligated to do so. It was most important to her personally to continue to learn the mathematical content more deeply and had that as her hope for those in her department as well. To Jeanne, pedagogical decisions should be left up to the individual teacher.

If you tell me how something works and get me into the details and nitty-gritty of a topic like rational functions or whatever, and I begin to understand all of that...teach me so I can understand it. Then I can take the pieces that I think will work and put it together rather than give me a worksheet and I’ll answer this, this, this, and this. That confines me too much. I can’t break out of that thing and see where kids are going to have trouble. If I have this worksheet, then I am tied to it. If another question then comes up, then we are stuck. Don’t put me in a box. I believe a lot of teachers aren’t confident with the content, so they feel more comfortable if they write out their worksheets—see this is me thinking as a department chair. If you work out your worksheet and have all the answers and the kids stay within this box, then I am safe. But if I go up there and make up a problem, I do make up problems on the board and may get myself in trouble. “Oh, okay, what did I do on this one?” Yeah, sometimes things don’t work out
obviously, but I was looking for the PLC to be more content. Teach me more and let us get engaged in the topic, struggle and work with problems. Let us fuss and muss about the problems so that we get a clearer understanding; so we then teach better.

The facilitator/researcher wondered in the post-PLC debriefing if Jeanne’s philosophy would work well with the less experienced teachers who were participating in the PLC. The facilitators agreed having deep conceptual knowledge is of paramount importance for teaching for student understanding, but they also pointed to their experiences with teachers. The facilitators also concluded that most novice teachers would need more structure to help them perform well in the classroom, and that experienced teachers would be a valuable asset to access in the PLC to assist these teachers. Upon considering the large number of local high school teachers he had observed teaching and referring to the large body of research literature, the facilitator/researcher noted that the vast majority of classroom instruction that takes place is not student-centered and focused on deepening students’ conceptual understanding. Rather, most mathematics instruction emphasizes students memorizing rules and performing procedural skills. More directly it was noted that the two classroom observations made of each participating PLC teacher by the facilitator/researcher showed that four of the six used primarily teacher-centered instructional methods. Additionally, participating teachers’ professed teaching practices show a heavy emphasis on memorization, quick recall, and teacher-centered instruction as well. In contrast to Jeanne’s proposal, this demonstrates the importance of not leaving pedagogical issues
entirely unattended to in a PLC and solely focusing on content knowledge. A more balanced approach seems appropriate and what is ultimately represented in the PLC structure that emerged.

Jeanne expounded on her vision for the PLC by incorporating things that she had utilized in the past as department chair for her teachers’ development. She values highly a teacher’s ability to teach as they wish.

We may want to just go in a room and have the five or six of us talk about a concept, try to understand it, play with some problems, and get to where everybody has got it. Lorene goes out, and she has got her three examples on the board that she teaches. Mary Beth, Loretta, and Diana work up a worksheet, and they do it with this step, this step, and this step and work through it. I’ll take five problems and put them up and ask the kids what are we going to do, and I think maybe Cooper would do that too. So you may have three different approaches, and we all have the same depth of understanding, but we are going to attack the problem or teach it in a way that is comfortable for us. Do we get the job done? Yeah. Maybe my emphasis is different from theirs but are the kids going to understand and be able to use it in the next course? I think so.

Jeanne’s suggestion of allowing the teachers the flexibility to choose their own classroom intervention and implementation techniques is a difficult issue that needs further study considering what is known about current teaching practices in the majority of American mathematics classrooms. The input provided by Jeanne, however, regarding placing more emphasis on content knowledge encouraged the facilitators to review the allocation of
time given to content exploration versus intervention development and provides support for the cycles of investigation (as detailed later in this chapter) that emerged during the year-long PLC implementation.

A contrary point to Jeanne’s was made by Lorene who mentioned in an early written email reflection that the views expressed by Jeanne and Cooper troubled her. She recognized that they did not want to feel obligated to use the PLCs research designed lesson plans or classroom materials. In Lorene’s opinion, the type of teaching that her colleagues promoted was a case of “flying by the seat of the pants”. Lorene’s contention with a teacher simply reacting in the moment without prior thought is supported by the research of Mason and Spence (1999). Recall that they found that being able to effectively act in the moment, a teacher needs to intentionally prepare for the moment. Lorene shared that she was experiencing building frustration with the attempts that had been made to craft collaborative teaching materials in the PLC due to this divide in the group’s goals. The lack of emphasis on producing well-thought-out classroom interventions by some participants has consequences for a PLC. This practice does not lend itself well to participants thinking deeply about mathematical concepts, studying obstacles to learning, and planning effective instructional methods which are all parts of the PLC organizational structure.

The facilitators strove to adapt the PLC structure and to make it more balanced between the two viewpoints as it was determined by the facilitators that there had been too much early emphasis placed on developing classroom interventions. The two facilitators worked diligently to address the concerns that were raised by altering the
scope of the PLC-developed materials in later sessions. The first sets of activities that were created in the PLC were manifested in lessons or entire units of study that got the participants bogged down in detailed curriculum development. After the first semester, the PLC chose to create only a few problems or a short problem set that addressed more narrowly defined mathematical concepts (such as the sessions described in Chapter 6). Time spent in the PLC by the participants creating curriculum diminished, and the participants’ collaborative efforts became more productive as is shown later in this chapter.

As a result of the early results and participant expectations, PLC sessions were directed by the facilitators toward a cyclical process of:

- learning the concepts more deeply;
- focusing on how students think about mathematical topics;
- creating conceptually-based classroom interventions that build upon student conceptions;
- working to improve participants’ instructional techniques;
- assessing more formally what students’ learn from classroom interventions; and
- reflecting on the effectiveness of PLC-employed methods and created materials.

_Norms of Collaboration_

Bellah, Madsen, Sullivan, Swidler, and Tipton (1985) define community as “a group of people who are socially interdependent, who participate together in discussion
and decision making, and who share certain practices that both define the community and are nurtured by it” (p. 333). According to Wenger’s (1998) “dimension of mutual engagement” teachers working in isolation keep aspects of their instruction private – such aspects as decisions made and tools used during planning, facilitation, assessment, and reflection. Conversely, as teachers are mutually engaged in the pursuit of a shared purpose, they need to develop norms of participation, which bring about the necessary consequence of teachers’ instructional practices becoming public. This in turn cultivates a mutual accountability of justifying and critiquing pedagogical justifications within a professional learning community.

In this study, “norms to live by” and mutually agreed upon techniques to assess how effectively the PLC functioned were established through a norm-setting process. Based on a constructivist philosophy, the facilitators agreed to formulate and utilize a process that would draw the norms out of the participants rather than dictating them. The observer/facilitator began the process by giving the participants small sheets of adhesive paper and asking that they write down as many responses as possible to two questions: From your prior experiences of working in groups, (1) What do you think has worked well?, and (2) What do you think has not worked well? The participants were then asked to post their responses on the board under the headings “good group characteristics” and “group characteristics that drive me crazy”. The participants were given time to reflect on the collective responses and were tasked with consolidating them into common themes by rearranging them into broad categories. From these broad themes two initial norms were set:
1. Do your part with a little bit of humor.

2. Don’t waste time.

The participants’ attention was then drawn to developing a technique for enforcing the norms. At first some members of the group didn’t feel this was necessary, but, in the end, the group accepted one of the “joking” suggestions of using a referee’s “yellow card” to warn someone of an impending infraction and a “red card” to let someone know they had violated a norm, and that they should stop. The community also agreed that the way in which the norms were to be enforced was for the facilitators to select a different person each meeting who would be in charge of watching for infractions. It is important to note that the members of this group are quite familiar with each other and have worked together before.

As a result, the initial sense was that they didn’t feel a need to set norms, but it became important, especially to a few individuals, as the year progressed. The facilitators observed during the meetings that the teacher personalities in the community varied from being quite outspoken and somewhat domineering (Cooper and Lorene) to being more reserved and laid back in group settings (Diane and Mary Beth). The initial two norms seemed to work well until the stronger personalities in the group started to inadvertently shut out the more reserved teachers. The participants found that norm enforcement is difficult with colleagues because of the nature of working together and not wanting to be seen as being malicious or unkind. Consequently, the teachers expressed their desire for the two facilitators to be responsible for norm enforcement.
Lorene: I don’t like this red card, yellow card thing because we know each other too well. We work together every day and like I’m not going to tell Mary Beth to shut up. (laughter) If I did, she would just laugh at me. So you guys have to be the enforcers whether you want to be or not. You have to be in charge.

F/R: I can definitely run the meeting and tell people what to do, but that isn’t the purpose of this. This is to be a collaboration of professionals with equal participation. You have to do it for each other because we want everyone to feel free to say things and not to be cut off by others. Also, remember that the observer/facilitator and I will not be here forever. You need to take the ownership if this community is to continue.

Diana: Yeah, but at some aspect it has to be you guys because if we do it too often here, we have to work with these guys, and then there will be some hurt feelings.

Lorene: That’s right. If she tells me to shut up, I’m going to come after her.

The facilitators resisted this request, however, and noted that this would be problematic as they planned for the PLC to continue without “outsiders” present. The participants were reminded that the plan was for the participants to gradually take the leadership of the PLC in future iterations. The teachers recognized that out of necessity the role of the facilitators would have to become increasingly more of an advisory role.
Therefore, in an attempt to keep the responsibility of the group with the PLC members, it was agreed to keep the norm-enforcing procedure in place.

For improvement of the group’s dynamics, the norms were revisited periodically by requesting that the participants reflect on how well the PLC was meeting them. Two additional norms emerged and were added after the two facilitators reviewed the end of the first semester email reflections. The process of increasing the number of norms began when Mary Beth shared through her email reflection that she did not feel like she was given equal opportunities to share her thoughts and views since she is not currently teaching Precalculus and also that other participants (Cooper and Lorene) are more assertive than she.

I have felt like I am a lesser member of the group more recently. There have been a couple of tasks that the group was asked to do that I could not do because I don’t currently have precalculus students. I was not given an alternative assignment or even a suggestion on what I might think about for the next class. At the beginning of the semester, I was a little disappointed with the time given to share our reflections and impressions on the articles we were asked to read and respond to. I felt that my response was not needed after a couple of others in the group jumped in and explained their points of views.

Diana shared in her end of semester email reflection that sometimes in the group discussions others would talk about issues or things that they do in their classroom or use terminology without being clear as to what is meant. This issue immediately came up again in a light-hearted way in the first PLC session of the second semester.
It was pointed out in one of your email reflections that sometimes people would introduce an idea, and there might be people who hadn’t heard the terminology before or weren’t familiar with the pedagogy that someone was describing, and we keep kind of moving past it…

Diana: What?

Lorene: Pedagogy?

(Laughter)

O/F: Yeah. There you go—a perfect example. We just want to keep that idea out in front of you.

Lorene: I don’t know what it means though O/F. (Laughter)

O/F: Oh, I’m sorry…

Diana: Yeah. (Laughter)

O/F: …Pedagogy means teaching methods….

As a result of the participants’ reflections and recommendations, two additional norms were added for a total of four. They were:

1. Do your part with a little bit of humor.
2. Don’t waste time.
3. Strive for equal participation in conversations.
4. Check for mutual understanding of new ideas.
Emergence of an Investigative Structure

Following the first three PLC sessions that dealt chiefly with administrative, informational, and organizational matters, the PLC structure began to take a cyclical form that provided coherence and direction to the meetings. This emerging creation was used by the facilitators to motivate all agenda construction and, subsequently, all PLC meetings theretofore. In this section, I explain the evolutionary process of formulating the cycle of investigation which is the PLC organizing structure developed out of this dissertation used for facilitating the professional development of the teacher participants. It was the driving force used by the two facilitators to help guide the participants through a logical and sequential study of mathematics content, student thinking, and pedagogy. The cycle of investigation was the vehicle through which the facilitators strove to promote deep conceptual knowledge and pedagogical content knowledge in the teachers as well as to challenge and confront their beliefs about learning and teaching mathematics. Once the emergence of the structure is depicted, I reveal the theoretical basis as well as the facilitators’ intended purpose for each of the six sessions within a cycle. Finally, as appropriate, the specific tools found to be effective in focusing the participating teachers on students’ mathematical thinking and on deepening their own knowledge are shared.

Initial Structure

The professional learning community structure did not initially start out in the six session cycle of investigation format. This more formal and refined mode of investigation emerged as structural modifications were made by taking into consideration the body of
research literature as well as the participants’ beliefs, desires, needs, and expectations for the PLC. One example of this is seen during development of the activities timeline. This process started with the observer/facilitator creating a list of all the weeks of the school year for the teachers to organize the year. The teachers eliminated the weeks of vacation and final examinations from the list and then clustered together three week intervals for topic investigation. The original plan was to have three three-session cycles to study each mathematical topic in the first semester with the final two meetings reserved for reflection and planning the second semester. As was shown previously in this dissertation, the first three weeks were used to establish the definition and goals for the PLC and the norms of collaboration. The plan for the second semester was to have five three-session cycles of study and two meetings at the end to reflect, assess, and plan for the following year. The week after the PLC concluded, the facilitator/researcher performed interviews with the participants and collected their written reflections. Table 6 shows the original outline of events and the topics to be studied.
Table 6

First Semester Planned Calendar of Events

<table>
<thead>
<tr>
<th>Date</th>
<th>Topic of Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 26(^{th})</td>
<td>Why are we here? What’s a PLC?</td>
</tr>
<tr>
<td>September 2(^{nd})</td>
<td>Establishing Group Norms</td>
</tr>
<tr>
<td>September 9(^{th})</td>
<td>What’s Important in Precalculus? (Setting a timeline)</td>
</tr>
<tr>
<td>September 16(^{th})</td>
<td>Function Transformations</td>
</tr>
<tr>
<td>September 23(^{rd})</td>
<td>Function Transformations</td>
</tr>
<tr>
<td>September 30(^{th})</td>
<td>Function Transformations</td>
</tr>
<tr>
<td>October 7(^{th})</td>
<td>Rate of Change</td>
</tr>
<tr>
<td>October 14(^{th})</td>
<td>(FALL BREAK)</td>
</tr>
<tr>
<td>October 21(^{st})</td>
<td>Rate of Change</td>
</tr>
<tr>
<td>October 28(^{th})</td>
<td>Rate of Change</td>
</tr>
<tr>
<td>November 4(^{th})</td>
<td>Logarithms</td>
</tr>
<tr>
<td>November 11(^{th})</td>
<td>Veteran’s Day</td>
</tr>
<tr>
<td>November 18(^{th})</td>
<td>Logarithms</td>
</tr>
<tr>
<td>November 25(^{th})</td>
<td>Logarithms</td>
</tr>
<tr>
<td>December 2(^{nd})</td>
<td>2(^{nd}) semester planning (reflection)</td>
</tr>
<tr>
<td>December 9(^{th})</td>
<td>2(^{nd}) semester planning (topic selection)</td>
</tr>
<tr>
<td>December 16(^{th}) – January 6(^{th})</td>
<td>SEMESTER BREAK</td>
</tr>
<tr>
<td>January 13(^{th}) – January 27(^{th}) (3 sessions)</td>
<td>Topic 1</td>
</tr>
<tr>
<td>February 3(^{rd}) – February 17(^{th}) (3 sessions)</td>
<td>Topic 2</td>
</tr>
<tr>
<td>February 24(^{th}) – March 9(^{th}) (3 sessions)</td>
<td>Topic 3</td>
</tr>
<tr>
<td>March 16(^{th})</td>
<td>SPRING BREAK</td>
</tr>
<tr>
<td>March 23(^{rd}) – April 6(^{th}) (3 sessions)</td>
<td>Topic 4</td>
</tr>
<tr>
<td>March 13(^{th}) – April 27(^{th}) (3 sessions)</td>
<td>Topic 5</td>
</tr>
<tr>
<td>May 4(^{th})</td>
<td>Year end reflection</td>
</tr>
<tr>
<td>May 11(^{th})</td>
<td>Plan for next year</td>
</tr>
<tr>
<td>May 18(^{th}) – 25(^{th})</td>
<td>Post PLC interviews, Teacher reflections</td>
</tr>
</tbody>
</table>

After studying function transformations for three sessions, a move was made to increase the time spent to four sessions because three members of the PLC (Mary Beth,
Loretta, and Lorene) stated that they wanted to be involved in the development of the instructional materials and not simply have them brought in. Utilizing their expertise, the two facilitators had met outside of the PLC to create a conceptually-based activity that the teachers could use in their classrooms. The facilitators had determined that developing curriculum during the PLC meeting would not be an efficient use of time. However, to be responsive to the participants’ needs and realizing that it would be beneficial for them to learn how to develop and adapt conceptual materials for their students, the facilitators agreed to add another session to the next topic of study. As an action item, the participants had been asked to reflect on the first “cycle” of the professional learning community that they had just experienced. More specifically, they were to think back to each of the first three meetings and comment on what they liked, what they gained, what frustrated them, and what they hoped would be different in future cycles. Upon arriving at the next PLC meeting, they were asked to share.

F/R: Who would like to share with the group your reflections?

Mary Beth:  I have really enjoyed the activity we developed, but I really don’t know how it was decided on. I mean you brought it; it was mostly done. We agreed it was a good activity, but I don’t know how you got there. That is something I am interested in. If I am going to be able to do this in the future, then I need to know how you get there—get to what you think is important.

F/R: That is what we (the observer/facilitator and facilitator/researcher) had discussed after last meeting. This group had agreed on three
meetings per cycle. That’s what we are calling them right now, but we are thinking about going to four.

Mary Beth: Yeah, that would be better.

F/R: With the added session we could do what you just described. We (the observer/facilitator and facilitator/researcher) would still bring something in to start with because there is no way that all of us can get an activity built from start to finish in an hour and finalize it. So what we thought was to have something to work with from the beginning, and you can trash it because it’s not her work (the observer/facilitator’s), it’s not mine, it’s ours (the PLCs). We (the observer/facilitator and facilitator/researcher) may have produced it to begin with, but it would be out there to be shot at and torn apart. People should feel free to say, “Let’s trash that part and do something different.” We are thinking about making these four-week cycles instead of three.

Mary Beth: I think it would be beneficial to just have the chance to see how it’s developed. I think we did a little of it, but we didn’t do nearly as much as you need to do to be able to produce an activity that is really going to benefit your students.

At this point Jeanne again stated her belief about how important it is to “see the kids”. This is her way of saying that she is hesitant about implementing anything in her classroom from outside sources.
Jeanne: I think it will be good to add the extra week, but it is really hard to try and make something up when you don’t see the kids. I have to see the kids and feel. When you are doing that kind of “blind”, then it is harder to do this.

F/R: That is why I like to come to your classes once in awhile to see what the kids are like in order to create some kind of contextual situation that they would enjoy and can relate to, yet still gets at the mathematical concepts.

In preparation for the second semester planning session, the participants were asked to write reflections on the first three cycles in terms of the process, content, and impact of the PLC. Jeanne and Cooper who desired to delve into the mathematical content more deeply did not feel the participants had been given an adequate amount of time to do so. In line with their personal beliefs, they felt that the PLC was getting too bogged down with curriculum development.

Cooper: When you have this number of people involved in a collective effort to pinpoint and prioritize concepts most difficult for students, you are bound to have discrepancies. Once we actually pinpoint a concept, to then agree upon the most effective way to demonstrate and evaluate it is yet another challenge. I believe that we are molding our process as we go. We may be learning a little bit more about process than we are content at this point. However, I believe this to be a natural phenomenon and to be expected. Like
starting a new job, you do not begin to be productive until you are comfortable with the new process and the people you work with.

At this point, one thing that could truly help our process is to have a longer block of time or more meetings per week.

Jeanne: When we first discussed this class, I had envisioned it a little different than it has turned out to be. I had hoped for more content, i.e. instruction in difficult math concepts. Some of us need more in-depth understanding of precalculus concepts, and I was thinking that would be something we would be doing more of. For instance, yesterday, five of us Math Topics 4 teachers got together after school and discussed the probability concepts we are currently teaching the students. We picked a few problems and argued and discussed how to do the problems until we all understood. We were actually learning ourselves so that we could better teach our students. I have been doing this kind of thing in the department for years, and teachers seem to think it is valuable. We have spent most of our time so far making worksheets. The problem I’ve found with the worksheets is that we haven’t identified clearly the concept we were trying to teach and couldn’t identify why the students were having problems.
Other participating teachers, however, appreciated the activities the PLC had created and were pleased with the results they had gotten. Loretta and Lorene, for instance, share their mid-year reflections.

Loretta: Sharing ideas and experiences in the classroom is enriching. The concepts we’ve covered have been wonderful. The conceptual logarithm problems we developed really benefited my students. I think they really helped the students get a handle on what we are doing. But will they retain it? I don’t know. I was very pleased with their progress during the chapter and thought they did better than previous year’s students. This class (the PLC) has helped me to be more thoughtful about my teaching. I’ve asked better questions of the students, and I think I am getting better at assessing where they are. The difficulties they conveyed with logs and exponents were surprising. I was very glad that I took the time to examine their learning rather than just grade the test.

Lorene: This class (the PLC) has provided an excellent opportunity to discuss relevant issues. It has helped me develop useful activities that directly tie to content areas of difficulty. My students enjoy doing group activities and self-discovery. The activities we have created are difficult to develop alone and very time consuming. I am hoping to develop and collect more of these activities in the coming months for future use.
The worthiness they both place on the PLC is noteworthy. It is also interesting to point out that they both independently refer to the professional learning community as “this class”. This choice of terminology indicates that they are there to learn, but the term “class” doesn’t seem to quite capture the community nature of the PLC.

In response to the participants’ written reactions, the move to add a fifth week of investigation was made. This additional week provided more time to both investigate mathematical concepts more deeply and to create conceptual activities or problems for classroom use. To address both of these participant goals, a revised five-session cycle was adopted for the second semester that included:

1. studying the mathematical concept,
2. looking at what the teachers had done before and what research says as far as teaching it,
3. developing and implementing an activity,
4. refining the activity and thinking about implementation, and
5. reflecting upon the intervention.

The participants appeared to like the changes and suggested reflecting multiple times if people implemented the PLC activity at different points of time in their classrooms.

Considering the movement by the group to increasing the time spent on investigation, it is interesting to remember that in one of the initial PLC meetings most of the participants were amazed that in the “lesson study” research format (Lewis, 2002) teachers “spend such a long time studying one concept”. Before experiencing the PLC,
the participants were shocked that so much time would be needed to investigate a single mathematical concept.

The call for expansion in the number of weeks in a cycle provides evidence that the teachers grew in their understanding of the complexity of teaching for student understanding. The teachers sought out longer time periods for the PLC to investigate and create together. They went further and recommended in the final cycle of the year to add a sixth week to the PLC structure in order to be able to focus more closely on lesson implementation and student assessment. Table 7 demonstrates how the time spent by the community studying their chosen mathematical topics gradually grew over time from initially three sessions of study to four, to five, and finally a recommendation made by the participating teachers for six sessions at the end of the year-long study.
Table 7

Second Semester Calendar of Events

<table>
<thead>
<tr>
<th>Date</th>
<th>Topic of Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 26th</td>
<td>Why are we here? What’s a PLC?</td>
</tr>
<tr>
<td>September 2nd</td>
<td>Establishing Group Norms</td>
</tr>
<tr>
<td>September 9th</td>
<td>What’s Important in Precalculus? (Setting a timeline)</td>
</tr>
<tr>
<td>September 16th</td>
<td>Function Transformations</td>
</tr>
<tr>
<td>September 23rd</td>
<td>Function Transformations</td>
</tr>
<tr>
<td>September 30th</td>
<td>Function Transformations</td>
</tr>
<tr>
<td>October 7th</td>
<td>Rate of Change</td>
</tr>
<tr>
<td>October 14th</td>
<td>(FALL BREAK)</td>
</tr>
<tr>
<td>October 21st</td>
<td>Rate of Change</td>
</tr>
<tr>
<td>October 28th</td>
<td>Rate of Change</td>
</tr>
<tr>
<td>November 4th</td>
<td>Rate of Change</td>
</tr>
<tr>
<td>November 11th</td>
<td>Veteran’s Day</td>
</tr>
<tr>
<td>November 18th</td>
<td>Logarithms</td>
</tr>
<tr>
<td>November 25th</td>
<td>Logarithms</td>
</tr>
<tr>
<td>December 2nd</td>
<td>Logarithms</td>
</tr>
<tr>
<td>December 9th</td>
<td>Logarithms</td>
</tr>
<tr>
<td>December 16th</td>
<td><strong>SEMESTER BREAK</strong></td>
</tr>
<tr>
<td>through January 6th</td>
<td></td>
</tr>
<tr>
<td>January 13th</td>
<td>2nd semester planning (topic selection)</td>
</tr>
<tr>
<td>January 20th</td>
<td>Radians/Fractions (study concept)</td>
</tr>
<tr>
<td>January 27th</td>
<td>Radians/Fractions (practitioner/research knowledge)</td>
</tr>
<tr>
<td>February 3rd</td>
<td>Radians/Fractions (activity development &amp; implementation)</td>
</tr>
<tr>
<td>February 10th</td>
<td>Radians/Fractions (refine activity)</td>
</tr>
<tr>
<td>February 17th</td>
<td>Composition of Functions (study concept)</td>
</tr>
<tr>
<td>February 24th</td>
<td>Composition of Functions (practitioner/research knowledge)</td>
</tr>
<tr>
<td>March 2nd</td>
<td>Composition of Functions (activity development &amp; implementation)</td>
</tr>
<tr>
<td>March 9th</td>
<td>Composition of Functions (refine activity)</td>
</tr>
<tr>
<td>March 16th</td>
<td><strong>SPRING BREAK</strong></td>
</tr>
<tr>
<td>March 23rd</td>
<td>Composition of Functions (refine activity – continued)</td>
</tr>
<tr>
<td>March 30th</td>
<td>Rational Functions (study concept)</td>
</tr>
<tr>
<td>April 6th</td>
<td>Rational Functions (practitioner/research knowledge)</td>
</tr>
<tr>
<td>April 13th</td>
<td>Rational Functions (activity development &amp; implementation)</td>
</tr>
<tr>
<td>April 20th</td>
<td>Rational Functions(refine activity)</td>
</tr>
<tr>
<td>April 27th</td>
<td>Reflect on Composition of Functions Lesson</td>
</tr>
<tr>
<td>May 4th</td>
<td>Reflect on Rational Functions Lesson</td>
</tr>
<tr>
<td>May 11th</td>
<td>Plan for next year</td>
</tr>
<tr>
<td>May 18th – 25th</td>
<td>Post PLC interviews, Teacher reflections</td>
</tr>
</tbody>
</table>
The Professional Learning Community Cycle of Investigation

I now turn to providing a clear description of the six sessions comprising the professional learning community cycle of investigation that emerged by the end of this study. Each cycle of investigation centers around a teacher-selected precalculus mathematical topic related to the function concept (e.g. rate of change, composition, transformations) that was perceived by the teachers as challenging for students or was one that they desired to learn more about. One of the unique characteristics of this model for professional development is the capability of the community to direct its investigative focus and collaborative efforts towards matters that the participants find problematic in their own teaching environment. In other words, the PLC is purposefully designed to be responsive to the practitioners’ needs and desires. Each cycle consists of meetings focused on the chosen mathematical topic and devoted to advancing the teachers’ conceptual knowledge, pedagogical content knowledge, beliefs, and classroom practices. Each of the sessions contributes to achieving the goal of understanding and enhancing students’ conceptual understanding. Figure 2 is a depiction of the components in pictorial form.
In this section, I provide the theoretical basis for each of the six sessions within the cycle and share, as appropriate, the specific tools and techniques used to focus the participating teachers on students’ mathematical thinking and on deepening their own knowledge. A careful detailing of one cycle and its impact on one of the participating teachers (Jeanne) is presented in Chapter 6.

Session 1: Conceptual knowledge. The researcher established earlier in this dissertation that the level of a teacher’s conceptual knowledge has a large impact on his/her ability to provide for students high-quality learning experiences that are conceptually based. To be effective, teachers must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks. Teachers also need to have a flexible understanding of the different representations of an idea, the relative strengths and weaknesses of each, and how they are related to one another (Wilson et al., 1987). Teachers need to understand the big ideas of mathematics and be able to represent mathematics as a coherent and connected
enterprise (Ma, 1999). Their decisions and their actions in the classroom— all of which affect how well their students learn mathematics— should be based on this knowledge. Since the ultimate goal of a professional learning community is to advance students’ learning, it is of paramount importance that the participating teachers’ conceptual knowledge be increased and deepened. Therefore, to initiate the cycle of investigation the facilitators led the participants through a process developed to focus on this significant aspect of teacher knowledge.

To prepare the participants for a successful dialogue in the PLC regarding conceptual knowledge, a variety of tools and techniques were employed by the facilitators. The strategies included assigning the participants individual and collaborative pre-meeting tasks or assignments including:

- recognizing and citing the central and peripheral ideas with respect to the function concept and their relative importance,
- expressing the prior concepts and knowledge that are required to understand a particular concept,
- demonstrating the different representations of a concept, the relative strengths and weaknesses of each, and how they are related to one another,
- articulating the contextual or “real world” meaning of an abstract mathematical concept,
- solving challenging mathematics problems including providing reasoned responses and justification of thought processes, and
- creating further extensions of a problem.
When the teachers convened again after completing their assigned tasks, the meeting’s objective was to have the teachers look deeply at the mathematical concept they had chosen to investigate. PLC activities that occurred during this part of the cycle included having discussions about content that centered on what it meant to understand, what was most important about a concept, or what made a concept challenging to learn. Other activities included observing video clips or reading interview transcripts of students solving mathematics problems, studying research articles, and envisioning how one could arrive at the PCA distracters (wrong answers).

Ball (2003) recommends that teachers’ opportunities to learn must equip them with the mathematical knowledge and skill that enables them to teach mathematics effectively. As a consequence, the process of focusing on teachers’ conceptual knowledge in the PLC was not clearly distinct from looking at the mathematics from the vantage point of students; the two interacted with, overlapped, and informed each other. Figure 2 shows this interaction and is represented as a double arrow between conceptual knowledge and pedagogical content knowledge.

**Session 2: Pedagogical content knowledge.** Research has shown that having a solid understanding of mathematical concepts and principles is not the only type of required knowledge for teaching mathematics successfully. Earlier in this report a second more practical form of knowledge, pedagogical content knowledge, is described as being used by teachers to guide their actions in the highly contextualized mathematics classroom. This knowledge entails envisioning what it looks like for someone else to have an understanding of a concept (Silverman, 2005), knowledge of how to structure
and represent academic content for direct teaching to students, knowledge of the common conceptions students have and the difficulties they encounter when learning particular content, and knowledge of the specific teaching strategies and interventions that can be used to address students’ learning needs in particular classroom circumstances.

Pedagogical content knowledge builds on the other forms of professional knowledge, and is, therefore, a critical—and perhaps even the paramount—element in the knowledge base of teaching (Shulman, 1987). Considering the extreme importance of this type of teacher knowledge, the objective of the second session of the PLC cycle of investigation was to study what is known about learning and teaching the chosen mathematical concept and to investigate various ways to help students understand the concept. Both research and practitioner knowledge was accessed in this session. Insights into the following questions were sought:

1. What does the available research say about the concept’s complexities and how to better teach it to students?
2. What have the participants done in the past to teach the concept? What has worked? What hasn’t worked? How do you know?

In an attempt to prepare the participants for this session, the facilitators in the study periodically asked them to read research articles that explained the complexities of a concept, provided a framework for developing a better understanding of a concept, or gave an idea of how to better teach a concept. Another technique used by the facilitators to focus the participants on pedagogical content knowledge was to ask them to give one or two conceptual problems to their students to solve and to then analyze the results and
report their findings to the PLC. This illuminated for the teachers one of their students’ thought processes about a concept. A final technique was to have the teachers critique the strategies used and examples chosen by the authors of their textbook to convey a concept. They were then to suggest possible gaps in the lesson and what they envisioned as possibilities to fill those holes.

The pedagogical content knowledge session focused on revealing and coming to an understanding of the kinds of conceptions the students have with regard to a concept. This was done in order to plan a roadmap for how students might acquire an understanding of a concept by building upon their existing knowledge base (Silverman, 2005). The participants were encouraged in this session to share with the PLC what they perceived to be their most powerful and effective analogies, metaphors, representations, models, verbal descriptions, and examples for facilitating and enhancing the learning experience of students. Chapter 6 provides some of the interesting results with regard to one of the participant’s beliefs about the learning and teaching of mathematics and, consequently, what she perceived to be “good” instructional materials and techniques. In the subsequent session of the cycle of investigation, the task of the PLC was to come to consensus on a classroom intervention (e.g. an activity, a problem set, a lesson) built upon the current level of their students’ understanding.

Session 3: Intervention. Effective mathematics teaching requires a serious commitment to the development of students' understanding of mathematics. Because students learn by connecting new ideas to prior knowledge, teachers must understand what their students already know (Ma, 1999; Mason & Spence, 1999; Schoenfeld, 1988;
Effective teachers know how to ask questions and plan lessons that reveal students' prior knowledge; they can then design experiences and lessons that respond to, and build on, this knowledge. The National Council of Teachers of Mathematics through the *Principles and Standards* (2000) lays out the agenda for secondary mathematics curriculum by stating that curricular materials must be coherent and focused on important mathematics.

A school mathematics curriculum is a strong determinant of what students have an opportunity to learn and what they do learn. In a coherent curriculum, mathematical ideas are linked to and build on one another so that students' understanding and knowledge deepens and their ability to apply mathematics expands. An effective mathematics curriculum focuses on important mathematics—mathematics that will prepare students for continued study and for solving problems in a variety of school, home, and work settings.

A coherent curriculum effectively organizes and integrates important mathematical ideas so that students can see how the ideas build on, or connect with, other ideas, thus enabling them to develop new understandings and skills. Researchers (Stigler & Hiebert, 1999) have shown that one important characteristic of the lessons they observed had to do with the internal coherence of the mathematics. The researchers found that typical Japanese lessons were designed around one central idea, which was carefully developed and extended; in contrast, typical American lessons included several ideas or topics that were not closely related and not well developed. The authors of *Principles and Standards* (NCTM, 2000) suggest in planning individual lessons, teachers should strive
to organize the mathematics so that fundamental ideas form an integrated whole. Big ideas encountered in a variety of contexts should be established carefully, with important elements such as terminology, definitions, notation, concepts, and skills emerging in the process. Mathematics curricula should focus on mathematics content and processes that are worth the time and attention of students.

Mathematics topics can be considered important for different reasons, such as their utility in developing other mathematical ideas, in linking different areas of mathematics, or in deepening students' appreciation of mathematics as a discipline and as a human creation. Ideas may also merit curricular focus because they are useful in representing and solving problems within or outside mathematics. The Precalculus Concept Assessment (PCA) Taxonomy (as was presented in Table 4) spells out foundational ideas that should have a prominent place in a precalculus course because they enable students to understand other mathematical ideas, connect ideas across different areas of mathematics, and have been found to be foundational to the study of calculus. Mathematical thinking and reasoning skills, including making conjectures and developing sound deductive arguments, are important as well because they serve as a basis for developing new insights and promoting further study. In addition, the curriculum should offer experiences that allow students to see that mathematics has powerful uses in modeling and predicting real-world phenomena.

The six topics chosen for investigation by the teacher participants during the year-long PLC are all represented in the PCA Taxonomy. Table 8 shows where each selected topic fits into the taxonomy and is represented in bold-face type. In this way the
participants of the PLC selected central ideas of the function concept to study. This
dissertation study focuses on the trigonometric cycle of investigation for careful analysis
in Chapter 6.
Table 8

**PLC Selected Topics for Investigation as Represented in the PCA Taxonomy**

<table>
<thead>
<tr>
<th>Reasoning Abilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
</tr>
<tr>
<td>Apply proportional reasoning</td>
</tr>
<tr>
<td>R2</td>
</tr>
<tr>
<td>View a function as a process that accepts input and produces output</td>
</tr>
<tr>
<td>R3</td>
</tr>
<tr>
<td>Apply covariational reasoning</td>
</tr>
<tr>
<td>• Coordinate two varying quantities while attending to how the quantities change in relation to each other</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Conceptual Abilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
</tr>
<tr>
<td>Interpret function information (table, formula, graph)</td>
</tr>
<tr>
<td>C1P</td>
</tr>
<tr>
<td>At points (t, f, g)</td>
</tr>
<tr>
<td>C1I</td>
</tr>
<tr>
<td>On intervals of the domain (t, f, g)</td>
</tr>
<tr>
<td>C1D</td>
</tr>
<tr>
<td>By attending to domain restrictions inherent in the function (t, f, g)</td>
</tr>
<tr>
<td>C2</td>
</tr>
<tr>
<td>Represent contextual function relationships using function notation (table, formula, graph)</td>
</tr>
<tr>
<td>C2V</td>
</tr>
<tr>
<td>Identify, define and relate variable quantities as functional relationships (t, f, g)</td>
</tr>
<tr>
<td>C2C</td>
</tr>
<tr>
<td>Use function composition to relate specified quantities (t, f, g)</td>
</tr>
<tr>
<td>C3</td>
</tr>
<tr>
<td>Perform function operations (table, formula, graph)</td>
</tr>
<tr>
<td>C3E</td>
</tr>
<tr>
<td>Function evaluation (t, f, g)</td>
</tr>
<tr>
<td>C3A</td>
</tr>
<tr>
<td>Function arithmetic (t, f, g)</td>
</tr>
<tr>
<td><strong>C3C</strong> Function composition (t, f, g)</td>
</tr>
<tr>
<td>C3I</td>
</tr>
<tr>
<td>Function inverse (t, f, g)</td>
</tr>
<tr>
<td><strong>C3T</strong> Function translations (t, f, g)</td>
</tr>
<tr>
<td>C4</td>
</tr>
<tr>
<td>Understand how to reverse the function process (table, formula, graph)</td>
</tr>
<tr>
<td>C4E</td>
</tr>
<tr>
<td>Solve equations that involve functional relationships (t, f, g)</td>
</tr>
<tr>
<td>C4IN</td>
</tr>
<tr>
<td>Solve inequalities that involve functional relationships (t, f, g)</td>
</tr>
<tr>
<td>C4IF</td>
</tr>
<tr>
<td>Understand the meaning of an inverse function (t, f, g)</td>
</tr>
<tr>
<td>C5</td>
</tr>
<tr>
<td>Interpret and represent function behaviors (table, formula, graph)</td>
</tr>
<tr>
<td>C5L</td>
</tr>
<tr>
<td>Linear (t, f, g)</td>
</tr>
<tr>
<td><strong>C5P</strong> Polynomial (t, f, g)</td>
</tr>
<tr>
<td>C5R</td>
</tr>
<tr>
<td>Rational (t, f, g)</td>
</tr>
<tr>
<td>C5E</td>
</tr>
<tr>
<td>Exponential (t, f, g)</td>
</tr>
<tr>
<td><strong>C5L</strong> Logarithmic (t, f, g)</td>
</tr>
<tr>
<td><strong>C5T</strong> Trigonometric (t, f, g)</td>
</tr>
<tr>
<td>C6</td>
</tr>
<tr>
<td>Interpret and represent <strong>rate of change</strong> information for a function (table, formula, graph)</td>
</tr>
<tr>
<td>C6C</td>
</tr>
<tr>
<td>Interpret the relative growth of the input and output variables (t, f, g)</td>
</tr>
<tr>
<td>C6A</td>
</tr>
<tr>
<td>Determine average rate-of-change (t, f, g)</td>
</tr>
<tr>
<td>C6I</td>
</tr>
<tr>
<td>Interpret rate-of-change information on intervals of the domain (t, f, g)</td>
</tr>
<tr>
<td>C6E</td>
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<tr>
<td>Understand multiplicative rate-of-change (exponential growth) (t, f, g)</td>
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In session three of the PLC cycle of investigation, the goal was to work together with other professional colleagues to develop or adapt a classroom intervention in the form of an activity, example, problem set, or adaptation of the textbook to enhance student learning. The facilitators prepared the participants for this task by asking them to:

- list the important ideas related to the mathematical concept,
- make suggestions for what the classroom intervention should address by considering what they believed was missing from or done inadequately in their textbook,
- determine how the concept fit into the “big picture” (i.e. other courses, previous courses, subsequent courses, applications, etc.), and
- bring in an activity, problem, or assignment that they had used to teach the concept and believed to be effective in building conceptual understanding.

The discussions from this session were used to determine what could be done in the classroom to help students learn. The focus was on curriculum selection/adaptation or problem development in order to help students build on or reevaluate their conceptions about a concept.

Session 4: Implementation. Students learn mathematics through the lens of their prior conceptions and the experiences that teachers provide (Piaget, 1933). Much is known about effective mathematics teaching, and this knowledge should guide professional judgment and activity. To be effective, teachers need to understand and be committed to their students as learners of mathematics and be skillful in choosing from and using a variety of pedagogical strategies (NCTAF, 1996). Pedagogical knowledge,
much of which is acquired and shaped through the practice of teaching, helps teachers understand how students learn mathematics, become proficient with a range of different teaching techniques and instructional materials, and organize and manage the classroom (NCTM, 2000).

Teachers have different styles and strategies for helping students learn particular mathematical ideas, and there is no one "right way" to teach. However, effective teachers recognize that the decisions they make shape students' mathematical dispositions and can create rich settings for learning. Teachers make many choices each day about how the learning environment will be structured and what mathematics will be emphasized. These decisions determine, to a large extent, what students learn. Effective teaching conveys a belief that each student can and is expected to understand mathematics and that each will be supported in his or her efforts to accomplish this goal (NCTM, 2000). Teachers' actions are what encourage students to think, question, solve problems, and discuss their ideas, strategies, and solutions. The teacher is responsible for creating an intellectual environment where serious mathematical thinking is the norm.

If students are to learn to make conjectures, experiment with various approaches to solving problems, construct mathematical arguments and respond to others' arguments, then creating an environment that fosters these kinds of activities is essential. One tool used to assess a teacher’s classroom practices is the Reformed Teaching Observation Protocol (RTOP) (ACEPT, 2000). The RTOP was designed to help one infer whether a teacher has integrated the elements of constructivism into their classroom practices and whether students in these classes are accustomed to contributing ideas (Lawson et al.,
In session 4 of the cycle of investigation, classroom implementation was discussed. The goal was to encourage the participating teachers to consider the potential instructional techniques and tools at their disposal. One way used in the study to accomplish this was to ask the teachers to prepare for the PLC session by thinking about and listing the different teaching techniques (i.e. lecture, group work, projects, experiments, etc.) that they believed could be used to teach the concept most effectively using the classroom activity that had been decided upon in the previous session. They were to also provide justification for why they had selected those that they did. The focus of the PLC meeting was placed on discussing and planning for the best teaching methods to use in order to reach the most students. Participants were encouraged to challenge each others’ beliefs regarding pedagogical issues in order to spur one another to think “outside-the-box” and to venture outside their “comfort zone”. To conclude the meeting, each teacher individually chose how to implement (or not implement) the activity in their own classroom.

Session 5: Assessment. When assessment is an integral part of mathematics instruction, it contributes significantly to students' mathematics learning. Assessment should be an integral part of instruction that informs and guides teachers as they make instructional decisions. Assessment should not merely be done to students; rather, it should also be done for students, to guide and enhance their learning (NCTM, 2000). Research indicates that making assessment an integral part of classroom practice is
associated with improved student learning (Black and Wiliam, 1998). To ensure deep, high-quality learning for all students, assessment and instruction must be integrated so that assessment becomes a routine part of the ongoing classroom activity rather than an interruption. To maximize the instructional value of assessment, teachers need to move beyond a superficial "right or wrong" analysis of tasks to a focus on how students are thinking about the tasks. Efforts should be made to identify valuable student insights on which further progress can be based rather than to concentrate solely on errors or to issue grades.

Good assessment can enhance students' learning in several ways. First, the tasks used in an assessment can convey a message to students about what kinds of mathematical knowledge and performance are valued. When teachers use assessment techniques such as observations, conversations and interviews with students, students are likely to learn through the process of articulating their ideas and answering the teacher's questions. This can also help students understand the characteristics of a complete and correct response. Similarly, classroom discussions in which students present and evaluate different approaches to solving complex problems can hone their sense of the difference between an excellent response and one that is mediocre (Cobb et al., 2001). Through the use of good tasks and the public discussion of criteria for good responses, teachers can cultivate in their students both the disposition and the capacity to engage in self-assessment and reflection on their own work and on ideas put forth by others (Schoenfeld, 1998).
In session 5 of the cycle of investigation, the objectives were to help teachers understand their mathematical goals and how their students may be thinking about mathematics, to learn possible means of assessing students' knowledge and how to interpret assessment information from multiple sources. For teachers to attain this type of important and necessary knowledge, assessment must be a major focus in the professional learning community.

In preparation for the session, the participating teachers were asked to collaborate with a PLC partner and to bring in assessment ideas and/or problems for determining the level of students’ understanding. Periodically student interviews were performed by the participating teachers to assess the level of understanding the student had. This led to some interesting findings and growth in one of the teachers (see Chapter 6). To facilitate growth in the teachers, the facilitators focused this session on discussing how to effectively evaluate student understanding as a result of the intervention and implementation that had been employed in the classroom. In addition, analysis of student work (e.g. a homework, quiz, or test problem) or student interviews was performed at times during this session as well.

Session 6: Reflection. Many current researchers agree that for meaningful and lasting changes in mathematical beliefs and practices to occur, teachers need to engage in practical inquiry—to move back and forth among a variety of settings to learn about new instructional strategies, to try them out in their own classrooms, and to reflect on what they observed in a collaborative setting (Stipek, Givven, Salmon, MacGyvers, 2001; Borko, Mayfield, Marion, Flexer & Cumbo, 1997; Franke, Fennema, Carpenter, Ansell &
Critical reflection, however, involves challenging the taken-for-granted assumptions of teaching and schooling practices and imagining alternatives for the purposes of changing conditions (Louden, 1992; Tabachnich & Zeichner, 1991). To von Glasersfeld (1991), reflection is the ability of an individual to “step out of the stream of directed experience, to re-present a chunk of it, and to look at it as though it were direct experience, while remaining aware of the fact that it is not” (p. 47). Cooney (1999) suggests that an effort to improve instruction begins with the teacher reflecting on what mathematics means to him/her and how he/she envisions its teaching: “What seems critical is that teachers see something problematic about the doing, teaching, and learning of math” (p. 175).

Effective teaching requires continuing efforts to learn and improve. These efforts include learning about mathematics and pedagogy, benefiting from interactions with students and colleagues, and engaging in ongoing professional development and self-reflection. Reflection and analysis are often individual activities, but they can be greatly enhanced by teaming with an experienced and respected colleague, a new teacher, or a community of teachers. Collaborating with colleagues regularly to observe, analyze, and discuss teaching and students' thinking is a powerful, yet neglected, form of professional development in American schools (Stigler & Hiebert, 1999).

Opportunities to reflect on and refine instructional practice—during class and outside class, alone and with others—are crucial in the vision of school mathematics...
outlined in *Principles and Standards* (NCTM, 2000). To improve their mathematics instruction, teachers must be able to analyze what they and their students are doing and consider how those actions are affecting students' learning. Using a variety of strategies, teachers should monitor students' capacity and inclination to analyze situations, frame and solve problems, and make sense of mathematical concepts and procedures. They can use this information to assess their students' progress and to appraise how well the mathematical tasks, student discourse, and classroom environment are interacting to foster students' learning. They then use these appraisals to adapt their instruction.

To prepare for session 6 of the cycle of investigation, the teachers were asked to implement and/or observe the PLC activity and prepare a written reflection on the learning and teaching of the activity. Those that taught the lesson asked any fellow participants that served as classroom observers to specifically critique one or two aspects of teaching or to assess one student’s learning during the lesson to narrow the scope of analysis. All were to bring back artifacts of student work (e.g. a homework, quiz, test problem) to the next PLC meeting for discussion. The objective of session 6 was to reflect on the classroom experience, to document important findings, and to revise the activity that was used to improve it for future use and to learn how to better teach the concept.

Some of the techniques and tools that were used during this session of the cycle included having the PLC participants:

- examine the artifacts from the classroom implementation (e.g. student work, interview, observation data),
• assess whether students’ demonstrated a deeper understanding of the material, and
• reflect upon the implementation and processing and to suggest changes that should be made to the activity and/or the teaching methods.

Conclusion

This chapter provides insights into the first two research questions of this dissertation study:

• What professional learning community attributes are associated with the development of conceptual knowledge and pedagogical content knowledge relative to the function concept in secondary mathematics teachers?
• What professional learning community support tools are associated with improvements in the facilitation of secondary teachers’ reflection on students’ thinking and reasoning relative to the function concept?

The data suggest that the sessions in the PLC cycle of investigation address six very important aspects of effective instruction. The teachers in this study appeared to benefit from exploring several different kinds of mathematical knowledge:

• knowledge of the central and important ideas of mathematics that is deep, connected, and flexible (Session 1: Conceptual Knowledge),
• knowledge about the challenges students are likely to encounter in learning these ideas and knowledge about how the ideas can be represented to teach them effectively (Session 2: Pedagogical Content Knowledge),
• knowledge to make wise curricular judgments, respond to students’ questions, and look ahead to where concepts are leading and plan accordingly (Session 3: Intervention),

• knowledge of pedagogy that helps teachers become proficient with a range of different teaching techniques and instructional materials, and organize and manage the classroom (Session 4: Implementation),

• knowledge about how students’ understanding can be assessed (Session 5: Assessment), and

• knowledge that is informed by engaging in reflective practice and continuous self-improvement (Session 6: Reflection).

In this section, the theoretical basis for each of the six sessions within the cycle was provided and some of the specific tools and techniques were shared that were used to focus the participating teachers on students’ mathematical thinking and on deepening their own knowledge. A careful detailing of one cycle (sessions 16 - 20) and its impact on one of the participating teachers follows in the next chapter.

The tools employed in the PLC to facilitate secondary teachers’ reflection on students’ thinking and reasoning included analyzing their students’ work (e.g. quizzes, tests, problem solving), interviewing students solving a conceptually-based problem, studying PCA distracters, observing video clips of students explaining their thinking, reading interview transcripts of students solving mathematics problems, and studying research articles.
CHAPTER 6: RESULTS

Introduction

From a constructivist view of knowledge development (von Glaserfeld, 1991), teacher knowledge is personal, in the sense that teachers formulate and draw upon their personal understandings of and experience with the practical circumstances in which they work (Elbaz, 1991; Russell and Munby, 1992). Consequently, armed with the best theories or techniques that mathematics education has to offer on teaching for conceptual understanding, the individual teacher is the one who makes inferences about when and how particular practices are appropriate for use. Thus, what those techniques or processes become when they are implemented is the product of the personal meaning and beliefs of the teacher (Chapman, 1997). Consequently, it is the personal growth in knowledge of one individual and the decisions that she makes as to what teaching techniques are appropriate for use in her classroom that this chapter seeks to reveal.

All six teachers in this research study were enthusiastic about discussing the learning and teaching of mathematics in the PLC sessions, but the teachers were not uniform when it came to their willingness and/or ability to alter their classroom practices based on their PLC experiences. Therefore, the decision was made to analyze one participant—Jeanne—because she was instrumental and supportive in helping the facilitators initiate and implement the PLC, was viewed as a leader in her department, and appeared to have the most potential for providing revealing findings. In addition, it was thought that by choosing a teacher resistant to changing her classroom practices and yet...
was supportive of the project’s goals, it would be possible to expose some of the barriers that can emerge while implementing a professional learning community in a secondary school.

In the following pages the impact that the professional learning community experience had on Jeanne is reported. Results are provided from investigating the effect that the PLC sessions had on the teacher’s growth in conceptual knowledge and pedagogical content knowledge. Data about the knowledge and beliefs that the teacher holds about the learning and teaching of mathematics are presented and personal behavioral traits that appeared to affect her instructional decision-making are shown. Finally, data are reported to reveal the impact that the PLC sessions had on her professed classroom practices. The chapter follows Jeanne through relevant portions of the cycle of investigation pertaining to trigonometry and reports passages from her reflections at the conclusion of the year-long professional learning community. Many excerpts are provided from the PLC-audiotaped data, interview transcripts, and periodic email reflections to document the professional growth of the subject.

The PLC Theoretical Framework provided the lens for analyzing and reporting the data. The framework’s three components and range of subdimensions are noted throughout this chapter by coding the data according to the notation presented in Chapter 3 (L1: Conceptual Knowledge, L2: Pedagogical Content Knowledge, L3: Beliefs about Learning and Teaching Mathematics). This section begins with the characterization of Jeanne in terms of her pre-PLC conceptual knowledge, classroom practices, and beliefs about the learning and teaching of mathematics. Analysis of the data involved
coding/noting the shifts in the subject relative to the three components of the framework over one cycle of investigation and at the conclusion of the participant’s year-long experience.

The primary goal of this chapter is to provide results pertaining to the following three research questions:

- What conceptual knowledge and pedagogical content knowledge are exhibited by a secondary mathematics teacher as she participates in a professional learning community?
- What beliefs about the learning and teaching of mathematics are exhibited by a secondary mathematics teacher as she participates in a professional learning community?
- What self-reported classroom practices does a secondary mathematics teacher display as she participates in a professional learning community?

The Case of Jeanne: An Experienced Secondary Mathematics Teacher

Jeanne is a case of an experienced teacher who has taught for over 15 years at the secondary level and yet was interested in advancing her knowledge base and working collaboratively with colleagues to learn about student thought processes. She reports that she is supportive of reform curriculum, but is hesitant to utilize pedagogical methods or instructional materials developed collaboratively by the PLC in her classroom. Finally, Jeanne details the reasons why she is not very receptive to changing her classroom practice.
Characterization of Jeanne

Jeanne is the school’s department chair and was the first mathematics teacher hired when the school was opened in 1993. The principal hand-selected her out of another district school to establish the Mathematics Department and is the only department chair the school has had. In her twelve-year tenure as chair, Jeanne has had the primary influence on the curriculum and personnel decisions of the school’s mathematics department. As was documented in Chapter 5, her strong desire for teachers to facilitate student understanding of mathematics and her commitment to ongoing professional development are set firmly in the philosophy of the department. Jeanne did not go right into teaching after graduating from college because she wanted to be a wife and stay-at-home mother first. As a result, her teaching career did not begin until she was age 40. She had sixteen years of teaching experience at the time of the study.

Conceptual Knowledge

Jeanne has attained a Master’s degree in Mathematics Education and is regarded by the faculty in the Mathematics Department as the resident expert. She is frequently sought out by teachers for her assistance in answering questions regarding mathematical content. In Jeanne’s pre-PLC assessment, she was shown to have a solid grasp of the function concept as measured on the Precalculus Concept Assessment (PCA). Recall that this quantitative instrument is a multiple-choice examination of 25 questions and was designed to measure students’ conceptual knowledge. Jeanne scored a perfect 25 out of 25 on this examination. The mean score for the 100 precalculus teachers who have taken
this exam is 19.24 and the standard deviation is 3.47. Comparing her test score to the other precalculus teachers’ scores, she performed at the 95th percentile. Based on these data, Jeanne is characterized as having well-above-average conceptual understanding of some aspects of the function concept in comparison to a similar population of secondary precalculus teachers who took the assessment. The PCA instrument is limited in its ability to assess the mathematical knowledge needed for teaching since it was designed to assess students’ understanding of the function concept. Therefore, this assessment is not an indication of the level of connectedness of Jeanne’s knowledge, nor does it provide insight into her understanding of the complexity of learning the function concept or her mathematical knowledge for teaching.

Classroom Practices

In an effort to characterize Jeanne’s classroom instructional practices, she was observed very early in this study before the PLC had been long established and again later in the year using the Reformed Teaching Observation Protocol (RTOP) instrument (Appendix I). As was shown in Chapter 4, the RTOP consists of 25 Likert scale items measuring the extent to which classroom practice is reformed and aligned with constructivist teaching principles. Each RTOP item is scored on a 0 (never observed) to 4 (very descriptive) scale. Thus, total RTOP scores can range from 0 to 100. Previous studies have shown the mean RTOP score among high school science and mathematics teachers to be 44.4 (Judson, 2002). A score above this infers the teacher has integrated many elements of constructivism, and students in these classes are accustomed to
contributing ideas. Scores from multiple classroom observations are averaged to assign each teacher a single RTOP score. Previous research has found science and mathematics achievement to correlate highly with RTOP scores (Lawson et al., 2002; Sawada et al., 2002).

The average score of Jeanne’s two RTOP observations was 36. Comparing this information to the statistics provided above, the researcher placed Jeanne into a wider population of teachers with regard to the level of reform instruction taking place in her classroom. Jeanne is characterized as a slightly below-average teacher with respect to the level of reform teaching that takes place in her classroom as described by the RTOP. Jeanne’s observed lessons placed value on a variety of student solutions to problems, focused on concepts rather than procedures, and often took their direction from ideas generated by the students as Jeanne responded to their remarks. However, while these are aspects of reform instruction as laid out in the RTOP instrument, the observed lessons did not begin with recognition of students’ prior knowledge and conceptions, nor did they attempt to engage students as members of a learning community. There was not much time allocated to student investigation of mathematics or student-to-student discussion. Rather, Jeanne’s classroom instruction is characterized as being teacher-centered and teacher-directed. The researcher’s notes from the first of Jeanne’s classroom observations showed evidence of this.

At the outset of the lesson, Jeanne was upset that the students didn’t do their assigned homework. She told them to do the work in class. The students worked on their own to complete the problems. After approximately ten minutes, Jeanne
asked the students to recall what they remembered about *average rate of change*. There was silence. One student brought up \( \frac{g(b) - g(a)}{b - a} \). Students did not appear to know what this notation meant. In an effort to initiate discussion, Jeanne fired off a lot of questions dealing with function notation and graphical interpretation, but did not provide adequate time for students to think nor did she encourage discussion among students. She then exhibited a sense of frustration with the students’ lack of knowledge: “Don’t you remember this? You should; you are in Precalculus! Have you ever done \( f(-3) \)? You should have. How do you find it on the graph?” She proceeded to work through some examples from the textbook while the students took notes. The problems she worked through on the board were conceptually-based and showed how to determine the average rate of change graphically and using the formula.

**Beliefs about the Learning and Teaching of Mathematics**

Jeanne frequently expressed strong beliefs about what and how mathematics should be taught. Jeanne began to disclose her beliefs early in the study during a pre-PLC interview designed to elicit teacher beliefs. Jeanne has long desired to “make sense” of mathematics, and the reason she gave for becoming a teacher of mathematics was her desire to “explain it better”.

I thought I could (laughter) explain word problems. I remember thinking that when I was in college. [L3.4b] My math teacher said to just wait until you take calculus, and all this will make sense. Well, I thought, “I **have** to take calculus
because I want to see the end of the story!” [L3.2d, L3.3b] I thought, “I can explain this stuff” and explain it better than I had it explained to me. [L3.4b]

These feelings have helped to establish Jeanne’s goals for the students at her school. She wants them to understand mathematical concepts and does not believe in “wasting time” with learning procedures or on what she perceives to be outdated topics. Jeanne seeks justification for what is taught to students and believes that warranted reasons for teaching a concept include its applicability in various contexts and students’ need for it in future academic courses—mathematics and other disciplines. She attributes this belief to the fact that she became a mathematics teacher later than most and after she had gained years of experience outside the classroom. She explained her rationale by answering the next interview question: “Listening to the description of your experiences, what convinced you to use ‘reform textbooks’ at this school? It appears to be a big change from what you experienced as a student.”

There were things out there that didn’t make sense. When I looked at kids when I was teaching and trying to teach the rational root theorem, I thought, “Why am I wasting my time? This doesn’t make any sense. Why are we doing this? Why am I teaching them to do things they are never going to use or need to do again? This is dumb. It doesn’t make any sense.” There were some things that just seemed like a waste of time. [L3.3b] I mean, I fought—I fought with people for ten years in this district, and there are still people around that still argue that these old-fashioned methods like Descartes’ rule of signs are useful. Maybe it is for some things, but I didn’t start teaching until I was 40, and I come in here and look at the
kids’ faces and think, “How can I justify this? How can I justify teaching this to kids? It just won’t work with my kids. What’s the point?” Well, there is no point. You know what? Let’s not do it—you know—let’s just not do it! There are so many other things that are useful that really have a real issue to them. So, I guess that is why I wanted to change the curriculum. I started looking at it with fresh eyes. [L1.1, L3.3b, BT1a, BT1b]

As a consequence, Jeanne has become a self-described “disciple” of the reform movement. She began a campaign to adopt curriculum for her department that she believed would help students “to think” and to understand mathematics. Over the five years prior to this study she had been teaching Honors Calculus using the Wiley Calculus textbook that heavily emphasizes mathematical concepts and their application. The text is more commonly known as “Harvard calculus” named after the consortium of Harvard University mathematics educators who constructed it. Through her teaching experiences with this “reform” material, Jeanne determined that she had learned the mathematical concepts more deeply. As Jeanne taught Honors Calculus with this curriculum and she saw how greatly her own conceptual knowledge was impacted, she started a drive to secure the same authors’ textbook for the precalculus level as well. She believed that using this type of curriculum would help students to learn mathematics more deeply just as she had. Jeanne tried out the reformed curriculum as a “pilot” in the year prior to the study with her Honors Precalculus class. After receiving what she determined to be favorable results, she independently decided to implement the curriculum in the regular Precalculus classes and to teach the course alongside the other teachers in the department.
Jeanne explains in her own words the rationale for this decision by answering the question, “When you decided to go to the Harvard precalculus book was that your decision or the department’s?”

I had been teaching out of the Harvard calculus book and I really liked that. I liked the approach. At the time, we had a traditional precalculus book, and when the students came into Calculus it was kind of like this (a blank stare). In the first section in the Harvard calculus book, the students didn’t know how to deal with the issues presented early on. It was practical stuff; it was good stuff. It was hard for me to deal with that. They had algebra skills coming into the class, but I haven’t seen much difference now that they are coming out of the Harvard book. [L3.4b] I thought that the traditional precalculus book wasn’t a good match for the Harvard calculus book, and besides that I knew we needed to get our kids thinking! [L3.3b, L3.4a] So I made the decision because I had worked out of the authors’ preliminary book. I used it with just my Honors Precalculus class. The department bought the texts out of our own money that we had earned. After that, I decided that we were going to go with it. Last year was the first year that we used it with all of the kids in regular Precalculus. I don’t know if the teachers were happy with me or not because I just told them we have got to do this! It’s the best thing because it follows our integrated program and all of that. The teachers struggled with it because when you ask the kids to think, boy, you have to think even twice as hard as you are asking them to think. My teachers were struggling with the concepts and how to help kids think. That’s a difficult thing to do, so
they probably swore at me. I was teaching the honors section so they were probably thinking, “Well, she’s up there, and I’m down here!” So now I am teaching the regular Precalculus class, and I did that on purpose so I could see what the problems are and what is going on there. I am wondering how we need to address those things. [L3.4a, L3.3b]

In an early PLC meeting (5), Jeanne talks about a section (chapter 3 section 2) out of the Harvard precalculus text that deals with function transformations and her recent experiences with it. She noted how it deepened her own knowledge and shares the value that she places on the way the text formulates questions that deal with understanding and places problem-solving in the context of a situation that enhances sense making.

This section is not about transformations, but more about meaning. In the first edition of this textbook, there was a great problem on a janitor who came in, turned up the heat, and then turned it off when he left. You were to then look at what happened if he came in five minutes late, or left early, or pushed up the temperature more than usual. [L3.4b, L3.3b] When I first did this kind of problem in this book, I struggled with it; so I understand how my students can struggle, but it still frustrates me when they do. [L2.2, L3.2d] I am telling you that when I had to go through that I struggled with that because you are moving the graph over and it is a plus, but you put in the minus into the formula. The whole thing about when it is a negative you move it to the right and when it is positive you move the graph to the left confounded me. [L1.4, L3.2d] It isn’t bad when you have an
actual situation that they can envision turning the temperature off and then raising it. [L3.4b, L3.3b]

Later in the same PLC session, she reiterated how much she learned about function transformations from using this curriculum. Jeanne understood from her own personal learning experiences how students can struggle with this topic, but she is often perplexed with why they do. Another PLC participant was discussing the results she had gotten from doing an interview with a student who was currently in Calculus and had gotten the interview question correctly when Jeanne commented.

This (function transformations) comes from chapter 3 in the precalculus book. If this student had been in a traditional book he wouldn’t have had a clue. [L3.3b] That whole chapter 3 deals with this issue, and when I went through that when I first taught it, I had trouble. I had trouble with, “What do I have and what is it asking me?” How can I have trouble and not expect them to? I’m telling you. I remember trying to think, “Is it going vertical or horizontal?” and just going crazy. [L1.4, L3.2d, L3.3b] I know kids don’t get it, but I don’t know why they don’t get it. [L2.2]

A transcript follows that both illustrates the importance Jeanne places on students understanding mathematics as well as her being in the role of authority. It shows the depth of her beliefs in reform curriculum and the extreme measures she took to insure that her school would be able to teach students the way in which she felt was best by attaining the Harvard textbook series in an atypical manner.
Imagine that you have just received notice that your district is transferring you from this school to another high school. You have no say in the matter and must go. What is your reaction? What will you miss most about this school? What will you not be looking forward to? What would you be excited about?

Jeanne: (While reading the question, she chuckles.) I am in a particular position that if I were transferred I wouldn’t be the boss. I like being the boss. [BT1a]

F/R: Why?

Jeanne: I feel like I have something to say. I think I have a sense about what you can teach and how you can teach it and what direction we want to go. [BT1b] (Jeanne leans in and whispers.) That is why the curriculum is so different here. We have got the reform curriculum because I have pushed it. I snuck it in when I secured that precalculus book.

F/R: How did that happen?

Jeanne: I got out of Calculus using Harvard, and I started teaching Precalculus. I took the book that the district mandated we use—it was a traditional book—and it looked like it had all kinds of things in it. But when I actually got into it, it was very traditional. We (the mathematics department chairs throughout the district) had adopted that book because we couldn’t find anything that we could
agree on. The whole district adopted it, but I didn’t use it. [BT1b] I couldn’t go back to a traditional book. [BT2b] I couldn’t go back to that old-fashioned stuff like the rational root theorem, all of that factoring, and no connection to the real world. [BT2b, L3.3b, L3.4b, L1.1] It was a total disconnect between the reform calculus we had put in place and the core mathematics curriculum. I thought, “These kids are going to start in math here, and they are just going to be manipulating values and algebraic symbols and not understanding it.” [L3.3b, L3.2a, L1.1] I said to myself that I was not doing that. We had extra money in our department, so we just bought these new books for the Honors Precalculus students. I did that with the honors kids. We spent our money on that and then the next year I asked the curriculum director—who was retiring—if we could pilot the book. Well, you know what happens when you pilot a book; this meant we had it. I knew that no school would have us not use books that we had just spent so much money on. A new lady came into that position and wondered why we were different, and I told her what we did. I snuck it in on purpose because I knew this new lady wasn’t going to let us do it. [BT1a] Up to that point, our curriculum had a reform beginning (integrated), a traditional middle (Precalculus), and a reform ending (Calculus). It just didn’t make any sense. [L3.4b] The other
district schools stayed with the traditional. One of the other schools is now using the Harvard. They have just broken away from the traditional, but we have been doing it for awhile now. If I leave to go to another school then I would be going back to a traditional program. If I was forced to go, I would do what I wanted to do, and I wouldn’t care what they said. I would teach how I want to teach. [BT1b, BT2b] I would not be a “happy camper” if I had to transfer. If I moved out of being department chair, I wouldn’t be a “happy camper” either. [BT1a, L1.1, L3.2a, L3.4a, L3.4b]

Sensing that it was important to determine why Jeanne believed so strongly about being in charge, the researcher continued with this theme. In the same pre-PLC interview, she demonstrated self-confidence in knowing what her students need in terms of preparation for calculus and in making connections of mathematical concepts. She shared the pleasure she finds in taking the responsibility for curricular decisions.

F/R: Why do you feel so strongly about reform curriculum? Why don’t you like the traditional material?

Jeanne: The traditional is easy to teach. I mean, I can teach it, but it’s just ugly; there’s just no point to it. You are just grinding these kids through. It’s a grind of algebra that doesn’t make any sense, and these kids are looking at you like you are “out of your mind” for asking them to do it! [L3.3b] Using reform materials our kids look at us like we are “out of our minds” too, but at least we’re trying to
make some kind of connection for them. I think it is better. [L3.3b] We threw out all of those things that I don’t think belong in a precalculus course. [L1.1, BT1a, BT1b] I know where they are going in calculus, and it is important to know where the kids are going. If teachers just see their own little world, just algebra and geometry, then they are going to teach it thinking that is the end, but it’s just the beginning! [L1.1, L3.4a]

F/R: Why do you think it is that people don’t want to switch to the reform curriculum?

Jeanne: Because it is hard. It’s hard to teach. It’s easier to do the way you have been taught. I just think that is what it is. If you look around this country and talk to teachers, most are doing the same mathematics that I was raised on in the 1950’s and 60’s. [L3.3b] With all of the stuff we have got going on, and all of the technology and all of the understanding that kids have to have—and we are still teaching that old-fashioned stuff? It just amazes me that things haven’t changed. It is just ludicrous. [L1.1, L3.3a, L3.4a]

In an attempt to move away from curricular issues and to try and understand Jeanne’s beliefs about the methods of teaching mathematics, the researcher asked her to give what she perceived to be the ideal teaching environment: “Let’s suppose for a moment that you could teach your math class any way you choose. It is totally up to you
what you teach and how you teach it. Describe this class in terms of what you would teach and how you would teach it.” Jeanne’s response showed her desire to have more time for her students to investigate mathematics and to be “exploring and touching and experimenting”. She believes that as a secondary school teacher, there is too much to cover to teach as she would like, but that teachers in earlier grades could be doing it because they are mostly reviewing previously covered topics.

I would like a longer period so we could get involved in the topics more where we don’t have to just get from here to here. That way I can stop and do the kinds of things that they really **should** be doing in junior high where they are touching and feeling. You see, in the junior high, they do a lot like the high school teachers do. They put it up on the board, do the problems, and are done with it. I think the junior high should be exploring and touching and experimenting and doing all of that kind of thing. They don’t teach the students anything new at the junior high. They do the same thing from 6th grade on. They could be doing all of that tactile stuff that we don’t have the time for at the high school because we have the material to cover. [L3.4d]

Another factor she saw hindering her efforts to teach using investigation and inquiry was the state’s standardized examination because it was viewed as another obligation to “cover the material”. She also felt pressure to cover a substantial amount of content due to her theory that doing so would better prepare her students for future mathematics courses.
F/R: What makes you feel like you have to get your students to a certain point?

Jeanne: Well, first of all, we have got the state examination. We have to get them so far for that test. Second of all, after the state examination, we have to cover the material so they can get to Precalculus. In Precalculus, we have to cover enough so that they can get to Calculus, and, you know, it just keeps going like that. [L3.4d]

F/R: What could change that?

Jeanne: I think the Harvard text has done a nice job of getting rid of some of the garbage. Those old-fashioned precalculus books had all kinds of stuff that was great for 200 years ago or even 100 years ago like the rational root theorem, Descartes’ rule of signs, or Cramer’s rule. All that stuff that used to be in those books the Harvard has taken out which is good. You don’t need any of that stuff because technology has made it so you don’t need that. [L3.3b, L1.1] That is why I went to these books. [BT1a] I said, “I refuse to teach the rational root theorem. [BT1b, BT2b] It’s just a waste of time. Let’s just get rid of that. [BT1a]” They have done a nice job of cleaning up things that are just vestiges—like an appendix—there is no good use for it. You still have it, but, you know, you are not using it for anything, so let’s get rid of it. [L1.1, L3.3b]
Jeanne pointed to the importance of spending time on what she perceived to be the chief ideas of mathematics. In her pre-PLC interview, she gave a response that shows she values getting students to understand and apply mathematical concepts, to have meaning, and to not just do calculations and computations. In the same transcript, however, she holds a seemingly contradictory position by stating that the way you accomplish student understanding as a teacher is to provide ongoing repetition and by “telling” students.

F/R: In your opinion, what does it mean to be a successful precalculus teacher?

Jeanne: You know, I don’t—I don’t—I don’t know. I guess if you’ve got some light bulbs going on once in awhile—it’s hard. It’s a hard course! If you can get kids coming and get kids engaged in it and thinking, “Oh, I get this—finally!” Because, you see, it is a matter of how many times you have to hit them over the head. One of those times it’s going to click. If you can get them to understand—if you can get them to understand and have meaning now, if you can get a student to say, “Ah, why didn’t my teacher last year tell me that?” [L3.4a] Teachers will say that the textbook doesn’t have a lot of practice. Well, you still need practice, but they’ll skip the word problems and the application problems because it is hard. They will concentrate on the skills. A couple of years ago I saw that evolving here so I reminded them that we’ve got to make sure
we don’t lose the explorations; we don’t lose the hands-on; [L3.4d] we don’t lose the concepts and understanding because it is so easy to go back to the drill-and-kill. [L3.4a] We have got to remember this other side of mathematics—the right brain—and they do it because it is easier than getting the kids to think. [L3.4a, L3.4c, L3.2a]

F/R: What’s easier?

Jeanne: It is easier to do the drill-and-kill. It is harder to get the kids to think: “If I can’t get them to think, at least let me get them to do.” And so it ends up that is all we do is have them “do”. I think that is why the traditional mathematics is where it is right now. It has evolved into all we have left is the “doing”, and we don’t have any of the thinking. [L3.3b] That is why when kids go into physics or engineering they don’t know how to apply it because it is always just the “doing” in their math classes. [L3.4a, L3.4c, L3.2a]

Jeanne expressed her desire to teach more in alignment with what research calls for in her post-PLC interview. She would like to do more group work and to get her students more actively involved doing mathematics, but stated that it was her lack of patience that holds her back.

I know that I should have the students work more in groups, to communicate, to reflect, but the reason for me personally not doing it, quite frankly, is impatience. I don’t go around and ask the kids and engage them. Instead, I say, “Okay,
everybody with me. We are going to do this together. Let’s answer the questions.”

I don’t get everyone involved because I am too impatient to wait. “So-and-so”, I
don’t ask, and “so-and-so”, I don’t ask. I don’t force them to participate, and I
know I need to. That’s a problem. I have got kids that I don’t know if they are all
with me or not, and I know I should be checking that. It’s my personality that gets
in the way. [BT2a, L3.4d, L3.4e, L3.4f]

Jeanne further expressed her philosophy regarding classroom instruction in an
e-mail reflection from early in the PLC when she concluded that students, classes, and
teachers are so different that each teacher must individually develop a lesson that “fits”
them and their students. To Jeanne, collaborating with colleagues in a PLC to prepare
lesson plans is fine, but when actual instruction takes place in the classroom, the teacher
must be able to react to the students. She frequently expressed her desire to feel
“comfortable” with what will transpire in her classroom.

So much of being successful in teaching mathematics is the person doing the
teaching. You could have a beautiful lesson that works with your kids, and I could
take that exact same lesson and go in there, and it just bombs because I can’t do it
or because it’s not right for me or my kids. [BT1c] The lesson has to be
something I am comfortable with. [BT2b] If you are teaching auto mans or robots,
then that is fine, but we are not. The teacher has to constantly monitor and adjust
and not just follow a prescribed worksheet. [L3.4b]

Similarly, in a mid-year email reflection, Jeanne expressed her lack of interest in
implementing the collaboratively developed PLC activities because of the discomfort she
felt with them and the philosophy she holds of reacting to her individual students’ needs by “seeing” them in a teachable moment and attempting to remedy the situation.

I haven’t felt comfortable with the activities we have developed so far, so I don’t think they helped me much in the classroom. [BT2b] I tend to be a loner when it comes to teaching. [BT1c] That is, I look at the students and see where they are struggling, and then I try to find a way to help them. It is, therefore, difficult for me to make up an activity one month in advance of instruction. [BT2b, L3.4b]

*Summary of beliefs.* The interview transcripts suggest that Jeanne has a litany of professed beliefs about the learning and teaching of mathematics that affect her instructional philosophy and pedagogical decision-making. The data reveal that she recognizes that there are barriers that exist to her being able to teach using reform instructional techniques. Table 9, as found on the following pages, shows a summary of the most common beliefs about learning and teaching mathematics that Jeanne professed to have, as well as the barriers and behavioral traits that emerged from the analysis. The table includes components of the PLC Theoretical Framework and related emergent factors that were evidenced in the data, page numbers from this chapter where supporting excerpts can be found, and representative quotes that summarize each of the findings. When there is possible confusion as to what aspect of the framework is being demonstrated by a quotation the relevant portion is presented in bold type.
## Summary of Jeanne’s Beliefs About Learning and Teaching Mathematics

### Beliefs about how students learn mathematics

<table>
<thead>
<tr>
<th>Beliefs about how students learn mathematics</th>
<th>Page Numbers</th>
<th>Representative Quote</th>
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<tbody>
<tr>
<td>For meaningful understanding of mathematics, <strong>one needs to concentrate more on the systematic use of general thought processes</strong> rather than on memorizing isolated facts and algorithms. [L3.2a]</td>
<td>169, 170, 175</td>
<td>p. 169: “I thought, ‘These kids are going to start in math here and they are just going to be manipulating values and algebraic symbols and not understanding it. ’ I said to myself that I was not doing that.”</td>
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### Beliefs about teaching mathematics

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<th>Beliefs about teaching mathematics</th>
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<th>Representative Quotes</th>
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<tr>
<td>For effective mathematics instruction to occur, a teacher needs to focus primarily on developing student understanding of principle concepts and ideas instead of on procedures and quick computation. [L3.4a]</td>
<td>165, 166, 170, 171, 174, 175</td>
<td>p. 174: Interview question: In your opinion, what does it mean to be a successful precalculus teacher? “I guess if you’ve got some light bulbs going on once in awhile... If you can get kids coming and get kids engaged in it and thinking, ‘Oh, I get this—finally!’ If you can get them to understand... and have meaning now...Teachers will...skip the word problems and the application problems ...and will concentrate on the skills. ...I saw that evolving here so I reminded them that we’ve got to make sure we don’t...lose the concepts and understanding because it is so easy to go back to the drill-and-kill.”</td>
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<td>For effective mathematics instruction to occur, a teacher needs to build upon students’ prior knowledge and personal experience rather than presenting material without regard to individual meaning and sense making. [L3.4b]</td>
<td>166, 166, 169, 170, 176, 177</td>
<td>p. 176: “If you are teaching auto mans or robots then that is fine, but we are not. The teacher has to constantly monitor and adjust and not just follow a prescribed worksheet.” p. 177: “... I look at the students and see where they are struggling, and then I try to find a way to help them. It is, therefore, difficult for me to make up an activity one month in advance of instruction.”</td>
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<td>For effective mathematics instruction to occur, a teacher needs to design instruction that requires active participation by students and encourages curiosity, creativity, investigation, collaboration, and healthy questioning rather than only expecting that students listen intently, take good notes, follow directions and examples, behave themselves, and do the assigned work [L3.4d]</td>
<td>169, 172, 173, 174, 176</td>
<td>p. 172: “… get involved in the topics more...”, “…do the kinds of things...are touching and feeling.”, “…exploring and touching and experimenting and doing all of that kind of thing.”, “... be doing all of that tactile stuff...”</td>
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Factors that hinder investigation and inquiry in the classroom:
- a large amount of material to cover
  - State assessment
  - Future course preparation
- inadequate classroom time
- it is hard work
- behavioral traits (see Table 10)

Beliefs about curriculum

<table>
<thead>
<tr>
<th>Mathematics should be studied more for personal benefit than for just fulfilling curriculum requirements. [L3.3b]</th>
<th>Page Numbers</th>
<th>Representative Quotes</th>
</tr>
</thead>
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<tr>
<td>Reform curriculum encourages students to:</td>
<td>163, 163, 164, 165, 166, 166, 167, 167, 167, 169, 169, 170, 171, 171, 173, 173, 175</td>
<td>p. 165: “...I knew we needed to get our kids thinking!”</td>
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<td></td>
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<td>p. 166: “It isn’t bad when you have an actual situation that they can envision...”</td>
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<td>Traditional curriculum is characterized by:</td>
<td></td>
<td>p. 173: “They have done a nice job of cleaning up things that are just vestiges—like an appendix—there is no good use for it. You still have it, but, you know, you are not using it for anything, so let’s get rid of it.”</td>
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<td>- not focusing on the primary concepts and ideas of mathematics,</td>
<td></td>
<td>p. 170: “...there’s just no point to it. You are just grinding these kids through. It’s a grind of algebra that doesn’t make any sense, and these kids are looking at you like you are ‘out of your mind’!”</td>
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<td>- being removed from mathematical applications,</td>
<td></td>
<td>p. 175: “I think that is why the traditional mathematics is where it is right now. It has evolved into all we have left is the ‘doing’, and we don’t have any of the thinking.”</td>
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Behavioral Traits that Affect Pedagogical Decisions

The analysis done thus far of Jeanne’s conceptual knowledge, pedagogical content knowledge, classroom practices, and beliefs is helpful in building a characterization of the teacher in order to track her professional development in the PLC. However, the researcher also saw in the data another important and influential factor emerging that appeared to be affecting Jeanne’s pedagogical decision-making—behavioral traits (i.e. personality). As the data were being analyzed it was noted by the researcher that certain portions of the transcripts could not appropriately be labeled as a “belief”. Rather, it was determined that such excerpts were primarily traits of Jeanne’s behaviors as Table 10 shows.

Jeanne could be characterized as one who likes to be in charge, is self-assured, and is confident in her belief system. She seeks to use teaching techniques that she perceives to be in alignment with her personality and make her feel comfortable in the classroom. Jeanne points to her personal impatience with the learning process as a major factor (in addition to inadequate time, too much material to cover, and the amount of hard work required) in her choosing not to employ more reform teaching techniques in the classroom.
Table 10

Jeanne’s Behavioral Traits

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<tr>
<th><strong>BT1:</strong> Issues of her self-identity</th>
<th>Page Numbers</th>
<th>Representative Quotes</th>
</tr>
</thead>
</table>
| a) role of authority                 | 164, 168, 169, 170, 170, 173, 173 | p. 168: “I like being the boss.”  
  p. 170: “I would not be a happy camper if I had to transfer. If I moved out of being department chair, I wouldn’t be a happy camper either.” |
| b) self-assurance                    | 164, 168, 168, 170, 170, 173 | p. 168: “I feel I have something to say. ...I have a sense about what you can teach and how you can teach it and what direction we want to go.”  
  p. 170: “If I was forced to go, I would do what I wanted to do, and I wouldn’t care what they said. I would teach how I want to teach.” |
| c) independence                      | 176, 177 | p. 177: “I tend to be a loner when it comes to teaching.” |

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<th><strong>BT2:</strong> Personality Characteristics</th>
<th>Page Numbers</th>
<th>Representative Quotes</th>
</tr>
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<tr>
<td>a) impatience</td>
<td>176</td>
<td>p. 176: “…the reason for me personally not doing it, quite frankly, is impatience. I don’t go around and ask the kids and engage them. Instead, I say, ‘Okay, everybody with me. We are going to do this together. Let’s answer the questions.’ I don’t get everyone involved because I am too impatient to wait. ‘So-and-so’, I don’t ask, and ‘so-and-so’, I don’t ask. I don’t force them to participate, and I know I need to. That’s a problem. I have got kids that I don’t know if they are all with me or not, and I know I should be checking that. It’s my personality that gets in the way.”</td>
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<tr>
<td>b) comfort level</td>
<td>168, 169, 170, 173, 176, 176, 177</td>
<td>p. 176: “So much of being successful in teaching mathematics is the person doing the teaching. You could have a beautiful lesson that works with your kids, and I could take that exact same lesson and go in there and it just bombs because I can’t do it or because it’s not right for me or my kids. The lesson has to be something I am comfortable with.”</td>
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Initial Profile of Jeanne

In this section, Jeanne has been characterized as having a solid conceptual knowledge of the function concept at the precalculus level by scoring at the 95th
percentile on the PCA. Due to the growth in conceptual knowledge that she experienced from using reform curriculum and her life experience outside mathematics, Jeanne appears to believe that “reform style curriculum” is beneficial to students for facilitating their understanding and application of mathematical concepts, advancing their thought processes, realizing its usefulness, and being able to problem solve. Despite this belief, Jeanne would not be considered a “reform” teacher as measured by the RTOP since she scored lower than average on this instrument. She reports using teacher-centered instruction and emphasizes repetitive practice in her teaching. Even so, Jeanne professed a desire and need to employ more inquiry-based and student-centered teaching techniques in her classroom, and shared reasons for why she perceived that she is not able to teach that way. These included the lack of time in class, too much material to cover, the amount of hard work required, and her impatience with the learning process. Finally, Jeanne prefers to not prepare materials for her classes ahead of time because of her desire to “see” her students, to be able to react to their immediate needs, and to be in charge of their learning. This detailed characterization is important for understanding the level of professional development that Jeanne experienced in the PLC, for seeing how her practice was influenced, and for documenting the interrelationships between the areas for professional growth.

Professional Learning Community Experience

In this section, I detail the professional growth that Jeanne exhibited during sessions 16 to 20 of the professional learning community. This cycle of investigation dealt with an aspect of trigonometry. More specifically, the cycle's focus was on
determining as a team of professionals how to help students evaluate trigonometric functions using a conceptual approach (i.e. the unit circle). During this cycle, Jeanne grew in her conceptual understanding, pedagogical content knowledge, and expressed her beliefs about the learning and teaching of mathematics. In terms of classroom practice, it is interesting to note that Jeanne did not demonstrate proficiency in implementing the research-based, conceptual activity despite the fact that she was instrumental in helping the community to develop it. Her beliefs, behavioral traits, and lack of preparation for the lesson appeared to be factors in her implementing it in a haphazard fashion and only as an afterthought. In this section, I share relevant data that provide evidence and support for these results.

Investigating Student Thinking

PLC Session 16

The objectives of session 16 were to enhance the teachers’ conceptual knowledge and pedagogical content knowledge regarding the evaluation of trigonometric functions in radian measure using a conceptual approach. The professional learning community determined in session 15 that when teaching students to evaluate trigonometric functions, accessing prerequisite knowledge of radian measure and fractions appeared to be a stumbling block for many of their students. To launch the professional learning community into a discussion on this topic, the facilitators assigned action items to pairs of participants to complete and present in session 16.

Pedagogical content knowledge and beliefs. The team comprised of Jeanne and Diana was given the task to reflect on the notion of radian measure and to consider
possible obstacles to learning and preconceptions that students have demonstrated in the past. The questions they were asked to ponder were:

From your experience, what challenges do you see students having with regards to being able to find trigonometric values of angles given in radians from the unit circle? What kinds of preconceptions do students have?

Jeanne was the spokesperson for the two and primary source of information in responding to the questions. Jeanne advanced the theory that students’ lack of understanding of fractions was one of the problems.

I think the problem is that they can’t express fractional values in terms of $\pi$. I think the fractions are the problem—things like two-thirds of a $\pi$ or three-fourths of a $\pi$. They don’t quite understand that. It is a fraction issue. [L2.2]

Jeanne continued by noting that she also thought the PLC needed to focus on helping the students gain an understanding of the ordering of radian measures. Cooper joined in the conversation and offered ideas for what type of intervention they could possibly make. Cooper relayed her position that giving the students a lot of problems to practice will help them learn, and Jeanne agreed with her.

Jeanne: The students need to see that $\pi$ is a number on the number line between 3 and 4, and I think we need to go further than that. We need to find two-thirds of a $\pi$ on the number line and $\frac{\pi}{2}$. I don’t see that whole thing. I see them getting $1 \frac{1}{2}$ and maybe 1.57, but
Cooper: Maybe we should give the students a bunch of problems and have the kids put them in order somehow. [L3.2a, L3.4a, L2.3]

Jeanne: Yeah, give them a number line with 1, 2, 3…10 on it and ask them, “Okay, where does this go? Where does that go? Where does that fit?” [L2.3]

Jeanne further reported that her students seemed to be confused by mixing whole number radian measures in with fractional radian measures.

Jeanne: Where is 2 radians? 3 radians?—whole numbers too. Let’s do more than what we normally do—fractions. For 1 radian or 2 radians, they have no idea! It’s weird, but after they have learned \( \frac{\pi}{3} \) and where it is located on the unit circle, and then you go to 1 radian, they don’t know where it goes! You are putting \( \pi \) and fractions together so you are putting two things that they really don’t get together. [L2.3, L2.2]

The participants noted on several occasions that they perceived attaining a solid understanding of fractions was a major challenge for their students. However, they were never able to articulate what it was about fractions that their students struggled with or how to help them overcome the challenge. To move the conversation in that direction, the facilitator/researcher attempted to tie Jeanne’s reported observations in with what the PLC had determined they wanted to study—how to help students evaluate trigonometric
functions given in radian measure. He tried to assist Jeanne in pinpointing the
preconceptions that students have. In this process, the belief that providing a lot of
practice aids in student understanding arose again as Jeanne proposed that the PLC create
a classroom activity focused on repetition.

F/R: Can the students find the $\sin(60^\circ)$ using the unit circle?

Jeanne: They will eventually get that. [L2.2]

F/R: Alright, so you think they can get that. Okay then, correct me if

I’m wrong. If we can get students to understand where $\frac{\pi}{3}$ is

located on the unit circle, then do you think they could evaluate

$\sin\left(\frac{\pi}{3}\right)$?

Jeanne: Yeah. Yeah, they can, but you have got to hammer them. I think

we need to drill them with a combination of radians with the trig

functions attached. [L2.2, L2.3, L3.2a, L3.4a]

A little later in the same session, the facilitator tried to determine what techniques
the teachers used to assess their students’ level of knowledge about evaluating
trigonometric functions in radian measure. Jeanne’s response is telling. Her technique
emphasizes repetition, memorization, and quick recall. Jeanne thinks that offering extra
credit and creating an atmosphere of pressure and fear will help motivate her students to
perform better.

F/R: What do you do to assess whether your students understand

evaluating trig functions in radians or not?
Jeanne: I want to drill this in to them so they can recall them. I’ll put a problem up on the overhead and use the random number generator to select the student to be called on. I give them three seconds—count to three—“One (bangs on the desk), two (bangs again), three (bangs a third time), you’re done!” and then go on to someone else. Everyone gets a turn when it comes around. They have to do it quick, and you know how hard that is? “Okay, \( \sin \frac{\pi}{6} \) is…” Boom!

You’ve got to give it to me right then and there, and everybody’s watching. [L2.3, L3.2a, L3.2d, L3.4a, L3.4b]

Mary Beth: That’s hard for me! (laughter by all) [L2.2, L2.3]

F/R: Me too!

Jeanne: The first time they’re really petrified, but you know what? They are all there! They are all absolutely there! I just go randomly around and even pick the same person if it comes up. The second time around it is for extra credit, so they get a little bit more relaxed. It takes about an hour to get around two rounds, so it is a chunk of time, but I think it is well worth it. [L2.3, L3.4d]

Alignment of curriculum. Later in the session, Lorene points out and is confirmed by Jeanne, the department chair, that this mathematical topic is mostly “new material” for the students. As a matter of fact, the PLC participants realize later in the cycle that the topic is totally new material.
Lorene: This is the most new material they have seen all year because up to this point everything really is review but a little bit more in-depth.

Jeanne: They have had some trig in Topics 5/6, but not a lot. This is important because it ties back to a statement Mary Beth, the only teacher not teaching Precalculus, had made in the prior week’s discussion in session 15. She had mentioned that she and the other Math Topics 5/6 teachers just barely touch on radians because they use degrees in introductory trigonometry activities. At the time that she mentioned this, it didn’t appear that she was heard by the others or given much credence because it becomes a big issue later (see session 20) to the teachers as they express their amazement that the students hadn’t done this type of work before when the teachers had been working under the assumption that they had.

In the post-PLC debriefing for this session, the facilitators wondered how the fact that the students were learning new concepts fit in with the teachers asking them for quick recall and not focusing on conceptual understanding. The facilitators thought that this must have been what Mary Beth’s point was when she said it was hard even for her (as a teacher) to do what Jeanne was requesting of the students. The facilitators emphasized that they thought students in the initial stages of learning should not be required to provide speedy answers, but rather be given a chance to understand the concept. They agreed to try and address that issue with the teachers as the cycle progressed and the classroom activity was constructed. The PLC experience had opened
lines of communication between teachers from different levels of mathematics which helped them better align their department’s curriculum (see session 20).

_Research, beliefs, and conceptual knowledge._ Recall that early in the first semester of the PLC, the teacher participants displayed their displeasure with mathematics education research. They perceived the material they had been exposed to as neither being responsive to the special needs of secondary mathematics teachers nor being written to them as the audience. The teachers’ were interested in research that would help them understand secondary mathematics students’ thinking and provide insight for overcoming the challenges they face in educating these students. They sought pragmatic information from the world of mathematics education.

Taking this into account, the facilitators in session 16 sought to provide relevant research that would illumine the issue that none of the participants had been able to address which was suggesting conceptions and thought-processes their students have with regard to fractions. The teachers had identified fractions as a general area of difficulty for their students, but not specifically the aspects of understanding fractions that posed the challenge. The observer/facilitator had located some research done by Lamon (1999) that was built around having high school calculus students solve conceptually-based fraction problems. The facilitators requested that the participants begin by taking some time at the beginning of the PLC session to work through selected problems and to then scan a summary of the research findings. This started a nice discussion and resulted in building up the participants’ conceptual knowledge through the process of thinking about students learning.
Observer/facilitator: We went back and found some research to share with you on the topic of students’ understanding of fractions. Let’s look at what research says about why it is that kids struggle with them and what can we do to kind of help them be able to deal with \( \pi \) when it is involved in a fraction. When we were talking about radians, we were saying there is a subset of radians that involve \( \pi \) and a fraction. We know that is not true of all radian measures, but we have stated that is one of the big things they have trouble with. I am going to have you do a few fraction problems so we can talk about them, and then I have put together a summary of the research for you to read. [L2.2, L2.3] Take the next five to ten minutes to try each of the three problems (Lamon, 1999) I have included, and then we will talk about how you did each one individually. (The problem set included the following fraction problem: Find 2 fractions between \( \frac{7}{12} \) and \( \frac{7}{11} \).)

As the participants worked through the problem set, the observer/facilitator noticed that most used a common denominator to find a fraction between \( \frac{7}{12} \) and \( \frac{7}{11} \), but the facilitator/researcher had put \( \frac{7}{11.5} \) for his answer. The facilitators had decided in the pre-session planning discussion that he would take this approach to solving the problem after the two had talked about the different possible techniques that could be used. The facilitators anticipated that the presentation of this technique would initiate some
discussion by the PLC participants because it is not a “standard” method. The resulting
talk among the participants centered on debating whether the approach should be
considered “correct” or not. [This entire excerpt is a combination of codes L1.4, L1.1,
L1.2, and L3.2b.]

Diana: What kind of math is that, math teacher (to facilitator/researcher)?

You can’t have a decimal in the denominator!

F/R: Why not?

Jeanne: Because you can’t have a decimal in a fraction.

Diana: You aren’t supposed to.

F/R: Who says?

Cooper: Everyone in math. (laughter)

Diana: Because that is the way I was taught in 2nd grade.

Cooper: A fraction is a ratio between two integers.

O/F: You can’t have $\frac{7}{11.5}$?

Jeanne: No.

Diana: That’s not an integer.

F/R: It doesn’t say it has to be a rational number it just says it has to be a fraction.

Cooper: Then, in that case, this is a really easy problem.

F/R: What’s a fraction? — A number with a numerator and a denominator. It can be thought of as the part of a whole or an
indicated division. A **rational number** has to be a ratio of two whole numbers.

Cooper: Well, I am sorry, but you guys (the facilitators) accept a fraction like that as an answer?

O/F: It depends what the question is that I am wanting answered.

Cooper: I wouldn’t accept that because that’s not a **simplified** fraction.

F/R: Would you accept $\frac{1}{\sqrt{2}}$ as an answer?

Diana: No.

Loretta: $\frac{1}{\sqrt{2}}$?...Yes.

Cooper: No, it’s not simplified as far as you can go.

F/R: Do the directions say “simplify” here?

Cooper: You…you are evil!

F/R: You need **me** in your class! (laughter by others)

Cooper: I don’t think you even need to say “simplify” when you work with fractions. That’s just a given, isn’t it?

This interchange had accomplished getting the teachers to question their thinking about what constitutes an acceptable answer, and the discussion exposed a belief by Jeanne and some of the others that one does certain things in mathematics simply because “that is how it is done”. The facilitators wanted to draw out the different techniques the teachers used to solve the problems so that they could see the many ways the problem
could be done. The facilitators hoped that the teachers would realize that their students might want to take different approaches too.

O/F: So do people want to share how they found two fractions between \( \frac{7}{12} \) and \( \frac{7}{11} \)?

Diana: I found a common denominator of 132. One is 79 over 132 and the other is 83 over 132, so you can pick a number anywhere from 79 to 83 over 132. [L1.2]

O/F: Is that what all of you did? (checks with the others)

Cooper: I changed it to \( \frac{7}{11\frac{1}{2}} \) and then I went through the whole spiel of changing it to simplest form. [L1.2]

O/F: So which “spiel” did you do? What did you turn that fraction into?

Cooper: I am going to confess and say that I first put \( \frac{7}{12.5} \) for an answer, but I saw that was wrong. So then the way that I had done it is I had \( \frac{7}{11.5} \) and then \( \frac{7}{11\frac{1}{2}} \) which would have been \( \frac{7}{23} \) and then multiplied to get \( \frac{14}{23} \). [L1.2] That is how I did it, but this guy (the facilitator/researcher) had \( \frac{7}{11.5} \), \( \frac{7}{11.6} \), \( \frac{7}{11.7} \)! [L1.4]

Diana: No! We’re not allowed to do that! (laughter) [L1.4, L3.2b]
Jeanne, while paying close attention, had been relatively quiet throughout most of this discussion until she was asked by Diana to share her method. Jeanne gave a different technique for finding the two numbers and then stated that she hadn’t thought of solving the problem the way that the facilitator/researcher had done it. She expressed her delight in it by calling it “cool” and valued how the technique arrived at a quick solution. Jeanne was not as receptive to the observer/facilitator’s suggestion of dividing the fractions out, however. This seemed to be too tedious for Jeanne, and the method would not take advantage of the tools of technology at one’s disposal. In this way, Jeanne expressed her belief that speed in arriving at a solution is an attribute of a good mathematical technique.

Diana: Jeanne, how did you do it?

Jeanne: I didn’t know at first, so I just averaged the two and found the value in the middle, but then I just put on another 0 and had \( \frac{70}{120} \). I realized I needed a number smaller than \( \frac{7}{12} \) so I made the denominator a little bit bigger and made it \( \frac{70}{121} \). Then I needed a number a little bit bigger, so I added a 0 on the other number and got \( \frac{70}{110} \) and did a similar thing. [L1.2] I never even thought to do it your (the facilitator/researcher) way (fractions with decimals). That was much easier, and I liked it. That’s cool! [L1.4, L3.2b]

F/R: Yeah, I wrote it as \( \frac{7}{11.5} \), and I didn’t even change it. [L1.2]
O/F: Did anyone convert the fractions over to decimals first? [L1.4, L3.2b]

Jeanne: Why would you ever do that—and without a calculator? [L1.1, L1.4, L3.2b]

O/F: Because I am an elementary school teacher, and I like long division. [L3.4b]

The two facilitators had anticipated this interchange in the pre-session planning meeting. By forcing the participants to struggle with determining whether an alternative method was acceptable, the facilitators had accomplished getting them to challenge their beliefs about how it is that they make this decision. Jeanne valued this discussion and learned different representations of the concept and problem-solving techniques. In this way, the interchange deepened her conceptual knowledge of the topic.

**Pedagogical content knowledge and conceptual knowledge.** After this discussion, the facilitator took the participants’ learning and connected it to the topic of radians again. The participants started to anticipate what students would do when fractions and $\pi$ are combined in a radian measure.

F/R: Now, if we stick $\pi$’s on these, this is how it applies to our earlier discussion, so we have $\frac{7\pi}{12}$ and $\frac{7\pi}{11}$. It’s knowing what that fraction part is, but also we have to deal with what $\pi$ is too. [L1.3]

Jeanne: This is what I was talking about earlier…fractions and $\pi$ together is hard. I think that kids would cry over this. They wouldn’t have a clue—even precalculus students won’t. [L2.2]
F/R: That’s what we need to do is to ask some students about this. Let’s try to find out what your own students will do with this. [L2.2]

Lorene: They will do exactly what she (the observer/facilitator) said. They will get their calculator out and divide it all out into decimals. [L2.2]

O/R: If so, when you get those decimals, it then becomes a place value question. You get .583 repeating and .636. What are numbers in between those? While it seems like a simple question, a lot of kids will have trouble answering that question: “What number comes between those decimal numbers?” [L1.3, L2.2]

The issue that the observer/facilitator raised about students having difficulty understanding an aspect of repeating decimals stirred a related content question within Diana’s mind. Her question initiated a discussion within the PLC as the others tried to understand and work through the dilemma that Diana had raised. In this instance Jeanne (and Loretta) took the role as a facilitator by trying to help the others to come to an understanding of the concept and by connecting it to calculus concepts such as limits and infinity. The conversation was cut short by the time constraints of the session.

Diana: You know. I have been wondering… I have a question: “What is a number between $4.\overline{9}$ and 5?” [L1.3, L3.2c]

O/R: Depends if you accept that they are equivalent or not. It depends if you think $\overline{9} =1$. [L1.2]

Cooper: There are an infinite number of values between them. [L1.2]
Lorene: You could make $4.\overline{9}$ into a rational number and then find one between. [L1.2]

Loretta: What is $4.\overline{9}$? $\overline{9}$ is $\frac{9}{9}$ or 1. [L1.2]

Cooper: Yeah… Wait… no, it isn’t! [L1.2]

Loretta: $\frac{1}{9} = .\overline{1}$. [L1.2]

Jeanne: $\frac{4}{9} = .\overline{4}$. [L1.2]

Diana: So you are saying that $4.\overline{9}$ is actually 5? [L1.2]

Jeanne: Now you are talking about infinity and limits. [L1.3]

Cooper: If you put it into a calculator 9 divided by 9 is 1…it is a geometric series…(Cooper fades off in thought.) [L1.2, L1.3]

O/R: I am very sorry that we have to stop now because we are just getting started on this nice conversation, but it is time to quit for today, and we want to honor the clock and wrap up our meeting.

The participants’ discussion concerning content matters did not stop at this point but, rather, continued for forty-five minutes after the official PLC meeting time had concluded.

At this point, the facilitators decided to ask the participants to give some of the fraction problems to their students as an action item for the next session anticipating that the teachers would see their students having the same kinds of issues with fractions that were presented in the research. At the conclusion of this PLC session, Jeanne predicts
that the students wouldn’t know how to do the problems because it had been too long since they had had these types of problems. She theorized that the cause for their lack of understanding was too much time elapsing and not that they never had learned the concepts deeply in the first place. Jeanne suggested that they narrow down the scope of what it was that the teachers were to analyze so they could more easily see what some of the obstacles were.

O/R: What we are going to do is give you an article that talks more about this fraction background and what is nice about the article is that the research was actually done with calculus students. What it is looking at is what older high school students can do with fractions. What we would like for you to do is to take two of the questions—either the questions we just did or others you found in the article that interest you—and just give them to one class of students or, if you want, give them to all of your classes just to see what your own students do with these fraction questions. [L2.2]

Jeanne: The kids will see the one with finding two fractions between two others and will kind of freak because they haven’t seen this for awhile. [L2.2] So let’s just give them a fraction question without \( \pi \) or anything, just so we can start to see how much they struggle with just the fractions. [L2.3]

The experience of taking the problems to their own classroom is a learning experience for the teachers as is seen in the following session (17).
Discussion

Important excerpts from session 16 are presented in Table 11 below to organize the findings that emerged in Jeanne and to demonstrate how they related to aspects of the PLC Theoretical Framework. Page numbers are cited in the table to offer evidence of the framework’s components that emerged in the session and a quote is provided that demonstrates the type of characterization the cited transcripts represent. Jeanne’s professional growth and a further characterization of her knowledge and beliefs are both manifested in the data.

Table 11

<table>
<thead>
<tr>
<th>Findings from Session 16</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PLC Theoretical Framework Components - CK</strong></td>
</tr>
<tr>
<td>L1.1: <strong>Conceptual Knowledge</strong> is characterized by having the ability to recognize the central and peripheral ideas of mathematics and their relative importance.</td>
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<tr>
<td>L1.2: <strong>Conceptual Knowledge</strong> is characterized by having the ability to provide reasoned responses and justify thought processes when problem solving.</td>
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<tr>
<td>L1.3: <strong>Conceptual Knowledge</strong> is characterized by having the ability to build a concept upon earlier ideas, formulate connections within and across domains, and</td>
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</tbody>
</table>
create further extensions.

Jeanne: \( \frac{4}{9} = \bar{4}. \)

Diana: So you are saying that \( \bar{4} \) is actually 5?

Jeanne: Now you are talking about infinity and limits.

### PLC Theoretical Framework Components – PCK

<table>
<thead>
<tr>
<th>Page Numbers</th>
<th>Representative Quotes</th>
</tr>
</thead>
<tbody>
<tr>
<td>191-192, 194, 195</td>
<td>p. 194: Jeanne: ...I never even thought to do it your (the facilitator/researcher) way (fractions with decimals). That was much easier and I liked it. That’s cool! F/R: Yeah, I wrote it as ( \frac{7}{11.5} ) and I didn’t even change it. O/F: Did anyone convert the fractions over to decimals first? Jeanne: Why would you ever do that—and without a calculator?</td>
</tr>
</tbody>
</table>

| 184 | p. 184: The students need to see that \( \pi \) is a number on the number line between 3 and 4 and I think we need to go further than that. We need to find two-thirds of a \( \pi \) on the number line and \( \pi \) over 2. I don’t see that whole thing. I see them getting \( 1 \frac{1}{2} \) and maybe 1.57, but where is this fraction located at? Where does that fit? |

| 184, 185, 185, 186, 186, 195, 198 | p. 184: I think the problem is that they can’t express fractional values in terms of \( \pi \). I think the fractions are the problem. Things like two-thirds of a \( \pi \) or three-fourths of a \( \pi \). They don’t quite understand that. It is a fraction issue. p. 185: Where is 2 radians? 3 radians?...whole numbers too. Let’s do more than what we normally do—fractions. For 1 radian or 2 radians, they have no idea! It’s weird, but after they have learned \( \pi \) and where it is located on the unit circle, and then you go to 1 radian, they don’t know where it goes! You are putting \( \pi \) and fractions together so you are putting two things that they really don’t get together. |

| 185, 185, 185, 186, 187, 187, 198 | p. 186: Yeah. Yeah, they can, but you have got to hammer them. I think we need to drill them with a combination of radians with the trig functions attached. |

| 186, 187 | p. 187: I want to drill this in to them so they can recall them. I’ll put a problem up on the overhead and use the random number generator to select the student to be... |
concentrate more on the systematic use of general thought processes rather than on memorizing isolated facts and algorithms.

called on. I give them 3 seconds—count to three—“One (bangs on the desk), two (bangs again), three (bangs a third time), you’re done!” and then go on to someone else. Everyone gets a turn when it comes around. They have to do it quick and you know how hard that is? “Okay, \( \sin \frac{\pi}{6} \) is…” Boom! You’ve got to give it to me right then and there and everybody’s watching.

L3.2b: For meaningful understanding of mathematics, one needs to examine situations in many ways, and not feel intimidated by committing mistakes rather than follow a single approach from an authoritative source.

L3.4a: For effective mathematics instruction to occur, a teacher needs to focus primarily on student understanding of principle concepts and ideas instead of on procedures and quick computation.

L3.4d: For effective mathematics instruction to occur, a teacher needs to design instruction that requires active participation by students and encourages curiosity, creativity, investigation, collaboration, and healthy questioning rather than only expecting that students listen intently, take good notes, follow directions and examples, behave themselves, and do the assigned work.

Conceptual knowledge. In terms of conceptual knowledge in this session, Jeanne demonstrated competence with the mathematical concept. She provided a reasoned response and justification for her thought processes as she explained her solution to one of the assigned fraction problems and was able to connect the topic to concepts from
calculus. In this session, Jeanne also demonstrated that she holds historical precedence in high regard as was evidenced in her recognition of the “rules” of simplification that have been accepted through the years. She was not amenable to accepting an answer that was not arrived at using more traditional techniques at first but later came to see the strength in a different representation (i.e. $\frac{7}{11.5}$). At the same time, she perceived a third technique (i.e. converting fractions to decimals) as being “weak” that did not take advantage of available technology and was much more time consuming despite the method’s potential instructional benefits.

*Pedagogical content knowledge.* In this session, Jeanne suggested that for students to understand the concept of evaluating a trigonometric function they need to be able to locate and order $\pi$ and fractions of $\pi$ on the number line and unit circle. Accessing her teaching experiences, Jeanne offered the common challenges that she perceived students having in this area as not understanding fractional parts of $\pi$, an inability to locate whole number radian measures on the unit circle, and struggling even further when these two notions are combined.

The findings from this session show evidence of the type of research that Jeanne finds relevant as she studied the Lamon (1999) article and research problems with great interest. This session demonstrates how a PLC discussion that focused on attempting to understand student’s thoughts about a concept was important in getting the participants to begin to grapple with their own conceptual knowledge regarding the topic.

Finally, it is important to note that it was in session 16 that Jeanne began to realize that there was a problem with the alignment of the department’s curriculum
through a brief discussion she had with Mary Beth, who teaches Math Topics 5/6 and not currently Precalculus. This issue is important enough for the PLC to again deal with in session 20.

Beliefs about the learning and teaching of mathematics. The findings reported from this session provide insight into Jeanne’s beliefs about learning and teaching. As has been shown earlier, Jeanne believes that quick computation and repetition are important components of effective mathematics instruction. She expressed the need to drill the students and to “hammer” concepts/skills into them. She described one of her teaching techniques that emphasized repetition, memorization, and quick recall. Expecting that it motivates her students to listen, pay attention, and perform better, Jeanne offered extra credit and created an atmosphere of pressure and fear in the classroom while employing the technique.

PLC Session 17

The purpose of session 17 was to continue investigating what students understand about fractions as they relate to radians and locating them on the unit circle and to then transition into creating an instructional intervention. This session was unique from the previous session because it started with the participants discussing what they learned about their own students’ knowledge. By recalling findings from the research base, the facilitators directed the participants through efforts to determine possible causes for their students’ lack of understanding. Finally, initial discussions begin about how to intervene and deepen the students’ conceptual understanding as newly informed mathematics educators.
Pedagogical content knowledge and research. Session 17 started with Jeanne reporting her findings from giving the research-based fraction problems to her students by describing what she thought her students were thinking as they solved the problems. Cooper discussed this with Jeanne and offered what she did in solving the problem to help elucidate the students’ thinking. The two teachers began to utilize the jargon offered in the research article to describe the different conceptions of fractions students can have. Through PLC discourse, Jeanne had come to the realization that students struggle with fractions because it is a non-trivial matter. She also suggested that the root of the problem lies in the inadequacy of how the concept has been taught to students in the past—not because students don’t try or are “bad”.

Jeanne: I had several students who came up with something like this, who in looking for a number between $\frac{7}{9}$ and $\frac{7}{8}$, they made $\frac{7}{9}$ into $\frac{14}{18}$, and then knew they needed something a little bigger, so they chose $\frac{14}{17}$. One kid explained that they “expanded the fraction to give them more room in between the numbers”. I had several kids do that. [L2.2]

Cooper: That’s what I did. I doubled both fractions (to get $\frac{14}{18}$ and $\frac{14}{16}$), but that only left one ($\frac{14}{17}$) in between, but I thought that wasn’t enough, which is what some of my honors students were doing, so
then I multiplied by 10 figuring that there would be a bunch in between. [L1.2, L2.2]

Jeanne: I never would have thought of that until I saw the students do it. [L2.2]

Cooper: The other students were like, “Why don’t you just find the common denominator?” That’s that part-whole thing from the research article we read. That’s the whole understanding that you have to break it into equal parts. [L2.1, L2.2]

Jeanne: But that’s what we deal with over and over with our students. [L2.2]

O/F: That is a student who understands the number of pieces compared to the size of the piece, you know. The numerator tells you the number of pieces, and the bottom tells you the size of the piece. They were able to look at that fraction and think, “… \( \frac{14}{18} \), well, I need a little more so I will make it bigger by making it \( \frac{14}{17} \).” They understood that an eighteenth of a piece is smaller than a seventeenth of a piece. [L1.2, L2.1, L2.2]

Jeanne: My student said, “I made the fractions so they are not reduced then I took numbers in between the denominator.” [L2.2]
Cooper: That’s why she multiplied by two. The first thing I actually did was make it $\frac{7}{8.5}$, but then I thought I have to mess with the wholes and halves and all that. I don’t want to do that. [1.2]

Jeanne: Yeah, I started to do that for them, and then the kids caught on right away with numbers, decimals like that. [L3.4d]

Cooper: I, myself, looked at that and thought, “I don’t know because I don’t understand it as well as I should.” I can do the part-whole stuff, but I don’t really have a great understanding conceptually of fractions. I don’t. [L1.2, L3.2c, L3.2d, L3.4b] That’s because we weren’t taught that way and that’s the same for the students. The teachers at the elementary level that are teaching this can’t teach the students because they don’t know how to do it any other way…. Some way we need to break the cycle of the way it’s being taught. [L3.4a]

Jeanne: And that is what that article was talking about. I was really impressed by that article! [L2.3]

Cooper: Yeah, me too.

Jeanne: There is one teacher I know of who says that she is not going to teach them any algebra until they can do fractions. I told her, “You know what? You will never get there, so forget it and try to teach them something else.” She is still yelling that they are not going to be able to do anything else until they can do fractions. The
point is that it’s not that they are bad; it’s that we are bad for not teaching them correctly so that they have a better conception of what is going on. I told my kids just today, “It’s not your fault; it’s our fault. We are not teaching right, but it’s too late for you anyway.” (lots of laughter by the others) [L3.4a, L3.2d]

The research presented in this session was deemed valuable by Jeanne and Cooper. They liked its conceptual focus and how it provided a framework for understanding some of the difficulties students have with fractions. Jeanne concluded from reading the research article and studying student thinking while solving problems that the reason why students were not understanding fraction concepts was because the methods the teachers had been using to instruct them were not working, and that it is a difficult idea to understand.

Later in this session Jeanne wanted to share more of the findings from her students’ work on another of the conceptual fraction problems (Lamon, 1999). While working out one of the problems on the board for her students in class, she learned how it could be looked at from two of the different perspectives reported in the research. (Figure 3 displays the problem.) After hearing her students discuss the problem, it was reinforced in her mind how hard it is for them to understand.

Does the shaded area in the figure show $\frac{3}{8}$ pie, $\frac{1}{8}$ pies, or $1 \frac{1}{2}$ $\frac{1}{4}$ pies? Does it matter?

Figure 3. Conceptually-based fraction problem
I went up to the board to work the problem and realized, “Now, wait a minute, you are looking at it as one of those bigger pieces \( \frac{3}{8} \) or three of the smaller pieces \( \frac{1}{8} \) or one and a half of the quarter pieces.” [L1.2] I had never ever thought about the different ways you can look at the size of the pieces before reading this article! [L3.4d, L1.4] As I listened closely to the students talk about this problem, one kid said, “I don’t know how to multiply by a mixed fraction so I have no idea what to do.” and another kid said, “Boy, I wish I knew!” [L2.2] (laughter by all)

As she tried to further investigate her students’ thinking, Jeanne found it fascinating that they got stuck on the question about finding two fractions between \( \frac{7}{9} \) and \( \frac{7}{8} \). She found it intriguing that students didn’t know that there were numbers “in between” the two fractions they had been given.

Most of my students didn’t understand the question. They were like, “What do you mean you want fractions in between?” I had to go up and explain that there is actually “stuff” in between those two numbers. They didn’t get that at all! They did lots of things that I had never even considered as possibilities or problems, so I thought it was really interesting, and I learned a lot. [L2.2]

Research to practice. In the same PLC session, Jeanne shared with the teachers some of what she had been doing in her Precalculus class and tied it into the discussions the group had been having about fraction research. Jeanne described her classroom
practice primarily by detailing what it was that she was doing and not what her students do. This is further evidence of her teacher-centered approach to instruction.

I have been working with fractions in completing the square. We are pulling a factor out front and determining how we get that middle term. I am explaining that I need to take $\frac{3}{4}$ out of 5 and they say, “I don’t know how to do that.” So I drew 5 pies on the board, and we counted it out. I did this whole thing, and I thought it was going to be “cake” for them because I had been playing with pies, and they had been laughing at me, but they have been paying attention. [L2.3, L2.4, L3.4d] I didn’t have very many kids that were doing that right so I had been talking about it for three days, and it hadn’t sunk in at all. [L3.4d]

The research article had caught Loretta’s attention and got her thinking about how the participating teachers could help the students overcome the obstacles that they had learned existed with fractions. She wanted clarification as to why it was that the author of the article claimed that students would find it difficult to overcome the challenges they face with fractions. The observer/facilitator offered the possibility that it was because teachers don’t spend enough time on it for students to come to an understanding. Meanwhile, Jeanne suggested that teachers may understand the concept because they have worked with it so frequently while teaching mathematics. This is the beginning of an important revelation for Jeanne as is shown later in this cycle.

Loretta: I didn’t get it if it was in the article, but did it say why students will never catch up conceptually? It pretty much said that they aren’t going to; yet, it really didn’t say why. [L2.2]
O/F: I think part of it is that it takes continuous thinking about these issues; so if teachers move on to different topics and use fractions, the students are not going to develop that conceptual base unless they have more time to think about the concepts. I am not sure if I agree with that whole thing because even as adults we revisit things and add on to our understanding. [L3.1a, L3.1b]

Jeanne: I think it is…it’s like I, as a teacher of mathematics, can find fractions between $\frac{7}{9}$ and $\frac{7}{8}$ because I have been exposed to it many times. I think it is because I’m involved in this game all the time. The students are not involved in it all the time like we are, so we need to keep that in mind. [L3.1a, L3.1b]

Loretta again referred to the research article and wondered why it was that the researcher reported that students differentiate between the different approaches to fractions (part-whole, quotients, operators, measures, and ratios and rates). She suggested that teachers should want their students to see and understand all the different ways to conceptualize fractions. Jeanne came to see through this interchange how the different approaches to understanding fractions can apply to locating fractional radian measures on the unit circle.

Loretta: I want to go back to the article because I didn’t think it mattered which way you do it (present the material to students), but the article says it does. The article states: “…children cognitively differentiate the three quantities named here.” Why does it matter?
I would think that you would want students to be more flexible in their thinking and see it in all three ways. [L3.2b, L3.4b, L2.2, and L2.1]

F/R: This is what the observer/facilitator and I were talking about. To tie our discussion of fractions back to trigonometry and the number $\pi$, we need to come to understand that some people understand fractions better one way over another. The observer/facilitator and I had a discussion about which conceptualization we thought was “best”. She counts the pieces of the circle out so is more of the “$\frac{1}{6} \pi$” person which is the operator approach that Lamon mentions. [L3.2b, L3.4b, L2.2, and L2.1]

Jeanne: I do the other one. I am the “$\frac{5}{6}$th of a $\pi$” person. [L1.2]

Cooper: Yeah, I think I do that too. [L1.2]

O/F: The researcher/facilitator likes the part-whole method. (At this point, each teacher mentioned which method they liked best.) The point is that students may like different ones, but how often do we show the kids a look at the same thing three possible ways? The article is reporting that if you were to ask your kids if the different representations are all equal, they would say no. [L3.2b, L3.4b, L2.2, and L2.1]
The connection between the research discussion and the teachers’ practice is initiated by the facilitators. Teachers are led into thinking about how they could use what they had learned from research and adjust what they do instructionally.

O/F: We have had this whole discussion about fractions in general, but let’s think about how all of it ties into trigonometry. [L1.3] For example, when locating \( \frac{5\pi}{6} \) on the unit circle, I think of it like \( 5\left(\frac{1}{6}\right)\pi \) because I think about it like this (draws Figure 4 on the board). I’ve got my \( \pi \) in 6 pieces and I want 5 of them…one, two, three, four, five…that’s how I internalize it. [L1.2] Where the researcher/facilitator thinks of it like \( \left(\frac{5}{6}\right)\pi \) because…

![Figure 4. Observer/facilitator’s drawing](image)

F/R: …because I like to base everything off of full or half turns in this case. (He draws Figure 5 on the board to demonstrate \( \frac{5}{6} \) th of \( \pi \).) [L1.2]
Figure 5. Facilitator/researcher’s drawing

Jeanne: Ah, I want a little bit less than a full $\pi$ so yeah—yeah—I see that. [L2.1, L1.4]

Cooper: I think of it the way you (the observer/facilitator) do ($5(\frac{1}{6})\pi$), but I write it like this (Cooper writes $\frac{5\pi}{6}$ on the board.) [L1.2]

O/F: One thing to consider, then, is to offer a way for kids to think about it and kind of translating for them—let them in on your thinking:

“Here’s how I am writing it, but I am thinking of it like this.” [L2.4]

Jeanne: I can see that because kids don’t realize it when you say “two-thirds times $x$ ($\frac{2}{3} \cdot x$) is equivalent to two $x$ over three ($\frac{2x}{3}$)”. They ask even at the precalculus level if they are the same! [L2.2, L1.4]

Cooper: Yeah, if you write $\frac{x}{3}$ down and ask what the coefficient is they don’t know. [L1.4, L2.2]

O/F: The reason we (the facilitator/researcher and observer/facilitator) brought the research up before launching into a whole discussion
of radians and developing an activity is it’s useful to look at things like this to get students to start thinking about the fractions. We know from what research shows us that not everyone is going to know what this notation (points to \( \frac{5\pi}{6} \)) means and that two of the ways we have seen to think about it is to break it up as \( 5\left(\frac{1}{6}\right)\pi \) or \( \left(\frac{5}{6}\right)\pi \). If they can internalize one of these then it might give them an edge. That’s the primary connection between the fraction stuff we have been reading and the trig radians. [L1.3, L1.4, L2.2, L2.4, L3.4b]

To conclude the session, the teachers expressed interest in securing more information of the type presented in the research article. They saw value in the conceptually-based fraction problems and wanted to see more because they had come to value the relevance of the material to them and their classroom needs.

O/F: If anyone is really interested, Susan Lamon, who wrote the article, has a whole book on teaching fractions. It has more information like we have looked at. [L2.2]

Cooper: Does she give more ideas on this and more problems? [L2.2]

O/F: She breaks it down into each of the different fraction representations and talks about understanding those; so if you are curious and are thinking about fractions in different representations, then it may be good for you. [L2.2, L1.4]
Jeanne: I think that this would be a good book for our Math Topics 1/2
teachers. That course is a nice place to work on this. I will order it
for our Math Department library. [L2.3]

Discussion

Important excerpts from session 17 are presented in Table 12 to organize the
findings that emerged with respect to Jeanne and to demonstrate how they related to
aspects of the PLC Theoretical Framework. Jeanne’s professional growth and a further
categorization of her knowledge and beliefs are discussed in the paragraphs following
the table.

Table 12

Findings from Session 17

<table>
<thead>
<tr>
<th>PLC Theoretical Framework Components - CK</th>
<th>Page Numbers</th>
<th>Representative Quotes</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1.2: Conceptual Knowledge is characterized by having the ability to provide reasoned responses and justify thought processes when problem solving.</td>
<td>208, 211</td>
<td>p. 208: I went up to the board to work the problem and realized, “Now, wait a minute, you are looking at it as one of those bigger pieces ((\frac{3}{8})) or three of the smaller pieces ((\frac{1}{8})) or one and a half of the quarter pieces.”</td>
</tr>
</tbody>
</table>
| L1.4: Conceptual Knowledge is characterized by having the ability to understand the different representations of a concept, the relative strengths and weaknesses of each, and how they are related to one another. | 208, 213, 213 | p. 208: I had never ever thought about the different ways you can look at the size of the pieces before reading this article!

p. 213: Ah, I want a little bit less than a full \(\pi\) so yeah—yeah—I see that. |

<table>
<thead>
<tr>
<th>PLC Theoretical Framework Components – PCK</th>
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<tbody>
<tr>
<td>L2.1: Pedagogical Content Knowledge is characterized by having the ability to imagine what it looks like for someone else to have an understanding of a concept.</td>
<td>212-213</td>
<td>p. 212 – 213: O/F: ...Where the researcher/facilitator thinks of it like ((\frac{5}{6}))(\pi) because...</td>
</tr>
</tbody>
</table>
Figure 4. Observer/facilitator’s drawing
F/R: …because I like to base everything off of full or half turns in this case. (He draws Figure 5 on the board to demonstrate $\frac{5}{6}$ of $\pi$.)

Figure 5. Facilitator/researcher’s drawing
Jeanne: Ah, I want a little bit less than a full $\pi$ so yeah—yeah—I see that.

L2.2: **Pedagogical Content Knowledge** is characterized by having the ability to articulate/reveal common conceptions students have regarding a particular concept.

204, 205, 208, 213

p. 204-205: I had several students who came up with something like this, who in looking for a number between $\frac{7}{9}$ and $\frac{7}{8}$, they made $\frac{7}{9}$ into $\frac{14}{18}$, and then knew they needed something a little bigger, so they chose $\frac{14}{17}$. One kid explained that they “expanded the fraction to give them more room in between the numbers”. I had several kids do that.

p. 208: As I listened closely to the students talk about this problem, one kid said, “I don’t know how to multiply by a mixed fraction so I have no idea what to do.” and another kid said, “Boy, I wish I knew!”

p. 208: They did lots of things that I had never even considered as possibilities or problems so I thought it was really interesting and I learned a lot.

p. 206:
Cooper: …That’s because we weren’t taught that way and that’s the same for the students. The teachers at the elementary level that are teaching this can’t teach the students because they don’t know how to do it any other way…. Some way we need to break the cycle of the way it’s being taught.

Jeanne: And that is what that article was talking about. I was really impressed by that article!

p. 215: I think that this would be a good book for our Math Topics 1/2 teachers. That course is a nice place to work on this. I will order it for our math department library.
<table>
<thead>
<tr>
<th><strong>PLC Theoretical Framework Components - Beliefs</strong></th>
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<th><strong>Representative Quotes</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>L3.1a: Mathematics is learnable by anyone willing to make the effort rather than by a few isolated people.</td>
<td>210</td>
<td>p. 210: I think it is...it’s like I, as a teacher of mathematics, can find fractions between $\frac{7}{9}$ and $\frac{7}{8}$ because I have been exposed to it many times. I think it is because I’m involved in this game all the time. The students are not involved in it all the time like we are, so we need to keep that in mind.</td>
</tr>
<tr>
<td>L3.1b: Achievement depends more on persistent effort than on the influence of the teacher or textbook.</td>
<td>210</td>
<td>p. 210: I think it is...it’s like I, as a teacher of mathematics, can find fractions between $\frac{7}{9}$ and $\frac{7}{8}$ because I have been exposed to it many times. I think it is because I’m involved in this game all the time. The students are not involved in it all the time like we are, so we need to keep that in mind.</td>
</tr>
<tr>
<td>L3.4d: For effective mathematics instruction to occur, a teacher needs to design instruction that requires active participation by students and encourages curiosity, creativity, investigation, collaboration, and healthy questioning rather than only expecting that students listen intently, take good notes, follow directions and examples, behave themselves, and do the assigned work.</td>
<td>206, 208, 209</td>
<td>p. 206: Yeah, I started to do that for them... p. 209: I have been working with... I am explaining that I... So I drew... I did this whole thing and...I had been playing with pies and they had been laughing at me, but they have been paying attention. I didn’t have very many kids that were doing that right so I had been talking about it for three days, and it hadn’t sunk in at all.</td>
</tr>
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*Conceptual knowledge.* The data from session 17 provided evidence that Jeanne advanced her conceptual knowledge by:

1. reading and studying the research article that incorporated conceptual fraction problems and a framework for understanding the different representations that students use to conceptualize fractions,

2. seeing how the conceptual fraction problems were solved by other PLC participants, and

3. working through one of the problems for her students in class.
The process of deepening conceptual knowledge in teachers is a slow process, but these findings show some progress being made and suggest possible techniques that can be used to advance the PLC teachers’ knowledge of content in a way that they find relevant and worthwhile.

*Pedagogical content knowledge.* The findings reported from session 17 show that Jeanne came to appreciate research findings as presented in the format of conceptually-based problems that help reveal student thinking. She had begun her year in the PLC perceiving the majority of mathematics education research as being irrelevant to her but had experienced through this session how it could be used to inform her teaching practice, and as a result, she valued it.

In this session, Jeanne learned from the other PLC participants’ problem-solving techniques, listening closely to her students’ thought processes, and investigating the research article’s findings and conceptual framework. Through these events, Jeanne began to see more clearly that many of her students did not have a conceptual understanding of fractions and how this gap in their knowledge hindered their ability to appropriately place radian measures on the unit circle. As she listened closely to what her students were saying as they worked on the conceptual fraction problems, Jeanne saw her students doing the problems in ways she never had considered before and was surprised at how poorly they did. She voiced the opinion that it was too late to make a difference with her students. Her perception was that she couldn’t go back and teach them what they were missing because there were too many other concepts and topics she had yet to teach. At the end of the session, Jeanne felt that the concepts they had studied about fractions
needed to be taught differently by teachers so that students could understand the concepts. She committed to purchasing the research book for her department and to try and share her new-found knowledge with the teachers in the courses that come prior to Precalculus in an effort to affect change.

Beliefs about the learning and teaching of mathematics. As has been the case throughout the study of her beliefs, Jeanne again emphasized the need for teachers to provide repetitive practice for students to learn. She based this on the fact that she realized that it took her a lot of exposure and times through the curriculum before she understood it. From the data, it is not clear whether Jeanne can see the distinction between providing more than one opportunity for a student to learn a concept and using drill or asking them for the quick recall of facts.

Classroom Intervention

PLC Session 18

The purpose of session 18 was to collaboratively develop a classroom intervention activity based on what the PLC participants had learned from the research on students’ understanding of fractions and the relationship that knowledge has to evaluating trigonometric functions using the unit circle. During the development process outlined in this section, Jeanne further reveals her deeply held belief that quick recall is important for precalculus students to be proficient at the next level—calculus. This belief lingers despite what she had come to learn and value in previous sessions and the professional learning community’s successful creation of a conceptually-based activity in this and subsequent sessions.
Activity development and beliefs. This session begins with the PLC participants trying to “unpack” what it means to understand evaluating trigonometric functions on the unit circle. The teachers attempted to plan a roadmap for how their students could acquire an understanding of the concept. They started this process by discussing how their textbook addresses the issue. Just as the community had noted in an earlier session, Lorene and Mary Beth point out in this transcript that the text emphasizes the importance of students understanding the ordering of radian measures and then finding the exact value without using a calculator. The difficulties that they see students having are then discussed.

Lorene: In the book they do ask on page 251 #8 and #9, “Without using a calculator rank the following (radian measures) in order from smallest to largest.” They are thinking about it on a number line but not on the unit circle, and number 8 has them find the sign of trig values without using a calculator. [L2.3]

Mary Beth: Number 15 on page 257 says: “Find the exact values for each: $\sin \frac{3\pi}{4}$, etc.” They have to know where those are on the unit circle to be able to find the exact value. [L2.3]

F/R: So what do you think, should we start by trying to get them to understand where to place the angles on the unit circle? [L2.3]

Jeanne: Determining the reference angle is perhaps the most important thing we have to help students learn. [L1.1] This obviously links to finding where the angles are located on the unit circle. [L1.3]
Lorene: We found reference angles today in my Precalculus class in degrees, and they caught onto it, but about the time they catch onto it in degrees, I’m going to be switching them over to radians, and they are going to freak out like we said earlier. What do we do? [L2.2]

Cooper: The radians are easier because if you wanted to know what the angle is in the third quadrant that has a reference angle of $60^\circ$, they have to count it out, but if they can do fractions, they don’t have to think. With degrees it’s a counting and adding thing, but with fractions if they can see the pattern, it’s easier. [L2.1, L3.4a, L3.4c]

O/F: I think it is also about being able to recreate the pattern. It takes awhile to get that quick recall, but if they have the ability to recreate the circle and get back to it, that’s a good starting point. [L2.1, L3.2a, L3.2d, L1.1]

As the discussion moved towards developing an activity, the teachers’ focus reverted back to procedures and quick computation rather than centering on student understanding of the principle concepts and ideas. The facilitators’ role is critical in the following interchange as the observer/facilitator perturbs Jeanne’s beliefs by asking thought-provoking questions. In addition, Diana, one of the precalculus teachers with little experience, questioned the other teachers about the strong emphasis placed on the speed tests that she was told to give her students. The PLC discussion promoted movement in Jeanne’s belief system.
The question I would raise is, “How quickly do they need to be able to recall it (answers) this year?” Maybe down the road it is more important after they understand it. [L2.1, L3.2a, L3.2d, L1.1]

Maybe our expectations are raised too high since it is only their first time through this. [L2.3]

Maybe our expectations are raised too high since it is only their first time through this. [L2.3]

Yeah, but will they be exposed to this again? [L2.3]

No, so this needs to be our big push for this. [L2.3]

I agree, because in Calculus we are like, “You should know this! Why don’t you know this?” [L3.2a] What might be a good thing to do is to have the overhead out with the unit circle, and you have the location of the special angles all marked. For number 1, you could point at the place on the circle for one of the angles and have them write which angle it is, and then quickly go to number 2. That might be helpful for them to find the location and write it down. If they can’t do it fast enough, I would take my pointer away, and then they wouldn’t know where I was pointing at. That might be a good memory recall technique. [L2.4, L3.2a, L3.4a]

You could do that, but again it comes down to when do students need to be able to know where the angle is from memory versus being able to recreate it. For me, I always have to redraw the circle and get centered again. [L3.4a]
Cooper: Yeah, but that’s because you (O/F) don’t do it all the time. [L3.4a, L3.4b]

Diana: But, Cooper, that’s like recreating the trig identities rather than memorizing all of them. [L3.4a, L3.4b]

O/F: Right, so that’s why I think that putting up the overhead and then pointing to it doesn’t give me time to get myself oriented. I need to think through it. [L3.4a, L3.4b]

Jeanne: Then there is no reason to do the memory tests on these. You should look at the worksheets we have and how we do our memory tests. If recalling it really quickly isn’t that important, then there is no reason to do those things, and we are wasting our time with those. [L3.4a]

O/F: I was just asking the question.

Jeanne: No, I agree with you. What I give them is a worksheet with many problems asking students to evaluate trig functions of special angles. I tell my students to basically memorize what’s on here. The question now is why are we asking them to memorize them all? [L3.4a, L3.4b]

After having her beliefs challenged, Jeanne proceeded to propose a different technique than the one she had presented. She described a “scaffold” approach – one that builds one concept upon another – to develop students’ thinking. This idea was initially discussed back in session 16 between Jeanne and the facilitator/researcher.
Jeanne: Maybe what they should need to be able to do is to recreate all this.
First of all, determine which reference angle it is you would use.

So, if I give you $\frac{5\pi}{6}$, you would find which quadrant it is in and also which sign it is—positive or negative. You would then go to your special triangle and then do your sine is opposite over hypotenuse. In other words, help the students know what all the components are to come up with that. [L2.3] If we don’t care whether they memorize where these are, then why don’t we just let them do this every time? (pause) Why are we doing memory tests? Because right now we are giving them tests on stuff like, “What is $\sin(\frac{5\pi}{6})$—real fast—let’s go!” Is that important? Is it important?

[L3.4a]

Diana: I don’t think so. [L1.1]

Cooper: I don’t think so either now, but is it important that they at least be able to find them? [L1.1]

Lorene: I’ll tell you what, we are doing differential equations right now in Calculus, and a differential equation could be defined as the $\sin(xy)$, and it separates into things like the $\sin(\frac{\pi}{2})$ or the $\sin(0)$. In that class, students need to just know arcsin($\frac{1}{2}$). [L1.1]

Cooper: I think in calculus it’s really important. [L1.1]
When the facilitators discussed this portion of the transcripts in the post-PLC debriefing, the observer/facilitator wondered how long the students had been using radians, and just because they may need to use them at higher levels, does that mean they have to memorize it now at the outset of learning it, or with use and practice will they internalize it? The facilitator/researcher added that “even in D.E. (differential equations), why can’t the students be allowed to be thinking, ‘What is the angle whose sine is $\frac{1}{2}$’, and then picture the unit circle and figure it out? Maybe in the department’s D.E. course speed is emphasized, too, so they have to have this memorized as well.”

Diana continued to challenge the others in their thinking as she wondered what it was that she should do with her students. She sought the input and advice from those who have taught Calculus in order to learn what it is that her students need to be successful.

**Diana:** Now, wait! Can they use their calculator (when evaluating trigonometric functions in Calculus)? [L1.1]

**Lorene:** Well, I assume that they know it. [L3.2a]

**Jeanne:** The thing is it will be a lot easier on them if they just knew it. [L3.2a]

**Diana:** I understand that, but are you giving them non-calculator tests or can they use them? [L3.2a]

**O/F:** Good question.

**Cooper:** Yes, but if the book wants them to give the exact value, that is a different story. Whether we want them to memorize them or teach
them the tools that they need to figure it out doesn’t matter to me.

[L1.1]

Diana: I think they need to be able to find where the angles are whether they memorize it eventually or come up with it or whatever, but they do need to know where they (radian measures) are at (on the unit circle). I think they need to know where they are at, regardless, right? (She checks for confirmation.) Where $\frac{\pi}{3}$ is—

where $\frac{2\pi}{3}$ is—I think they then need to know the reference angle.

[L3.4a]

As this session ended, the action items given by the facilitators for session 19 included having smaller teams work independently of each other. The group consisting of Jeanne and Diana agreed to draft an initial activity for the PLC to begin analyzing based on their discussions.

In the post-PLC debriefing, the facilitators noted how surprised they were that quick recall and performing calculations was associated with student understanding by the teachers and appeared to be a major driving force behind their instruction. The observer/facilitator wondered, “If students are just beginning to understand reference angles with degrees, wouldn’t one want to spend at least another day consolidating that, so it can be a bridge for students to using radians?” The facilitator/researcher agreed with her assessment by stating, “I get the distinct feeling that the speed of getting answers and the pace of covering everything is most important and valued the most. I don’t know if
they are aware of how that contrasts so starkly from their stated goal of getting students to understand mathematics.”

Discussion

The following table organizes the important findings from session 18 and ties them to the PLC Theoretical Framework. A discussion of the findings is found after the table.

Table 13

**Findings from Session 18**

<table>
<thead>
<tr>
<th>PLC Theoretical Framework Components - CK</th>
<th>Page Numbers</th>
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<tbody>
<tr>
<td>L1.1: <strong>Conceptual Knowledge</strong> is characterized by having the ability to recognize the central and peripheral ideas of mathematics and their relative importance.</td>
<td>220</td>
<td>p. 220: Determining the reference angle is perhaps the most important thing we have to help students learn.</td>
</tr>
<tr>
<td>L1.3: <strong>Conceptual Knowledge</strong> is characterized by having the ability to build a concept upon earlier ideas, formulate connections within and across domains, and create further extensions.</td>
<td>220</td>
<td>p. 220: This obviously links to finding where the angles are located on the unit circle.</td>
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<tr>
<td>L2.3: <strong>Pedagogical Content Knowledge</strong> is characterized by having the ability to plan a roadmap for how others might acquire understanding of a concept by building upon their existing knowledge base.</td>
<td>224</td>
<td>p. 224: Maybe what they should need to be able to do is to recreate all this. First of all, determine which reference angle it is you would use. So, if I give you $\frac{5\pi}{6}$, you would find which quadrant it is in and also which sign it is—positive or negative. You would then go to your special triangle and then do your sine is opposite over hypotenuse. In other words, help the students know what all the components are to come up with that.</td>
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<tr>
<td><strong>L2.4</strong>: Pedagogical Content Knowledge is characterized by having the ability to use powerful and effective analogies, metaphors, representations, models, verbal descriptions, and examples in facilitating and enhancing the learning experience of others without taking over the process of thinking for them and thus eliminating the challenge.</td>
<td>222</td>
<td>p. 222: What might be a good thing to do is to have the overhead out with the unit circle, and you have the location of the special angles all marked. For number 1, you could point at the place on the circle for one of the angles and have them write which angle it is, and then quickly go to number 2. That might be helpful for them to find the location and write it down. If they can’t do it fast enough, I would take my pointer away, and then they wouldn’t know where I was pointing at. That might be a good memory recall technique.</td>
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| **L3.2a**: For meaningful understanding of mathematics, one needs to concentrate more on the systematic use of general thought processes rather than memorizing isolated facts and algorithms. | 216, 219 | p. 216: I agree, because in Calculus we are like, “You should know this! Why don’t you know this?”
p. 219: The thing is it will be a lot easier on them (in Calculus) if they just knew it. |
| **L3.4a**: For effective mathematics instruction to occur, a teacher needs to focus primarily on student understanding of principle concepts and ideas instead of on procedures and quick computation. | 222, 222-223, 223, 224 | p. 222: (same as quote for L2.4) p. 222-223: O/F: ... I think that putting up the overhead and then pointing to it doesn’t give me time to get myself oriented. I need to think through it.

Jeanne: Then there is no reason to do the memory tests on these then. You should look at the worksheets we have and how we do our memory tests. If recalling it really quickly isn’t that important, then there is no reason to do those things, and we are wasting our time with those.

O/F: I was just asking the question.

Jeanne: No. I agree with you. What I give them is a worksheet with many problems asking students to evaluate trig functions of special angles. I tell my students to basically memorize what’s on here. The question now is why are we asking them to memorize them all?

p. 218: If we don’t care whether they memorize where these are, then why don’t we just let them do this every time? (pause) Why are we doing memory tests? Because right now we are giving them tests on stuff like, “What is \( \sin(\frac{5\pi}{6}) \)—real fast—let’s go!” Is that important? Is it important? |
Conceptual knowledge. The purpose of session 18 was not specifically designed to deepen the participating teachers’ conceptual knowledge, but rather to begin developing a classroom intervention. Therefore, there was not much data from this section to offer much insight into this component of her professional growth. However, two excerpts do relate to this area, and in them Jeanne demonstrated the willingness to state what she perceived to be the most important concept and saw the connection between what the PLC was studying about fractions and the unit circle.

Pedagogical content knowledge. The findings from this session showed Jeanne weighing her beliefs about learning and teaching mathematics against her classroom practices. Jeanne was not being consistent in that she had stated that she valued students understanding mathematical concepts, and yet, her professed classroom practice heavily emphasized rote memorization and quick recall of facts. This PLC session showed Jeanne in the initial stages of attempting to come to terms with these two contradictory issues. After her beliefs were challenged by Diana and the two facilitators, Jeanne shared her idea for a potential classroom activity that led a student through steps for building a conceptual understanding of how to evaluate trigonometric functions from the unit circle. These ideas became the outline for the final PLC activity and after further discussion in the PLC and revision resulted in a conceptually-based classroom activity. In session 19, Jeanne and her team present this activity.

Beliefs about the learning and teaching of mathematics. In this session, Jeanne again expressed her belief that “just knowing” the answers is what she expects, and it is “easier on them (in Calculus)” if they have the values for the trigonometric functions
memorized. She begins to question the value of speed and memorizing as her beliefs are challenged, and she is required to give a justification for why she emphasizes these things so much. It has not occurred to her that she is not appreciating the learning process that students must go through to attain a conceptual understanding and how difficult and detailed it is.

PLC Session 19

Activity development and pedagogical content knowledge. The purpose of session 19 was to review the draft of a classroom activity as created by Jeanne and Diana. The PLC participants were asked to critique the activity in an effort to ensure that it encompassed what had been learned about students’ challenges with understanding fractions and that it focused on evaluating the trigonometric functions conceptually. Diana began by sharing what she and Jeanne had decided regarding the makeup of the activity. They had agreed that it would be important to only include the sine and cosine functions initially because they believed that the other trigonometric functions could be learned with relative ease after the students had a foundational understanding of these two functions. Diana, as a new precalculus teacher, followed Jeanne’s lead in this matter, and it is shown that the others respect Jeanne’s views as well.

Diana: Jeanne and I worked on this, and she didn’t want anything besides sine and cosine. She said that those are the things that we need to develop, and the other functions are not that important in this case. We are not saying that they aren’t important, but she says if we
build up the sine and cosine, then all the others can be built off of that. [L1.1, L2.3]

Lorene: Maybe students are confused because they don’t really get a chance to understand sine and cosine before having to move on to the other ratios that rely on sine and cosine. [L2.2] Maybe that is why we are having so much trouble. We are trying to throw in all these other things, and if we’d just stay with sine and cosine so they get that solid. [L2.3]

Jeanne: I think that once they get these—sine and cosine—tangent comes to them just like that (snaps her fingers), and the other ones come easy as well if they have sine and cosine down. [L2.3]

The activity that Jeanne and Diana created began by asking the students to locate on a unit circle where different radian measure angles should be placed (see Figure 6).
Figure 6. Part 1 of the activity developed by Jeanne and Diana

After studying the activity and working through it for a few minutes, the observer/facilitator asked the team to explain how they had thought that students would go about trying to do what the two teachers had asked them to do. This spurred a discussion about mapping out a plan for how students might acquire an understanding of the concept. The transcripts reveal that the teachers were not used to thinking about how students would proceed through a problem.

O/F: How do you envision students doing Part 1 of the activity? Do you remember that last week we talked about how each of us placed the angles? I am guessing that I did this part differently because I don’t do it as often as you all do who teach it. I may have done it more
like a student would have, so I am just curious as to how you all did it. [L2.1]

Diana: What?

O/F: How do you expect students to place $\frac{7\pi}{4}$? [L2.3]

Jeanne: Okay so there is 1, 2, 3, 4… (She broke the unit circle into eighths.) [L1.2, L2.3]

O/F: So you broke it up. I am just saying that if we think about how we want the students to do it… [L2.3]

Lorene: I have my kids make up one circle that has all the angles on it, and I expect them to break up the one unit circle with all the key pieces first. [L2.3]

O/F: So you would expect them to mark up the circle first. [L2.3]

Lorene: Yeah, I would have it all created first by marking all of the special angle locations and then use it. [L2.3]

R/F: I would draw a little circle for each one separately and then use it for just that one angle. [L1.2]

O/F: When I did it, I looked on the worksheet for all the fourths. I didn’t go a, b, c, d. But a, e, and g are all fourths, so I placed those. So I drew the fourths in because later I knew that I would need the reference angles. [L1.2]

Diana: But the point is we want them to have to put the angles all on the unit circle in order. [L2.3]
O/F: Right, but then I moved to the $\frac{\pi}{3}$, and then I had to think about where it goes because I had $\frac{\pi}{3}$, $\frac{2\pi}{3}$, and $\frac{4\pi}{3}$. For those, I had to think about which side of my $\frac{\pi}{4}$ I wanted to go on. I ultimately put them on the same circle. [L1.2]

The next three parts (2, 3, and 4) of the activity created by Diana and Jeanne were designed to lead students into evaluating sine and cosine by determining the reference angle, the appropriate sign, and finally the exact value (see Figure 7).

2. Write the reference angles of the angles above.

3. What is the sign (+ or -) value of the following for the angles in question #1.
   
   I. sin                      II. cos

4. Find the exact value of the following trig functions.

---

Figure 7. Parts 2, 3, and 4 of the activity developed by Jeanne and Diana

After studying the entire activity, the teachers anticipated that it would be easy for their students and that the students would already know how to work through it all. They brought this up even after they had expressed in prior sessions how difficult they had found this type of problem to be for students.

R/F: I think it (the activity) is a little confusing right now, and we need to figure out a way to help students walk through it more clearly. [L2.3]
Jeanne: But we have been already doing all of this with our kids in class. [L2.2]

Lorene: The activity is cumulative. The students have already done this circle building before. You guys (Observer/facilitator and Researcher/facilitator) haven’t done it much, so it’s confusing for you, but my kids have already done this. [L2.2]

Diana: Mine, too. [L2.2]

Jeanne: We have gone over this circle 10 times by now. They are sick of the circle. [L2.2]

O/F: Then I guess the question is whether this is a hard activity for them or not. [L2.2]

Jeanne: I think this will be very easy for students. [L2.2]

Diana: Me, too. [L2.2]

In the post-PLC debriefing the facilitators noted that the goal of topic selection in the PLC was to target something participants perceived would challenge the students. Over the course of the cycle of investigation, the participants had seen that their students do, in fact, have a hard time with locating a radian angle on the unit circle, and, yet, most in this session expressed the view that they thought it would be easy for their students. It does not appear that the participants recognized this conflict.

At this point, the observer/facilitator reminded the participants of what they had learned from research about how difficult students find this. The discussion turned back to focusing on what specifically it was that the teachers needed to address in the activity
and how to accomplish doing so. Notions similar to what had been mentioned in earlier sessions were reiterated by Jeanne and others with regard to what they believe challenges their students. The idea of making thought-provoking or conceptually-based problems “bonus” or “extra-credit” arises. The facilitator/researcher cautions the participants about doing so because it was his belief that many students would not attempt the problems because of the label they were considering giving these problems. He believed that it would leave the impression that challenging problems are not the norm but “extras” to be done only by the most gifted or talented students.

O/F: Do you remember when we discussed that students struggle with knowing where these (radian measures) are on the circle? That is why I was asking how you think the students would do it. My impression was that they have difficulty placing it (radian measures) on the circle, but now if you are saying it is easy for them, then that is something different. [L2.2]

Jeanne: What’s really hard for them is to find where 2 radians is located. How does that fit into this whole thing? [L2.2]

Diana: I thought you wanted special angles in this activity. [L2.3]

Jeanne: I want that, too. (laughter) I do. I want that too—to name the quadrant where the terminal side is for an angle of 2 radians. [L1.1] Well, what happens is they want to know where the π is on the 2. [L2.2] I try to remind them of what 2 radians means. I take the radius and go along the edge two times to see where it lands on
the unit circle. [L2.4] They lose sight of that as soon as we start doing this \( \frac{\pi}{4} \) business. They know that \( \left( \frac{\pi}{4} \right) \) is radians, but they forget that 2 can be a radian measure too. The connection is missing. [L2.2]

Diana: Okay, so you don’t want just all exact angles? [L2.3]

R/F: You could add problems k and l since the activity stops at j and put the whole number radians in a separate box or something if you don’t want them to try and find the reference angle. [L2.3, L2.4]

O/F: Yeah, you wouldn’t have to do parts 2 – 4 (see Figure 7) for those. [L2.3]

Lorene: You could and make them bonus: What is the reference angle? You know \( \pi – 2 \), and if we want to know what the sign is, they can still tell us if it is plus or minus. [L2.3]

R/F: I think that is a good question to ask them because it will help them see the conceptual meaning. I think asking the students to find the reference angle for 2 radians is a good way to get at whether the students understand the concept of a reference angle because it makes them connect the concept with radian measure and not just degrees. [L2.4, L3.2d]

Lorene: The only thing you couldn’t do is find the exact value. [L2.3]
R/F: If we are focused on getting them to a conceptual understanding
and they can tell us that the reference angle is $\pi - 2$, that is
evidence that they have the concept down. [L3.2d]

Lorene: Do you want to make it number five and make it bonus? [L3.1a,
L3.3b]

R/F: I would rather they just be labeled problems k and l and just ask
them to do them. [L2.3, L3.1a]

Jeanne: You could put a star on it if you wanted to indicate bonus. [L3.4c]

Diana: Do you want me to?

Lorene: Yeah.

R/F: Here is a word of caution: If you put the word bonus next to it or a
star or something, the students who don’t think that they are good
at math are just going to say to themselves, “I can’t do regular
problems let alone do a **bonus** problem!”, or they will think it is
optional and most will not want to do more than they have to do to
get by. [L3.1a]

The facilitator/researcher raised a formatting issue as he considered how students
would be working through the problem. This transcript reveals another instance of when
Jeanne suggested that students should “just know” how to do mathematical steps and not
necessarily demonstrate the thinking process. Jeanne demonstrated this belief
consistently throughout this study and was found to be one hard to get her to confront and
change. The two facilitators make an effort to move her belief (and others) to valuing a more conceptual approach in the following interchange.

R/F: On problem 2, where do you want students to do the work? [L2.3]

Jeanne: There is no work. They should just know them. They know that the reference angles of all the $\frac{1}{4}$’s are 45. [L3.2a]

O/F: Do you want the reference angle in radians or degrees? [L2.3]

Lorene: Radians of course. [L2.3]

Diana: I wrote them all in degrees when I did it because that is where I am at right now (in the book). [L1.2]

Jeanne: But that’s the idea; we don’t want them to do that. [L2.3]

F/R: Why? Do most people think of 10 meters as: “How many feet is that?” first to understand that measurement? Don’t they work from where they can relate or have the most experience with? [L3.4b]

Diana: Okay, so do we want the reference angle in radians? [L2.3]

O/F: Then the point would be that part of why we are doing this is so they know which special triangle to pull. [L2.3] Do they think of the triangles in terms of degrees or radians? [L2.2]

Diana: I think they think of them in degrees. [L2.2]

Jeanne: The idea is for them to become synonymous because really we need to get them off degrees. We need to wean them off degrees because everything that they do from here forward is going to be in radians. [L2.3, L1.1]
O/F: …except those special triangles. [L2.1]

Diana: Okay, in Calculus do you still use the 30-60-90 triangle, or do you use it in $\frac{\pi}{6}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$ radians? [L1.3]

Jeanne: Actually, we still use the 30-60-90. [L1.3]

Diana: You see what she (observer/facilitator) is saying. That is the same thing. [L1.3]

Jeanne: I know. I know. [L1.3]

Loretta: Then we go into chapter 7 which is in degrees. Isn’t it—the right triangle trig? [L1.3]

Diana: Okay, so do we want the angles in degrees or radians? [L2.3]

O/F: It could be both. If you want them to be fluent then that is what we should do. [L2.3, L2.1]

Diana: I thought of just degrees when I did it myself because I was thinking that $\frac{7\pi}{4}$ is a 45. The reference angle is 45. That is exactly what I thought of—not even thinking about $\frac{\pi}{4}$. [L1.2]

Jeanne: It’s really easier to understand in degrees. [L2.1]

Loretta: Yes, it is. [L2.1]

Lorene: Maybe we want both. [L2.3]

O/F: For me, it was easier to do it in radians. I would just look at the denominator. I don’t even care what size it is. [L2.2]
Jeanne: It looks like it depends on the student as to which way they prefer
to work through it. Let’s include both in the revised activity. [L2.3]

The facilitators conclude this part of the session by raising questions about parts 3
and 4 of the activity. They are concerned that the creators may not have considered how
students would need to work through the thought processes in order to arrive at the
answer. The facilitators thought that the activity was designed more for asking students to
provide a correct solution and not to provide evidence of their thinking.

O/F: On #3, what about student organization? Would they be able to
record their answers in such a way that they could identify the
patterns you are going for? [L2.3, L2.4, L3.4a]

R/F: Let’s go back to problem h \( \left( \frac{3\pi}{2} \right) \). Asking what is its reference
angle and what the signs are for the trig values. It may seem like a
strange question at first to ask them for the reference angle because
there technically isn’t one, but I think that it is still a good question
to have on there to discuss. It makes them grapple with the notion
of what a reference angle is used for. [L3.4a, L3.4c, L3.2c, L2.3]

Loretta: I know what they are going to say; they are going to say it is 90°.
[L2.2]

Jeanne: I do think it is a good question. We will need to be aware of that
one when we go through it in class. [L3.4c]

R/F: For #4, do you want blank triangles on here? [L2.3]

O/F: Is the hope that they draw one triangle for each? [L2.3]
Jeanne: I expect my kids to draw it in. Aren’t we guiding the kids too much? [L2.3, L3.4c]

R/F: Remember, when we set this up, we wanted to take them step-by-step through what they had to do. This isn’t a quiz; it is to be a learning tool. We are helping to unpack what the \( \sin\left(\frac{3\pi}{2}\right) \) is and how to find it conceptually.[L2.3, L2.4, L3.2d]

The revised activity included the changes suggested by the participants and the facilitators in the session. When the two facilitators discussed after the session that there should be a problem that would somehow connect the different parts of the activity together in order to help students consolidate their learning, the activity also included one final part (5). The two facilitators emailed the PLC participants their idea, and it was agreed by mutual consensus to add part 5. The activity as it was to be implemented in the classroom is shown in Figure 8. Each teacher was given the leeway as to how she would use it with her students.
1. Mark the circumference of where the terminal side is located for each of the following

   a. \( \frac{7\pi}{4} \)  
   b. \( \frac{\pi}{3} \)  
   c. \( \frac{11\pi}{6} \)  
   d. \( \frac{2\pi}{3} \)  
   e. \( \frac{5\pi}{4} \)  
   f. 1

   g. \( \frac{7\pi}{6} \)  
   h. \( \frac{9\pi}{4} \)  
   i. \( \frac{3\pi}{2} \)  
   j. \( \frac{4\pi}{3} \)  
   k. \( \frac{5\pi}{6} \)  
   l. 4

2. Write the reference angles for each angle, using degree and radian measure.

<table>
<thead>
<tr>
<th>Degree</th>
<th>Radian</th>
<th>Degree</th>
<th>Radian</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( \frac{7\pi}{4} )</td>
<td>( \frac{7\pi}{6} )</td>
<td>g. ( \frac{7\pi}{6} )</td>
<td></td>
</tr>
<tr>
<td>b. ( \frac{\pi}{3} )</td>
<td></td>
<td>h. ( \frac{9\pi}{4} )</td>
<td></td>
</tr>
<tr>
<td>c. ( \frac{11\pi}{6} )</td>
<td>i. ( \frac{3\pi}{2} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. ( \frac{2\pi}{3} )</td>
<td>j. ( \frac{4\pi}{3} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e. ( \frac{5\pi}{4} )</td>
<td>k. ( \frac{5\pi}{6} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f. 1</td>
<td></td>
<td>l. 4</td>
<td></td>
</tr>
</tbody>
</table>
3. What is the sign value (+ / -) of the following for the angles in question #1.

<table>
<thead>
<tr>
<th></th>
<th>sin</th>
<th>cos</th>
<th>sin</th>
<th>cos</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>( \frac{7\pi}{4} )</td>
<td>( \frac{7\pi}{6} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>( \frac{\pi}{3} )</td>
<td>( \frac{9\pi}{4} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td>( \frac{11\pi}{6} )</td>
<td>( \frac{3\pi}{2} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td>( \frac{2\pi}{3} )</td>
<td>( \frac{4\pi}{3} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e.</td>
<td>( \frac{5\pi}{4} )</td>
<td>( \frac{5\pi}{6} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f.</td>
<td>1</td>
<td></td>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>

4. Find the exact value of the following trig functions using special triangles.

<table>
<thead>
<tr>
<th></th>
<th>sin</th>
<th>cos</th>
<th>sin</th>
<th>cos</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>( \frac{7\pi}{4} )</td>
<td>f.</td>
<td>( \frac{7\pi}{6} )</td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>( \frac{\pi}{3} )</td>
<td>g.</td>
<td>( \frac{9\pi}{4} )</td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td>( \frac{11\pi}{6} )</td>
<td>h.</td>
<td>( \frac{3\pi}{2} )</td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td>( \frac{2\pi}{3} )</td>
<td>i.</td>
<td>( \frac{4\pi}{3} )</td>
<td></td>
</tr>
<tr>
<td>e.</td>
<td>( \frac{5\pi}{4} )</td>
<td>j.</td>
<td>( \frac{5\pi}{6} )</td>
<td></td>
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</tbody>
</table>

5. Describe the process for evaluating \( \sin\left(\frac{5\pi}{6}\right) \). Give a step-by-step process.

*Figure 8. PLC-developed activity for evaluating sine and cosine conceptually*
**Discussion**

The following table organizes the important findings from session 19 and ties them to the PLC Theoretical Framework. A discussion of the findings is found after the table.

Table 14

**Findings from Session 19**

<table>
<thead>
<tr>
<th>PLC Theoretical Framework Components - CK</th>
<th>Page Numbers</th>
<th>Representative Quotes</th>
</tr>
</thead>
</table>
| **L1.1: Conceptual Knowledge** is characterized by having the ability to recognize the central and peripheral ideas of mathematics and their relative importance. | 236, 239 | p. 236: I want that, too. (laughter) I do. I want that too—to name the quadrant where the terminal side is for an angle of 2 radians.  
| | | p. 239: The idea is for them to become synonymous because really we need to get them off degrees. We need to wean them off degrees because everything that they do from here forward is going to be in radians. |
| | 240, 240 | p. 236: Diana: Okay, so do we want the reference angle in radians (in the activity)?  
| | | O/F: Then the point would be that part of why we are doing this is so they know which special triangle to pull. Do they think of the triangles in terms of degrees or radians? |
| | | Diana: I think they think of them in degrees.  
| | | Jeanne: The idea is for them (radians and degrees) to become synonymous because really we need to get them off degrees. We need to wean them off degrees because everything that they do from here forward is going to be in radians.  
| | | O/F: ...except those special triangles.  
| | | Diana: Okay, in Calculus do you still use the 30-60-90 triangle, or do you use it in \(\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}\) radians? \[L1.3\] |
| | | Jeanne: Actually, we still use the 30-60-90.  
| | | Diana: You see what she (observer/facilitator) is saying. That is the same thing.  
<p>| | | Jeanne: I know. I know. |</p>
<table>
<thead>
<tr>
<th><strong>PLC Theoretical Framework Components – PCK</strong></th>
<th><strong>Page Numbers</strong></th>
<th><strong>Representative Quotes</strong></th>
</tr>
</thead>
</table>
| **L2.2: Pedagogical Content Knowledge** is characterized by having the ability to articulate/reveal common conceptions students have regarding a particular concept. | 235, 235, 235, 236 | p. 235:  
Jeanne: We have gone over this circle 10 times by now. They are sick of the circle.  

p. 235:  
Jeanne: I think this will be very easy for students. |
| **L2.3: Pedagogical Content Knowledge** is characterized by having the ability to plan a roadmap for how others might acquire understanding of a concept by building upon their existing knowledge base. | 231, 233, 239, 240, 241, 242 | p. 239:  
Jeanne: I don’t want to have to convert these back and forth every time. I want to know them (snap, snap, snap). So what is π? Let’s write it down. Okay, we divided the circle into sixths—I mean twelfths—so we are going to be counting by..... How many degrees is that? And, then they can (snaps her fingers three times) knock them off. |
| **L2.4: Pedagogical Content Knowledge** is characterized by having the ability to use powerful and effective analogies, metaphors, representations, models, verbal descriptions, and examples in facilitating and enhancing the learning experience of others without taking over the process of thinking for them and thus eliminating the challenge. | 236-237 | p. 236-237: I try to remind them of what 2 radians means. I take the radius and go along the edge two times to see where it lands on the unit circle. |

<table>
<thead>
<tr>
<th><strong>PLC Theoretical Framework Components - Beliefs</strong></th>
<th><strong>Page Numbers</strong></th>
<th><strong>Representative Quotes</strong></th>
</tr>
</thead>
</table>
| **L3.2a:** For meaningful understanding of mathematics, one needs to concentrate more on the systematic use of general thought processes rather than on memorizing isolated facts and algorithms. | 2339 | p. 239:  
F/R: On problem 2, where do you want students to do the work?  

Jeanne: There is no work. They should just know them. They know that the reference angles of all the \( \frac{1}{4} \)’s are 45. |
| **L3.4c:** For effective mathematics instruction to occur, a teacher needs to purposefully provide tasks that offer diverse and appropriate challenges rather than striving to alleviate the struggle and ambiguity of doing mathematics. | 238, 241, 242 | p. 238:  
F/R: If we are focused on getting them to a conceptual understanding and they can tell us that the reference angle is \(-2\pi\), that is evidence that they have the concept down.  

Lorene: Do you want to make it number five and make it bonus?  

R/F: I would rather they just be labeled problems k and l and just ask them to do them.  

Jeanne: You could put a star on it if you wanted to indicate bonus. |
Conceptual knowledge. In this session, Jeanne showed confidence in her knowledge of the important ideas for attaining an understanding of the process for evaluating trigonometric functions conceptually. She insisted that the students be asked to find where both whole number and fractional radian measures are located on the unit circle. She acknowledged that students tend to use degrees even when a problem is written in terms of radians but wanted to discourage that because she believed that students would not be working with degrees in higher level courses. As it was pointed out to her that degrees is the way that the special triangles are given in textbooks and generally referred to, Jeanne realized that students are encouraged in some ways to continue to depend on their conception of degrees and to not transfer to radian measure. This caused a dilemma for Jeanne which she did not settle within herself.

Pedagogical content knowledge. The findings from session 19 show the process by which the participants of the PLC came to a consensus on a conceptually-based activity for developing a students’ ability to evaluate the sine and cosine functions for a variety of radian angle measures. Throughout the session it was shown how Jeanne still possessed a belief that quick recall is of paramount importance for students to be
proficient and to be prepared for calculus. The frequent reoccurrence of this belief is interesting to see because of how it contrasts with her strongly stated beliefs that she desires for students to attain conceptual understanding. With Jeanne’s beliefs reemerging throughout the process, the PLC succeeded after much work in developing an activity that builds upon students’ prior knowledge and focuses primarily on principle concepts and ideas instead of on procedures and quick computation. This was a large accomplishment considering the type of professed classroom practices that were described by Jeanne (and others) earlier in this cycle that solely relied on quick recall and computational skills.

Even after a large amount of time had been spent in the PLC investigating the foundational concepts involved, studying the many nuances in gaining a conceptual understanding, and designing a classroom intervention that was to develop students’ understanding, some of the teachers felt that it would be too easy. Jeanne agreed with this premise because the activity was designed to develop the process sequentially (it “babied” them), and she had already gone through evaluating trigonometric functions using the unit circle many times with her class.

Finally, it is important to note that when Jeanne referred to her classroom practice she described it in terms of what she does: “I take the radius and go along the edge two times to see where it lands on the unit circle.” This is consistent with previous accounts when she described her classroom practice as being teacher-centered.

Beliefs about the learning and teaching of mathematics. A consistent theme that runs through the data collected on Jeanne is her deeply held belief that for students to
understand mathematics they need to concentrate on memorizing facts and algorithms. This session provides further evidence of this belief as she expressed that there was no need for the PLC-created activity to provide more space for student work. When questioned about this Jeanne replied, “There is no work. They should just know them.”

Another interesting belief of Jeanne’s that emerged in this session is that it is good for teachers to ask students to be challenged. She suggested that students be given “bonus” points if they do a more challenging problem. The facilitators pointed out that this may leave the impression with the students that not all students are expected to be challenged but only a few isolated people who are more “gifted” in mathematics. After this case was made, the PLC agreed to not offer bonus for the more difficult problems in the activity.

PLC Session 20

Jeanne’s classroom implementation. The action item for the participants from session 19 was to implement the PLC-developed activity in their classrooms in the manner in which each individual teacher chose to do so. The experience that Jeanne had in implementing the lesson showed how her beliefs about the learning and teaching of mathematics and her behavioral traits affected her classroom practice. Despite the fact that Jeanne was one of the original two framers of the PLC activity and was heavily involved in the refinement process, she did not employ it in the classroom enthusiastically or give it much forethought. Jeanne had already been teaching her students how to evaluate trigonometric functions for a period of time using instructional techniques that emphasized memorization and repetition before employing the PLC
activity. Her chosen course of instruction was to drill the students and to give them practice tests as well as “real” tests. According to Jeanne, this was done in order to provide them with many opportunities to learn the material. Jeanne reported that she had taught the concept, reviewed, and tested it and that her students “did well” receiving extra credit for having done so. In class the day of her reporting, she decided to use the PLC activity just briefly before actually executing it in class because she thought it would help her students to make connections.

I used it today in class. I didn’t realize I was going to use it until two minutes before because I had been doing other things with the kids. I thought that this would help because I am having them memorize this material. We’ve worked through all this already; we had a practice test last Friday, and everyone did really well, got extra credit. Today was the real test on sines and cosines, and tomorrow we will have another real test, and then Thursday and Friday I will drill them.

[L3.2a, 3.4a] Today I was thinking that not all of them were getting it—getting the connections between all of these steps—so I gave it today for in-class work. I told them that if they did really nicely, they’d get extra credit. I was hoping that this activity would help them make the connections because I was saying to them, “You have got to memorize this, and I don’t care how you do it.” [L3.2d, L3.2a] I tried to make the connections for them so I’ll see tomorrow because I’ll let them take the test again, and we’ll go over it and see how they do.

Despite the work that the PLC had done to understand student thinking, Jeanne was surprised that her students had a difficult time after she had instructed them. Some of
the areas she noted as being problematic from analyzing her students’ work on the PLC activity were ones that the PLC had discussed. She told how the whole number radian measures (such as 1 and 4 radians) are still difficult for her students to place on the unit circle and as a result concluded that more advanced students may be the only ones capable of doing that type of problem. This belief is contrary to the belief that all students can learn the material if the instructor adjusts his/her teaching methods. She wasn’t sure what else to try. Jeanne explained the results that she saw.

I was surprised that they got stuck. They want to die on the 1 radian and the 4 radians—even though when we worked in class with the unit circle, we found where 1, 2, 3, 4, was. [L2.2] They still don’t get those, but it may keep the real bright kids going. [L3.1.a] They get used to working with radian measure with π in it and then forget what they are doing. [L2.2] I am not sure how to get the kids to understand this, but we have spent a lot of time on it, and I have given them many opportunities to get it. [L2.3]

Beliefs about the learning and teaching of mathematics. After Jeanne reported her findings, Lorene shared that she, too, thought that the activity would be a good way to recap what she had been doing in class with her students. She came to see that the students had not learned how to evaluate the functions very well using the techniques that she had used. After analyzing her students work on the activity, she found that some of the areas the PLC had discussed as being problematic were, in fact, true with her students as well. Following Lorene’s revelation, Jeanne again brought up how she thinks memorization can help students learn the process. Lorene and Cooper agreed with her.
This interchange between three teachers who have each taught Calculus for over ten years is interesting in how it shows them speaking from a high level of mathematical understanding. It appears from their discussion that they think their students should be able to converse in a similar fashion until Cooper briefly pointed out that they may be assuming too much; the three of them put a lot of emphasis on how much the students should already know.

Lorene: I thought this activity would be a recap, but it apparently wasn’t. 1 radian was okay for my kids, and when they worked with 4 radians, they went to degrees. They know 1 radian is about 57°, so 4 radians are about 228°. The activity took a lot longer than I expected. It was a challenge. [L2.2]

Jeanne: I think that some of learning how to do this is if you can memorize the things that connect, then you put the angle in the right quadrant, and so you memorize half of the unit circle and then figure out the rest. [L3.2a]

F/R: What do you mean?

Lorene: For instance, they should know by now that for 45°, sine and cosine are the same value. They should know by now that for 30° and 60° the sine and cosine values change places. So if you know that one answer is $\frac{1}{2}$, then the other has to be $\frac{\sqrt{3}}{2}$. [L2.3, L3.2a]

Cooper: And for the tangent, undefined, 0. You know. [L2.3, L3.2a]

Jeanne: Yeah, I talk about the slope when I work on tangent. [L2.3, L3.2a]
Cooper: And then for sine and cosine, the quadrantal angles are easy. [L2.3, L3.2a]

Lorene: Students just need to know there are 3 key values: $\frac{1}{2}$, $\frac{\sqrt{2}}{2}$, and $\frac{\sqrt{3}}{2}$. 30° comes first, so it must be $\frac{1}{2}$, then 45° is next so it must be $\frac{\sqrt{2}}{2}$, and then 60° is last, so it’s $\frac{\sqrt{3}}{2}$. [L2.3, L3.2a]

Cooper: But remember, Lorene, you get it. [L2.1]

Jeanne: They need to memorize these things, and then know which one is coming up now, and also $\sin^2 x = \frac{3}{2}$ they should know, and tan 90° they should just know. [L2.3, L3.2a]

Pedagogical content knowledge and curriculum alignment. The following conversation shows how the assumptions that Jeanne, Cooper, and Lorene had been making about their students’ prior knowledge were not based on fact. Note the conflicting positions that they were holding without realizing it. First, as was shown in the previous excerpt, they held that their students should “just know” a lot of the material they were teaching, but, on the other hand, they admit in the following discussion that the concepts they were trying to teach is “new information”. They had been working under the assumption that their students had a solid foundation for this material, but, in reality, the teachers learn from Mary Beth, a Math Topics 5/6 teacher, that their students had not previously learned any of the concepts that they were assuming they had. Recall that
Jeanne is the Mathematics Department chair, and she too was working under this assumption.

Jeanne: There is a lot of newer information in this unit on trigonometry that they haven’t done much before—radians. They don’t remember any of this stuff from their Topics 5/6. When you (Mary Beth) taught this, did you do $\frac{\sqrt{3}}{2}, \frac{1}{2}$, and the other values? [L2.2, L3.4b]

Mary Beth: Do you mean the special triangles? The last time that they would have done special triangles was Topics 3/4. [L2.2]

Jeanne: You don’t use the special triangle values in Topics 5/6? [L2.2, L2.3, L3.4b]

Mary Beth: No, we don’t talk about it in terms of those fraction values. [L2.3]

R/F: So it is the first time that they are getting this material? [L2.2, L3.4b]

Jeanne: Yeah, is it? I thought that was one of the things that they do in the Topics 5/6 course. Mary Beth, for a student leaving Topics 5/6, could they tell us what the cos 30° is or what the sin 30° is? [L2.2, L2.3]

Mary Beth: Yes. They could give you the decimal approximations but not the exact values. [L2.2]

Jeanne: Why not the exact values? [L2.2]

Mary Beth: Because we don’t do the exact values in that class. [L2.2]
The revelation that Mary Beth shared starts a discussion between the PLC participants that motivates them to want to learn more and to work towards better alignment of their curriculum. The teachers and especially Jeanne, the department chair, want to know why it is that the Math Topics 5/6 teachers don’t teach the special triangle values.

Lorene: Why not? [L2.3]

Cooper: Yeah, why not? Why not? [L2.3]

Jeanne: That’s a problem. They’ve never seen those! They are getting .866 and .5, but they are not seeing $\frac{\sqrt{3}}{2}$ and $\frac{1}{2}$? [L2.2, L2.3]

Mary Beth: No. [L2.2]

Jeanne: But why? Where is that in here? (She pulls out the Topics 5/6 textbook.) [L2.3]

Mary Beth: So you think, Jeanne, that we should use special right triangles in Topics 5/6. [L2.3]

Jeanne: Yes, we need to have those kids work with those! [L2.3] I am always so surprised that they forget everything, but they say, “No, we haven’t done this before.” I’m like, “Haven’t you ever seen the $\frac{\sqrt{3}}{2}$?” Because you would think that would stick in their minds, but they have never seen it. They would say, “We just use graphs.”, and I would say, “Who did you have (as a teacher)?” [L2.2]
Mary Beth: Yeah, that is what we do. [L2.3]

O/F: So maybe in Topics 3/4 they must have just used the $\frac{\sqrt{3}}{2}$ in right triangle trig and kept it at that and not ever worked with the unit circle. [L2.3]

Mary Beth: Exactly. [L2.3]

The teachers had assumed the students’ previous knowledge base was something it wasn’t. As a consequence, Cooper and Jeanne began to discuss how their students’ lack of prior knowledge affected their attitude about mathematics. They convinced Mary Beth that the Math Topics 5/6 teachers needed to make teaching special triangles and their related fractional values a high priority.

Cooper: This is the thing with the radians. This year my honors kids are saying: “We hate radians!” and “We are done with radians, right?” They have got this wall up about radians. So if you are going to teach it—you know what I am saying—I think it might be more of a disturbance to go through it briefly in Topics 5/6. [L2.3]

Jeanne: But they aren’t supposed to go through it briefly, they are supposed to deal with it in the trig section. [L2.3]

Cooper: Well, in honors, all I know is they are coming in and saying, “I hate radians!” [L2.2]

R/F: Why do you think they say that?

Cooper: I don’t know, but I think it must have something to do with how they were exposed to it to begin with. Mary Beth, how do they (in
the Math Topics 5/6 text) do trig without the special triangles?

[L2.3]

Mary Beth: If you look at the circle, you can tell the horizontal distance is increasing at a different rate than the vertical over certain parts of the domain. [L2.1]

Cooper: You guys should put those right triangles in—not as much as we are but… [L2.3]

Jeanne: They should at least be doing those ratios with the two special triangles. [L2.3]

Mary Beth: Yeah. It wouldn’t be hard to add. I just had never thought of it before now. [L2.3]

This discussion demonstrates the importance of Mary Beth’s role in the group—she is an essential link to the reality of the Math Topics 5/6 curriculum. The interchange also shows that in order for a teacher to help students to make mathematical connections, the teacher must possess knowledge of what is to come. Jeanne, in a previous PLC cycle, had commented that a lot of the Math Topics 5/6 teachers have never taught Precalculus, so they don’t necessarily know what to emphasize in their classes to help their students to be better prepared for future courses. She felt that they simply followed the textbook. This transcript, however, shows how important it is for the teachers at the higher levels of mathematics to know what their students’ prior knowledge base truly is and to not make assumptions about such. As a consequence of this session, Jeanne expressed in a PLC email reflection that she had planned a future meeting with the Math Topics 5/6 teachers.
Her goal for the meeting was to discuss how the special values of right triangle trigonometry are developed and their connection to evaluating trigonometric functions using the unit circle.

Discussion

The following table organizes the important findings from session 20 and ties them to the PLC Theoretical Framework. A discussion of the findings is found after the table.

Table 15

Findings from Session 20

<table>
<thead>
<tr>
<th>PLC Theoretical Framework Components – PCK</th>
<th>Page Numbers</th>
<th>Representative Quotes</th>
</tr>
</thead>
</table>
| L2.2: Pedagogical Content Knowledge is characterized by having the ability to articulate/reveal common conceptions students have regarding a particular concept. | 251, 251, 254, 254, 255, 255 | p. 251: I was surprised that they got stuck. They want to die on the 1 radian and the 4 radians—even though when we worked in class with the unit circle, we found where 1, 2, 3, 4, was.  

p. 254: Jeanne: There is a lot of newer information in this unit on trigonometry that they haven’t done much before—radians. They don’t remember any of this stuff from their Topics 5/6. When you (Mary Beth) taught this, did you do , and the other values?...  

Jeanne: ...You don’t use the special triangle values in 5/6?  

Jeanne: ...Yeah, is it? I thought that was one of the things that they do in the 5/6 course... |
| L2.3: Pedagogical Content Knowledge is characterized by having the ability to plan a roadmap for how others might acquire understanding of a concept by building upon their existing knowledge base. | 251, 252, 253, 254, 255, 255, 256, 257 | p. 251: I am not sure how to get the kids to understand this, but we have spent a lot of time on it, and I have given them many opportunities to get it.  

p. 254-255:  

Jeanne: Why not the exact values?  

Mary Beth: Because we don’t do the exact values in that class.  

Lorene: Why not? |
Cooper: Yeah, why not? Why not?

Jeanne: That’s a problem. They’ve never seen those! They are getting .866 and .5, but they are not seeing \(\frac{\sqrt{3}}{2}\) and \(\frac{1}{2}\).

Mary Beth: No.

Jeanne: But why, where is that in here? (She pulls out the Topics 5/6 textbook.)

Mary Beth: So you think, Jeanne, that we should use special right triangles in 5/6.

Jeanne: Yes, we need to have those kids work with those! I am always so surprised that they forget everything, but they say, “No, we haven’t done this before.” I’m like, “Haven’t you ever seen the \(\frac{\sqrt{3}}{2}\)?”

Because you would think that would stick in their minds, but they have never seen it. They would say, “We just use graphs.”, and I would say, “Who did you have (as a teacher)?”

Mary Beth: Yeah, that is what we do.

p. 256: But they aren’t supposed to go through it briefly, they are supposed to deal with it in the trig section.

p. 257: They should at least be doing those ratios with the two special triangles.

<table>
<thead>
<tr>
<th>PLC Theoretical Framework Components - Beliefs</th>
<th>Page Numbers</th>
<th>Representative Quotes</th>
</tr>
</thead>
<tbody>
<tr>
<td>L3.1a: Mathematics is learnable by anyone willing to make the effort rather than by a few isolated people.</td>
<td>251</td>
<td>p. 251: I was surprised that they got stuck. They want to die on the 1 radian and the 4 radians—even though when we worked in class with the unit circle we found where 1, 2, 3, 4, was. They still don’t get those, but it may keep the real bright kids going.</td>
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| L3.2a: For meaningful understanding of mathematics, one needs to concentrate more on the systematic use of general thought processes rather than on memorizing isolated facts and algorithms. | 250, 250, 252, 252, 253 | p. 252: I think that some of learning how to do this is if you can memorize the things that connect,… and so you memorize… then figure out the rest. 

p. 253: They need to memorize these things and then know which one is coming up now and also \(\sin^2 x = \frac{3}{2}\) they should know, and tan 90° they should just know. |
<table>
<thead>
<tr>
<th>L3.2d: For meaningful understanding of mathematics, one needs to reconstruct new knowledge in one’s own way instead of <strong>memorizing it as given.</strong></th>
<th>250</th>
<th>p. 250: <em>I told them, “You have got to memorize this, and I don’t care how you do it.”</em></th>
</tr>
</thead>
<tbody>
<tr>
<td>L3.4a: For effective mathematics instruction to occur, a teacher needs to focus primarily on student understanding of principle concepts and ideas instead of on <strong>procedures and quick computation.</strong></td>
<td>250</td>
<td>p. 250: <em>Today was the real test on sines and cosines, and tomorrow we will have another real test, and then Thursday and Friday I will drill them.</em></td>
</tr>
</tbody>
</table>
| L3.4b: For effective mathematics instruction to occur, a teacher needs to **build upon students’ prior knowledge and personal experience** rather than presenting material without regard to individual meaning and sensemaking. | 254 | p. 254: *There is a lot of newer information in this unit on trigonometry that they haven’t done much before—radians. They don’t remember any of this stuff from their Topics 5/6. When you (Mary Beth) taught this, did you do \( \frac{\sqrt{3}}{2}, \frac{1}{2} \), and the other values?*  
*Researcher/facilitator: So it is the first time that they are getting this material?*  
*Jeanne:  Yeah, is it? I thought that was one of the things that they do in the 5/6 course. Mary Beth, for a student leaving 5/6, could they tell us what the cos 30° is or what the sin 30° is?*  
*Mary Beth: Yes. They could give you the decimal approximations but not the exact values.*  
*Jeanne: Why not the exact values?*  
*Mary Beth: Because we don’t do the exact values in that class.* |

**Pedagogical content knowledge.** Session 20 was designed by the facilitators to provide a vehicle for the participants to share the results of implementing the PLC-developed activity on evaluating sine and cosine using the unit circle. The data support that Jeanne, though instrumental in developing and refining the activity, did not employ it in her classroom using a carefully planned approach. Rather, she decided to use the activity as she was teaching a lesson. Jeanne’s findings showed that she was still
frustrated that her students did not perform well in this area even after all of the work she had done with and for them.

The frustration that Jeanne expressed led the PLC into a discussion regarding possible causes for the students’ lack of understanding. Through an extended interchange with Mary Beth, the Math Topics 5/6 teacher, Jeanne came to learn that the curriculum of her department was not properly aligned for the topic they had been investigating. Jeanne reported that she had assumed that her students possessed the necessary prior knowledge about the trigonometric concept. However, the students did not have the prerequisite knowledge because the Math Topics 5/6 teachers had not taught the concepts.

As an outcome, the session displayed the merit of having teachers from different mathematics courses working alongside one another. The collaborative environment of the PLC promoted the advancement of Jeanne’s pedagogical content knowledge by helping her to better understand her students’ prior knowledge base. As a result of her learning, Jeanne planned to address the curriculum alignment issue with her Math Topics 5/6 teachers and to teach them the importance of the special triangles for their students’ future mathematical learning.

*Beliefs about the learning and teaching of mathematics.* In this last session of the cycle, Jeanne restates her beliefs that challenging problems are only for bright kids and that students learn by memorizing or “just knowing” the concepts. She reports that she plans to “drill” her students in class to help them learn the material.
Post-PLC Reflection

At the conclusion of the professional learning community, the researcher conducted post-PLC interviews with each participating teacher asking them to reflect upon and assess their PLC experiences. This section presents the most interesting information from Jeanne’s post-PLC interview that provided insight and evidence for the results described in this chapter and helps to provide insight for the research questions. In the interview, Jeanne self-reported the learning that she attributed to the PLC with regard to pedagogical content knowledge and conceptual knowledge. She also shared additional insight into her system of beliefs and how they affect her decisions regarding classroom practice. When prompted to describe the valuable aspects of the professional learning community, she provided her perceptions and gave suggestions for improvements in future iterations. Because of the large amount of data collected, details are presented only for selected interview questions. The data were coded according to the PLC Theoretical Framework. The presentation of results in this section includes a statement of the interview question and transcripts of Jeanne’s subsequent response. This is followed by a discussion of the interview results for that question.

Reflections on Pedagogical Content Knowledge

Share with me some of your reactions to the experiences you had this year as a participant in the PLC.

What has been interesting about the work that we have done here is it has really made me understand when I sit and talk with the other teachers how little we understand about how people learn and what kids are actually getting. That has been a revelation to
me this year. [L2.2, L3.4b] For example, take fractions, we talked about fractions in one of the cycles, and when you gave out the article that discussed different ways students could look at a fraction, I thought, “Ah, that is why fractions are hard!” [L1.4, L2.1] That is one thing that was really cool was when you would bring research and information in about what has been found about how people learn and in particular with fractions. [L2.1] I realized that they (the students) have just learned a small piece (she gestures a little bit with her two fingers) so that when we (high school teachers) expect them up here, (she raises her hand over her head to demonstrate a high level of understanding) they’ve never been there because we’ve never brought them or never taken them there. I’d never explored that before. [L2.2, L3.4b]

Is there anything else you learned about student thinking from your experiences this year?

Oh, yes. I saw how important it is to look closely at the assumptions that you make about what students know [L2.2], and, as the department chair, I realized that we needed to look more closely at the alignment of our curriculum. [L2.3] That whole discussion about special triangles was “scary” for me—that we were assuming so much about what our students knew. Having Mary Beth there so that we could learn from her what was going on in Topics 5/6 was huge. [L2.2] After that PLC session, I went right to the Topics 5/6 teachers, and we discussed how important it is to have the kids learn the special triangles and the fractional values for the triangles’ sides. They had no idea because they hadn’t been teaching the upper levels or had even thought about it for a long time. [L2.3, L3.4b]
As a teacher of secondary mathematics, what are your thoughts regarding mathematics education research literature or materials?

The observer/facilitator was cool bringing in that stuff, but I don’t know where to find that or look for that or know of anything that is very good. When we sit down at lunch and talk about kids—and I did this, and it worked, and this didn’t work…. That’s more about how we can help each other teach better. But we don’t have access; I don’t have access to a lot of research that is reliable. Like that stuff we were screaming about at the beginning of the year—you know. This isn’t any good. You are not solving our problems. Solve our problems! This is what kids do [L2.2]; this is how you get them to do what you want them to do or understand. [L2.3, L2.4] Don’t just tell me there is a problem. We know all the problems. We need some solutions here. [L2.3, L2.4] That is part of the problem with research. They aren’t always giving us solutions. They are discovering problems. [L2.2, L2.3, L2.4]

Can you give me an example of some research we shared that you felt helped you “solve a problem”?

Again, I have to go back to fractions. I felt reassured when the observer/facilitator came back with that research on fractions that showed us what has been found to be hard about it or what the major issues are. That helped me. [L2.2] I see my students doing things wrong and wonder what is wrong with me. I try to make them understand, and I do all kinds of things to help them see it, but they don’t. [L2.4] Then I think back to how many times I have taught this and how much better I see it now than when I taught it the second or third time. We are expecting our kids to understand it at the same level that we
do despite the fact that we have been teaching it for ten years or more! That was really revealing to me, and I appreciate that. [L2.2]

**In your opinion, was the PLC worthwhile? If so, in what ways?**

What I got out of this was to look at what students are doing more. [L2.2, L3.4b] Yesterday, I was going over as $x$ goes to infinity, $f(x)$ goes to where? The kids said that they didn’t get it. Okay, I said, “Let’s think of $\frac{1}{x}$ cubed. What does it look like?” They graphed it in the sky so they already have done it, but, of course, making no connection between the graph and the question. I told them, “But, guys, didn’t I talk about what is happening to $x$? What is the horizontal asymptote so as $x$ is going to infinity where is the function going?”—no connection—This $f(x)$ notation is still not a vertical value for them even after all of this time! That is what is really enlightening to me. This book has been more powerful to me this year because of the PLC; this thing that we have been doing had me look at the students. [L2.2] I was thinking, “Why is this precalculus curriculum doing all of these baby things with horizontal and vertical all year long?” Then finally I realized, “But wait a second, isn’t this asking where are the $y$-values going? They knew when I showed them that as it was going towards the $x$-axis, it is going to 0. Now what is happening when I go to the left?” Then finally one girl goes, “I’ve got it!” What is amazing to me is this—The whole year we have been going through this, and just now, finally, she is making a connection between the graph and the notation. It is kind of like when Ann Sullivan and Helen Keller kept pounding the fingers. They were pounding and pounding to teach Helen, and then, finally, she made the connection
between the water and this stuff. Why did it take so long? Because learning takes time! [L2.3]

What comes to mind is that whole fraction thing (cycle of investigation) that we did was great! I am looking at my little sophomores and thinking, “Holy cow! They don’t have a clue about fractions. How can I teach them anything, and it is not their fault.” I always thought fractions were easy, and that certain kids just don’t get it. [L3.1a] No, fractions are fundamentally hard. They are hard! It must be hard considering that even I didn’t get them until I had gone through a whole bunch of math. [L2.2] I started wondering, “How can I make them see the connection?” [L2.3, L2.4] I learned that things are more difficult than we think they are. That is the problem with math teachers; they think, “How can they not get this?” I don’t know, but it’s not because they are stupid. It’s because it’s hard! [L2.2, L2.3]

Another thing that has shaken me up this year is the realization of why my tests are so hard. Students are always saying my tests are way too hard, but I think they are so easy that I can’t even make them any easier! I now know it’s because I am looking at my level of understanding, and I don’t see how complicated things are for students. [L2.1] I think this is true for all teachers. We need to quit being impatient with kids not getting it and do something else. I don’t know what, yet. I don’t know what. It is just hard. It gives me something to think about. [L2.3, L2.2]

Discussion. An accumulation of the results from Jeanne’s post-PLC interview are displayed in Table 16. The data are compiled according to the relevant components of the PLC Theoretical Framework.
Table 16

Reported Pedagogical Content Knowledge Growth

<table>
<thead>
<tr>
<th>Pedagogical Content Knowledge</th>
<th>Page Numbers</th>
<th>Representative Quotes</th>
</tr>
</thead>
<tbody>
<tr>
<td>• the ability to imagine what it looks like for someone else to have an understanding of a concept. [L2.1]</td>
<td>263, 263,</td>
<td>p. 263: “That is one thing that was really cool was when you would bring research and information in about what has been found about how people learn and in particular with fractions.”</td>
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<td>p. 266: “Another thing that has shaken me up this year is the realization of why my tests are so hard. Students are always saying my tests are way too hard, but I think they are so easy that I can’t even make them any easier! I now know it’s because I am looking at my level of understanding, and I don’t see how complicated things are for students.”</td>
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<td>• the ability to articulate/reveal common conceptions students have regarding a particular concept. [L2.2]</td>
<td>262-263, 263</td>
<td>p. 265: “…this thing that we have been doing had me look at the students.”</td>
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<td>p. 266: “What comes to mind is that whole fraction thing (trigonometry cycle of investigation) that we did was great! I am looking at my little sophomores and thinking, “Holy cow! They don’t have a clue about fractions. How can I teach them anything, and it is not their fault.” I always thought fractions were easy and that certain kids just don’t get it. No, fractions are fundamentally hard—they are hard! It must be hard considering that even I didn’t get them until I had gone through a whole bunch of math… I learned that things are more difficult then we think they are. That is the problem with math teachers; they think “How can they not get this?” I don’t know, but it’s not because they are stupid. It’s because it’s hard!”</td>
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<td>• the ability to plan a roadmap for how others might acquire understanding of a concept by building upon their existing knowledge base. [L2.3]</td>
<td>263, 263,</td>
<td>p. 263: “…I realized that we needed to look more closely at the alignment of our curriculum.”</td>
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The findings presented in Table 16 reveal that Jeanne reported that she grew in pedagogical content knowledge from her experience in the year-long professional learning community. Analysis of the data indicates that Jeanne believes she learned that:

- she understood very little about how people learn and what kids are “getting”,
- studying student thinking and basing classroom instruction upon it are important parts of being an effective teacher,
- mathematics education research can be useful to the practitioner by illuminating how students think about a concept, clarifying the challenges and obstacles students face, and providing suggestions of teaching techniques or good problems for classroom use in order to help overcome students’ challenges,
- teachers can make too many assumptions about what students’ prior knowledge is,
- it is important to monitor the alignment of the curriculum to insure that it establishes the necessary concepts for students,
- the process of coming to understand a mathematical concept is a challenging, time-intensive, and difficult process for many students,
• teachers need to appreciate the difficulty of the learning process and to try and be patient through it, and

• as a teacher, she should try to not get frustrated, but to think about trying a different approach.

Reflections on Conceptual Knowledge

If this PLC were to continue next fall here at your high school, what suggested changes and improvements would you have in terms of engaging each other in more beneficial discussions of mathematics and in teaching practices?

I’d like to get into content more. I really appreciated a lot of the discussion because it made me feel better—that it wasn’t just my kids, but it’s that all kids look like this. Okay, now that all kids are like this. Let’s try to find a solution. That is why I think that it might be better if we had some instruction so that we have a common goal. I know this topic, or, no, I don’t know this topic, and I am going to learn it, and if people saw other people learning…. How do people get to know the material? They do by seeing it, working with it, and actually teaching it. That is this whole thing about why I believe in rotating teachers through the curriculum. If you just leave teachers at one level, then they will look stupid all of the time. They are, but that is because they haven’t been exposed to the content enough, yet.

That’s what I would like to see in a PLC: Here is the topic, let’s work on these problems, and relatively difficult stuff so you can get at where you don’t understand so that everybody has to struggle so that some don’t think, “Well, this is easy, and you are stupid because you don’t get it.” Make it so even the Calculus teacher struggles as well as
the Math Topics 1/2 teacher. I don’t know if you were thinking about coming back, but I would like to open it up to more people rather than keep it at the precalculus level and a narrow thing. If you wanted to do it again, I would like to see a broad spectrum because these topics are not just precalculus topics; they are topics all the way through the curriculum. Everybody could benefit from this type of thing as opposed to writing a lesson plan or a discovery for one day.

Teachers need to go to a thing (session) thinking I am going to learn something here today. It is important that I am going to bring some knowledge back rather than just sit and talk about what we are going to create for the kids. That is important because if they don’t think they are learning anything in this process, then they will back out.

What would be the most beneficial thing(s) that universities/professional development organizations could do to help you grow as a high school math teacher?

I would love to have courses called: “Teaching Precalculus for Teachers”, “Teaching Functions for Teachers”, “Teaching Trigonometry for Teachers”, and teach me the content. Before I taught from the Harvard calculus textbook, I went to a special course for teachers on how to teach from it. There were five of us. We went through that book as students who were going to be teaching it to kids. It was really cool! It probably would have been even better going through it again the next year after having taught it first and being able to come back and say, “Explain this to me and explain that to me—teach me how to teach out of that content.” There were some good problems, and there are some that I still use every year from that. When I had a problem, and I didn’t
understand something, I would call the instructor up and had him explain it to me because he knew the textbook. What I thought was cool was to have someone teach me the textbook before I had to teach kids.

*Discussion.* The results from Jeanne’s post-PLC interview regarding content suggested that she desired an increase in the amount of time spent on working through and solving mathematics problems. Jeanne’s opinion was that an increased focus on conceptual understanding would be advantageous for the teachers so that they can gain a deepened understanding of the mathematics that they teach. Jeanne suggested the following changes for future PLC iterations regarding conceptual knowledge:

- emphasize the content aspect more,
- focus on content for *teachers*,
- challenge the teachers with difficult problems they must work together on, and
- include levels other than precalculus in the PLC because the mathematical concepts are addressed throughout the curriculum.

*Beliefs about Learning and Teaching Mathematics*

*It seems that getting students to solve math problems quickly is valued by some teachers. What are your thoughts about getting your students to quickly solve mathematics problems? Is this important to you? Why or why not?*

I ask them to memorize the cosine and sine of the regular angles. This is so that they will learn them. I put time constraints on it because the only way that kids are going to learn it and not go to a cheat sheet every time is if I pressure them into that. That is a lower level thing like learning your multiplication tables. Those things you should have
to know to learn them. After you learn them, then you can figure out okay 5 times 6 is…

I am real slow at math. I have to write it down. I do very little math in my head. I need a piece of paper because I guess that is how I learned it. [L3.4a]

I don’t think it is important to be fast, but to understand and to know where to start. Speed isn’t understanding. Those that turn their test in early either don’t have a clue or don’t know it well. [L3.2a]

**It has been shown by research that many teachers do not utilize teaching methods that support students acquiring an understanding of the important concepts of mathematics. As a consequence, we (the facilitators) believed it was important to try and affect the PLC teachers’ classroom instruction. How would you suggest trying to impact participants’ teaching?**

Probably what would have been interesting would have been to say, “Okay now, Lorene, what would you do to teach this?” And have each person do it the way they think it should be done— “I’ll do it my way and you do it your way.” Then let’s have a common assessment where we all understand how we are going to assess; let’s write up an assessment for understanding. We all teach it, and we come back to see which parts each group of students did better on. We could then see if we can put the good parts of each into one lesson. That would be different than what we did here where we had one way to teach it. Everybody was supposed to go out and use what we created and then come back and say it did or didn’t work in different aspects. Well, how do we know how to fix that? If we had that way to do it and several other ways, then you could have come
back and said, “We have all of these different techniques. Maybe we should combine the
good aspects of all four ways.” [BT1c]

Discussion. An overview of the results from Jeanne’s post-PLC interview is
displayed in Table 17. The data are compiled according to the relevant components of the
PLC Theoretical Framework.

Table 17

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<th>Reported Beliefs About Learning and Teaching Mathematics</th>
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<td><strong>Beliefs About Learning and Teaching Mathematics</strong></td>
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<td><strong>Representative Quotes</strong></td>
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<tr>
<td>L3.2a: For meaningful understanding of mathematics, <strong>one needs to concentrate more on the systematic use of general thought processes</strong> rather than on memorizing isolated facts and algorithms.</td>
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<td>L3.4a: For effective mathematics instruction to occur, a teacher needs to focus primarily on student understanding of principle concepts and ideas instead of on <strong>procedures and quick computation</strong>.</td>
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<td><strong>BT1: Issues of her self-identity</strong></td>
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<td>c) independence</td>
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Jeanne does not seem to see the dichotomy of her stated beliefs and practice in
this interview. In her teaching practice, she has students memorize the trigonometric
values as a way to learn them and to be able to recall them quickly. She does not appear
to see the level of understanding involved in the concepts and calls it a “lower level
thing”. On the other hand, she professes to believe (in the same interview questions) that
speed doesn’t equate with understanding. However, this notion is what her practice encourages and relays to her students. As the interviewer seeks her input as to how to affect teachers’ classroom practice, Jeanne, as was reported in earlier findings (see Chapter 5), believes that should be left up to each individual teacher to decide.

Summary of Results

This section is a general summary of the results as reported in this chapter. It lays the groundwork for providing insight into the research questions relative to Jeanne in the following chapter. The summary is divided into the four areas of Jeanne’s professional growth that were researched: conceptual knowledge, pedagogical content knowledge, beliefs about the learning and teaching of mathematics (including behavioral traits), and professed practices.

Conceptual Knowledge

At the outset of this study, Jeanne was shown to have a relatively solid conceptual knowledge of the function concept at the precalculus level as measured by scoring at the 95th percentile on the PCA. This report has documented that through her experiences in the PLC, Jeanne deepened her understanding of the process of evaluating trigonometric functions using the unit circle and in particular learning the importance of having a conceptual understanding of fractions. Analysis of the data reveals that Jeanne:

- provided a reasoned response and justification for her thought processes as she explained her solution to one of the assigned fraction problems related to evaluating trigonometric functions and was able to connect the topic to concepts in calculus,
• was not amenable to accepting a “non-traditional” solution (i.e. \( \frac{7}{11.5} \)), at first, but came to see and appreciate the strength of the technique used to get the answer after discussion in the PLC,

• perceived an alternative technique suggested by one of the facilitators for solving a fraction problem (i.e. converting fractions to decimals) as being “weak”, because it did not take advantage of available technology and for being much more time consuming than the method she supported,

• showed confidence in her knowledge of the important or “big” ideas for attaining an understanding of evaluating trigonometric functions conceptually,

• insisted that the students be asked to find where both whole number and fractional radian measures are located on the unit circle because of her perception that both were important for students to know,

• acknowledged that students tend to use degrees even when a problem is written in terms of radians but wanted to discourage that because she held that students would not be working with degrees in higher level mathematics courses,

• saw the connection between the students’ conceptual understanding of fractions and being able to evaluate trigonometric functions using the unit circle,

• advanced her conceptual knowledge by reading and studying the research article that incorporated conceptual fraction problems and a framework for
understanding the different representations that students use to conceptualize fractions, and

- broadened her conceptual knowledge regarding fractions during a PLC discussion that focused on attempting to understand students’ thoughts about the concept, by seeing how the conceptual fraction problems were solved by other PLC participants, and by working through one of the problems for her students in class.

**Pedagogical Content Knowledge**

Pedagogical content knowledge was the area of professional growth that Jeanne showed the most advancement in through her experience participating in the precalculus PLC. Analysis of the data indicates that Jeanne:

- began her year in the PLC perceiving the majority of mathematics education research as being irrelevant to her but had come to learn how it could be used to inform her classroom practice and better understand students’ thinking,

- saw more clearly that many of her students did not have a conceptual understanding of fractions and how this gap in their knowledge hindered their ability to appropriately place radian measures on the unit circle,

- came to appreciate the level of difficulty and challenge that the fraction concept posed for students,

- learned additional approaches to a problem from seeing the other PLC participants’ problem-solving techniques, listening closely to her students’
thought processes, and investigating the research article’s findings and conceptual framework,

- had assumed that her students possessed the necessary prior knowledge about a trigonometric concept and was surprised to learn how poorly her students did on problems that she felt would be “easy” for them,
- realized that there was a problem with the alignment of her department’s curriculum with regard to the topic the PLC studied,
- felt it was too late for her to help her students overcome the conceptual obstacles they faced with fractions because there were too many other concepts and topics she had yet to teach,
- committed to purchasing the research book the PLC had studied regarding fractions for her department and to sharing her new-found knowledge with the teachers in the courses that come prior to Precalculus,
- felt that the concepts they had studied about fractions needed to be taught differently by teachers so that students could understand the concepts, and the teachers needed a better understanding of the concept themselves,
- was instrumental in developing a classroom activity that led students through steps for building a conceptual understanding of evaluating trigonometric functions using the unit circle, and
- wondered how to change her teaching practices to better facilitate student conceptual understanding.
In terms of her beliefs about the learning and teaching of mathematics, analysis of the data reveals that Jeanne believes that:

- quick computation, drill, repetition, and memorization are important components of effective mathematics instruction and are important skills for students to possess in order to be proficient and to be properly prepared for Calculus,
- some problem solving techniques are more “correct” than others due to their being more widely accepted traditionally,
- offering extra credit and creating an atmosphere of pressure and fear in the classroom motivates students to listen, pay attention, and perform better,
- “just knowing” the answers is important for students,
- it is important for students to have a conceptual understanding of mathematical topics,
- there is inadequate time and too much material to cover to use the reform teaching techniques she professed she would like to use,
- it is good for teachers to ask students challenging questions, but it may be necessary to offer “bonus” points in order to help motivate students to attempt the problems, and
- “reform curriculum” is beneficial to students for facilitating their understanding and application of mathematical concepts, advancing their thought processes, realizing its usefulness, and being able to problem solve.
Behavioral Traits

An unexpected result of this study was that Jeanne’s behavioral traits played a major role in determining the level of her professional growth in the PLC and her willingness to employ different techniques and activities in the classroom. The researcher expected that Jeanne’s knowledge and beliefs would be significant factors in her development, but the degree to which her behavioral traits played a role was surprising to the researcher. The traits that data analysis showed were the most instrumental in the growth process were Jeanne’s:

- desire to be the one in authority and making decisions,
- self-assuredness and confidence in her belief system,
- independent nature,
- need to feel “comfortable” with what she was teaching, and
- impatience with the learning process.

Professed Practices

Jeanne would not be considered a “reform” teacher as based upon the RTOP instrument. Data analysis from this study reveals that Jeanne’s teaching practices would be characterized as:

- being teacher-centered,
- emphasizing repetitive practice, and
- being reactionary and not planned out in advance,

Jeanne’s professed practice is an intriguing aspect of this study. There is a conflict between her beliefs and professed practice. She has a stated belief that getting her
students to have a conceptual understanding of mathematics is very important to her, yet in her teaching methods, she drills students so they can memorize and recall answers quickly. Jeanne does not appear to see the level of understanding involved in learning the concepts and calls it a “lower level thing”. On the other hand, she professes to believe that speed doesn’t equate with understanding. However, this notion is what her teaching practice encourages and relays to her students as being most important.

Even so, Jeanne professed a desire and need to employ more inquiry-based and student-centered teaching techniques in her classroom, and shared reasons for why she perceived that she is not able to teach that way. These included the lack of time in class, too much material to cover, the amount of hard work it required, and her impatience with the learning process.

Another important finding from this study is that Jeanne, though instrumental in developing and refining the activity developed by the PLC, did not readily employ it in her classroom. Instead, she decided to use the activity at the last minute as she was teaching a lesson. This demonstrated that her behavioral traits of wanting to be the one in charge, independent, and comfortable with what she teaches took precedence over the learning that she had experienced in the PLC. After Jeanne had taught the lesson, Jeanne expressed her frustration that the students did not perform well. This experience resulted in her beginning to wonder how she could teach the concept differently in the future. Her discussions with the other PLC participants caused her to question and weigh her beliefs about learning and teaching mathematics against her classroom practices and further encouraged her to look closely at her classroom practices.
Finally through the PLC experience, Jeanne considered what she could do as department chair to better prepare the students in her department. She took action by meeting with the teachers in the prerequisite courses to Precalculus in an effort to communicate the concepts that the students were not adequately prepared in and how the teachers were not teaching some necessary topics.
CHAPTER 7: CONCLUSIONS AND DISCUSSION

Introduction

I now present the major conclusions as supported by the development of the professional learning community structure described in Chapter 5 and the results presented in Chapter 6. In this chapter, I discuss these conclusions by providing insight into the research questions that guided this study. When appropriate, I link the conclusions with findings from relevant research studies reported in Chapter 2. Next, I present a brief discussion of the PLC Theoretical Framework used in this study for analyzing participant data. The chapter concludes with a discussion concerning the limitations of the study and recommendations for further research.

Recall the research questions that guided this study:

- What professional learning community attributes are associated with the development of conceptual knowledge and pedagogical content knowledge relative to the function concept in secondary mathematics teachers?
- What professional learning community support tools are associated with improvements in the facilitation of secondary teachers’ reflection on students’ thinking and reasoning relative to the function concept?
- What conceptual knowledge and pedagogical content knowledge are exhibited by a secondary mathematics teacher as she participates in a professional learning community?
What beliefs about the learning and teaching of mathematics are exhibited by a secondary mathematics teacher as she participates in a professional learning community?

What self-reported classroom practices does a secondary mathematics teacher display as she participates in a professional learning community?

Question 1 - The Structure of a Professional Learning Community

What professional learning community attributes are associated with the development of conceptual knowledge and pedagogical content knowledge relative to the function concept in secondary mathematics teachers?

As was discussed in Chapter 2, the literature is quite extensive in suggesting that a promising strategy for sustained, substantive teacher improvement is the creation of professional learning communities among teachers within a school. Although there have been many calls for employing PLCs that are designed to focus on building teacher knowledge and are structured to incorporate how teachers learn, not much is currently known about how to initiate, implement, and sustain such programs. This dissertation study responded to these calls.

In this study, the six secondary precalculus teachers who participated were instrumental in directing the development of the professional learning community structure. As was presented in Chapter 5, one of the unique characteristics of this model for professional development is the capability of the community to direct its investigative focus and collaborative efforts towards matters that the participants find problematic in their own teaching environment. In other words, the PLC is purposefully designed to be
responsive to the practitioners’ expressed interests and needs. As the process of
development unfolded in this study, the PLC structure evolved into six cyclical sessions
that focused on investigating important aspects of learning and teaching.

Recall the conflict described in Chapter 5 between those in the PLC who wanted
to focus primarily on developing their conceptual knowledge (Jeanne and Cooper) and
those who wanted to spend time creating materials for their classrooms (Loretta, Lorene,
Diana, and Mary Beth). The facilitators determined that both of these goals for the PLC
were legitimate and would be worthwhile endeavors. However, the facilitators also knew
it was essential to incorporate into the model’s structure the elements of effective
teaching that have been revealed to be significant in the body of research literature. A
semester of working through PLC session design and implementation was required
before the facilitators arrived at a workable approach to achieving the two group’s goals
while maintaining a foundation based on solid research. The resulting PLC organizational
structure consisted of the six sessions formulating the cycle of investigation.

Each cycle of investigation centered on a teacher-selected precalculus
mathematical topic related to the function concept (e.g. rate of change, composition,
transformations) that was perceived by the teachers to be challenging for students or was
one that they desired to learn more about. Each cycle consisted of sessions focused on the
chosen mathematical topic and devoted to advancing the teachers’ conceptual knowledge,
pedagogical content knowledge, beliefs, and classroom practices. Each of the sessions
contributed to achieving the goal of developing conceptual knowledge in both teachers
and students. The structure of the cycle of investigation that emerged in this study and was outlined in great detail in Chapter 5 includes sessions that build and focus on:

1. conceptual knowledge – knowledge of the central and important ideas of mathematics that is deep, connected, and flexible;

2. pedagogical content knowledge – knowledge about the challenges students are likely to encounter in learning these ideas and knowledge about how the ideas can be represented to teach them effectively;

3. intervention – knowledge to make wise curricular judgments, respond to students’ questions, and look ahead to where concepts are leading and plan accordingly;

4. implementation – knowledge of pedagogy that helps teachers become proficient with a range of different teaching techniques and instructional materials, and organize and manage the classroom;

5. assessment – knowledge about how students’ understanding can be assessed;

6. reflection – knowledge that is informed by engaging in reflective practice and continuous self-improvement.

In addition to erecting the formal structure of the professional learning community model, the results from this dissertation study revealed other key findings regarding related PLC structural issues. The facilitators learned during the development process in early sessions that there was a need to establish a clear definition for what a PLC is and to build an understanding in the participating teachers of what would be expected from them. It could be that during the development of a PLC, teachers may
necessarily experience ambiguity. After all, how do teachers really know what they are to experience until they have actually done so? Naturally uncertainty will exist.

Nonetheless, the early PLC sessions revealed (as described in Chapter 5) that the participants experienced some frustration with not being initially provided with a clear description of what constituted a professional learning community. The vagueness that existed made it difficult for the facilitators to enlist the help of the community members in creating an environment that was effective in developing their knowledge base and improving their classroom practice. A better approach in future PLC iterations may be to more clearly communicate to the teachers the goals and outcomes of the project early in its development. It is noted, however, that this was difficult to do in the current study due to the fact that it was the first effort to implement such a model, and the facilitators were formulating the PLC and its structural organization throughout the process. This finding, however, should help other PLCs get started in a better fashion.

Another issue related to the process of initiating and structuring a PLC that arose in this study was regarding the perceptions that the participants held towards those they would be working with from the supporting university. As was revealed in Chapter 5, some of the participating teachers perceived that they were not viewed as professionals by others and resented that university professors/researchers were. This revelation came from the discussion that the participants had regarding what it meant to be a “professional”. Recall that Cooper expressed her disdain for college professors and their being considered “professional” when she, as a secondary school teacher, was generally not. She demonstrated her angst by sharing what she felt was the widely-held perception
that what practitioners do isn’t “professional”. Furthermore in her pre-PLC interview, Cooper exhibited her lack of respect for college professors and stated that she considers them to be poor teachers who “just present information to students and don’t care if they get it”. She suggested that college professors are generally not concerned about student learning per se but, rather, are immersed in their own research interests. Although Cooper was the most outspoken person on this issue, others expressed similar notions.

*Discussion of Question 1*

In response to these findings, I believe that it is important for those working with secondary mathematics practitioners in PLCs to be keenly aware of this perception and to not perpetuate it by appearing to be condescending or to not appreciate what these teachers have to offer. In addition, to win the respect and trust of secondary teachers, I contend that it is essential for the facilitators of PLCs to be current, or at the very least, former practitioners. I believe this will help to build a bridge between the two groups involved in structuring and implementing the professional learning community. I also postulate that a carefully planned effort to construct PLCs where researchers and practitioners meet and work together and where benefits to both sides are readily apparent will aid in changing these perceptions and starting the process of building better relations.

*Question 2 – Components and Support Tools*

**What professional learning community support tools are associated with improvements in the facilitation of secondary teachers’ reflection on students’ thinking and reasoning relative to the function concept?**
Since teaching mathematics for student understanding presumes the importance of the teachers’ understanding of students as learners, permeating and evolving through this PLC is the focus on student thinking. The results from this study provide evidence that maintaining this focus is critical to the viability and effectiveness of a professional learning community. The underlying assumption of the support structure of the PLC (i.e. cycle of investigation) is a thorough investigation of students’ thinking to initiate the emergence of improved classroom practices. Each means of support builds on this idea of focusing on students’ thinking.

Analysis of the data collected during the interviews and weekly sessions revealed that the PLC participants believed that the most useful components and tools of the PLC were activities that directed them to focus on student thinking and required them to: (1) analyze student work that was acquired by the teachers’ conducting individual interviews with their students; (2) examine the distracters (incorrect answers) from the PCA instrument; (3) investigate student thinking using previously developed conceptual frameworks; (4) analyze student written work in the form of solving research-based problems; (5) listen closely to their students, discuss, and work through assigned problems; and (6) solve conceptually-based problems and discuss alternative problem-solving techniques with other participants in the PLC.

An illustration of how powerful focusing on student thinking can be for a teacher was demonstrated during session 17. After watching and listening to her students try to solve some of the conceptual fraction problems that had been discussed in the PLC, Jeanne reported, “I never would have thought of that (alternative solution method) until I
saw the students do it.” As she further investigated her students’ thinking, Jeanne found it fascinating that they got stuck on a question she had predicted would not be problematic. In the same session, Jeanne exclaimed that “They didn’t get that at all! They did lots of things that I had never even considered as possibilities or problems, so I thought it was really interesting, and I learned a lot!”

I conclude that an emerging focus while looking at student work is debating students’ thinking behind the diversity of their solution procedures. In order to support students’ learning of the mathematical ideas, the PLC participants must first attempt to understand students’ reasoning. Teachers can then build from the students’ thinking about a problem to support students’ learning of the mathematic ideas instead of trying to “fix” the students’ methods of problem-solving. Since the PLC participants are looking at the student work together, they can hold conversations which focus explicitly on their interpretations of student thinking and how to support it.

The results from this study suggest that lesson creation may not be a useful activity during the early implementation of a PLC. Initially, the researchers provided all of the research-based activities necessary for the group to adapt and implement a research-developed lesson. But after the first cycle, the group requested that they create some of the student activities themselves so that they could learn how to design them. When we switched to this model, we saw that the products that emerged from the lesson creation were not designed to promote deep understandings of the concepts in their students, nor was the time spent in creating these activities particularly fruitful in improving the teachers’ conceptual or pedagogical content knowledge. Analyses of the
transcripts from the PLC sessions and end-of-semester interviews revealed that teachers learned more about their students from the conceptual activities we provided than the activities they created. As a consequence, a problem that needs to be addressed in the future is how can a research institution provide support to participants as they take on the ownership and facilitation of the PLC and still ensure that the sessions are organized around productive activities and tasks.

Discussion of Question 2

As the participants of the PLC plan, teach, and revise classroom activities together, decisions about modifications to the activity and teaching practices can be made based on students’ thinking. I believe what makes this process so valuable is its unrelenting focus on student learning. As collaboration continues to develop within the professional learning community, teachers can learn to hold each other accountable for focusing on student thinking. Focusing on student thinking when planning is a fundamental shift in reasoning about teaching for most teachers. A focus on students’ thinking should permeate all aspects of teaching: task selection, planning, classroom environment, discourse, informal assessment, formal assessment, etc. This means decisions made in the PLC are to be based on student thinking, but that there must still be an overarching mathematical agenda and a sense of direction that the professional mathematics educators set.

Furthermore, I maintain that having the participating teachers conduct interviews with individual students would be a powerful agent for initiating teacher change. The interviews of students can be a successful catalyst for initiating conversations about
current practices being problematic, because the students can frequently be found to be thinking differently about the tasks than what teachers assume or predict. Recall that Jeanne expressed how revealing it was for her to hear students discussing the problems she had assigned to them and to see her students not “getting it” even after she had taught them the concepts. The learning Jeanne experienced started her wondering about what she could do differently as a teacher to help the students. If Jeanne had been asked to conduct an individual interview that was designed to probe student’s thinking, I believe that she would have learned more precisely some of the conceptual obstacles that students face. I also predict that she would have learned to become better at revealing her students’ thinking by employing better questioning techniques during classroom instruction. As a result, I believe that Jeanne would have been even more motivated to adjust her teaching practices. As was described in this study, affecting Jeanne’s classroom practices was not an easy thing to do, and this may have been another factor in stirring her to change.

Question 3 – Conceptual Knowledge and Pedagogical Content Knowledge

What conceptual knowledge and pedagogical content knowledge are exhibited by a secondary mathematics teacher as she participates in a professional learning community?

Conceptual Knowledge

At the outset of this dissertation study, the facilitators had thought that building conceptual knowledge in the PLC participants would be a key aspect of change, but Jeanne was found to already possess a relatively strong understanding of the function
concept and, consequently, did not exhibit large-scale professional growth in this area during the year-long study. Even so, there was evidence of some significant improvement in her conceptual knowledge.

During the cycle of investigation (sessions 16 – 20) studied in this dissertation, Jeanne demonstrated growth by deepening her understanding of evaluating trigonometric functions conceptually by learning different representations for fractional radian measures. For instance, Jeanne learned some of the prerequisite conceptual knowledge students need regarding fractions. The Lamon (1999) research article studied in the PLC sessions reported a framework that demonstrated some of the different ways that students can view and conceptualize fractions. This investigation deepened Jeanne’s own conceptual understanding and encouraged her to formulate connections and create further extensions/questions.

An illustration of this finding came from session 17. The PLC teachers had been asked to consider how they could use what they had learned about the conceptual understanding of fractions to teach the concept of evaluating trigonometric functions. Recall that the observer/facilitator had noted that when locating $\frac{5\pi}{6}$ on the unit circle, she thought of it as $\frac{1}{6}(\pi)$ because she preferred to look at the circle “in sixths of $\pi$”.

Alternatively, the facilitator/researcher conceptualized $\frac{5\pi}{6}$ as $(\frac{5}{6})\pi$ due to his desire to base everything off full or half turns in the unit circle. After seeing these two ways of thinking about $\frac{5\pi}{6}$, Jeanne stated that she hadn’t ever considered these different
possibilities. From what she had learned, Jeanne proceeded to make the connection to other related ideas she had noticed in her students. Jeanne pointed to how she had seen her students having trouble seeing the equivalence of $\frac{2x}{3}$ and $\frac{2x}{3}$. As a consequence of the study of research and PLC discussions, Jeanne had deepened her conceptual knowledge and made connections of the concept to other related contexts.

Another way that Jeanne learned was through the process of the participants sharing the techniques they used to solve research-based problems. Recall in session 16 the facilitator/researcher shared his answer for finding a number between $\frac{7}{12}$ and $\frac{7}{11}$ as $\frac{7}{11.5}$. This initiated a lot of discussion among the teachers due to the solution’s unconventional nature and caused them to confront some of their beliefs about the teaching of mathematics and what constitutes a “correct” answer. Jeanne came to see the value in the approach used and expressed her satisfaction in seeing its relevance and power by responding, “I never even thought to do it your (facilitator/researcher) way. That was much easier, and I liked it. That’s cool!” This is evidence that solving conceptually-based problems can deepen the conceptual knowledge of PLC participants by building connections, eliciting different representations, and demonstrating alternative solution paths.

In addition, Jeanne’s relatively deep understanding of the function concept proved beneficial to others in the PLC. The results illustrate that she facilitated advancement in Diana’s conceptual knowledge by helping her to make the connection between a content issue that had arisen to related concepts in calculus. Recall that Diana had wondered
about finding a number between $4.\overline{9}$ and 5 when Jeanne offered some techniques she could use to see that the two were, in fact, equal. Jeanne also noted for the members of the PLC that during this discussion they had ventured into the concepts of infinity and limit that appear in calculus. Although the interchange that took place did not show Jeanne building her conceptual knowledge, it displayed the importance of having her in the PLC to help develop the conceptual knowledge of her colleague.

**Suggestions for Improvement**

The results of this study reveal that Jeanne’s conceptual knowledge grew from her year-long PLC experience, but was not characterized, however, by marked change. This could partly be due to the relatively strong knowledge base that Jeanne had regarding the function concept prior to the implementation of the PLC and her many years of Precalculus and Calculus teaching experience. In an attempt to improve the PLC structure, Jeanne suggested that additional emphasis be placed on building conceptual knowledge in the teachers. Jeanne’s opinion was that this would be advantageous for many of the teachers in her department by deepening their understanding of the mathematics that they teach. I concur with Jeanne’s recommendations and, therefore, suggest the following four changes for future PLC iterations regarding conceptual knowledge:

1. require participants to solve conceptually-based problems that challenge teachers at all levels and encourage them to work collaboratively,
2. focus on the mathematical concepts the participants study from the angle of learning how to teach more effectively by framing and representing the concepts for students,

3. include teachers from various levels because by integrating the different levels of mathematics, all teachers’ conceptual knowledge can be enhanced by improving their ability to see connections up and down the curriculum, and

4. incorporate a course of study linked to the PLCs that focuses primarily on developing teachers’ conceptual knowledge regarding the function concept.

Discussion

A comment I would like to make here is that it is supposed that if this study had focused on a different teacher, more considerable growth in conceptual knowledge could possibly have been exhibited. It is expected that each teacher in a PLC will benefit to differing degrees in the area of conceptual knowledge. I would suppose that work with novice teachers and those without much experience teaching a specific course would be two types of teachers that would demonstrate greater degrees of movement. This point, however, does not diminish the fact that Jeanne, too, developed her conceptual knowledge in significant ways.

Pedagogical Content Knowledge

This aspect of professional development was the area in which Jeanne demonstrated the most growth. In the PLC, the facilitators directed Jeanne to look at her students’ conceptions and to try and build instruction upon student knowledge. This forced her to focus on student learning and pedagogical issues. While eliciting
pedagogical content knowledge, the facilitators were able to impact Jeanne’s conceptual knowledge (as was described in the last section) and challenge her beliefs and professed classroom practices (described in a later section).

The results regarding the growth of Jeanne’s pedagogical content knowledge provide further evidence that having an undeviating focus on student thinking is essential to developing a successful PLC. Jeanne was most responsive to PLC discussions that focused on her students’ mathematical understandings and learning of concepts using research tasks designed to reveal student thinking and understanding of an aspect of the function concept. Analysis of the PLC sessions and interview transcripts revealed that these tasks promoted meaningful discussions about the mathematics and the process of knowing and learning the mathematics. Further coding of these activities revealed that they were effective in: (1) raising Jeanne’s awareness of the sometimes impoverished understandings possessed by her students; (2) imparting pedagogical content knowledge relative to the common misconceptions that her students encounter; (3) promoting Jeanne’s engagement in deeper discussions of her students’ thinking and reasoning; (4) elevating the value that Jeanne placed on mathematics education research by showcasing its relevance to her classroom practice; and (5) challenging Jeanne’s current classroom teaching practices. The evidence for these findings consists of a collection of excerpts from the PLC sessions, individual interviews, and self-reflections.

Results from this study reveal that Jeanne had a weak understanding of what is involved in learning some function concepts. Jeanne didn’t appreciate the complexity of the learning process but came to better understand it through her participation in the PLC.
Through her investigation of student thinking, Jeanne came to see the degree of challenge that evaluating trigonometric functions conceptually and specifically understanding prerequisite fraction concepts posed for students. By studying a research article that laid out a conceptual framework for understanding fractions (Lamon, 1999), she learned that mathematics education research can be useful to the practitioner by illuminating how students think about a concept, clarifying the obstacles students face, and providing suggestions of teaching techniques or good problems for classroom use in order to help overcome students’ challenges.

An example of this is found in Jeanne’s post-PLC interview when she shared her thoughts:

What has been interesting about the work that we have done here (in the PLC) is it has really made me understand when I sit and talk with the other teachers how little we understand how people learn and what kids are actually getting. That has been a revelation to me this year. For example, fractions, we talked about fractions in one of the sessions, and when you gave out that article that discussed different ways students could look at a fraction, I thought, “Ah, that is why fractions are hard!” That is one thing that was really cool was when you would bring research and information in about what has been found about how people learn and in particular with fractions ...They (the students) are just learning a small piece so that when we expect them up here (at a high level of understanding), they’ve never been there because we’ve never brought them or never taken them there. I’d never explored that before.
Another aspect of Jeanne’s learning with respect to pedagogical content knowledge had to do with the alignment of her department’s curriculum and the assumptions she had been making as a classroom teacher about her students’ prior knowledge. Recall that through her discussions with Mary Beth, a Math Topics 5/6 teacher, Jeanne had come to see that she had been incorrectly trying to build upon a knowledge base that didn’t exist in her Precalculus students. As an outcome, the session displayed the merit of having teachers from different levels of mathematics working alongside one another. The collaborative environment of the PLC promoted the advancement of Jeanne’s knowledge by helping her to more accurately understand her students’ prior knowledge base. As a result of her learning, Jeanne planned to address this curriculum alignment issue with her Math Topics 5/6 teachers by teaching them the prerequisite concepts required for their students’ future mathematical learning.

Question 4 – Beliefs About the Learning and Teaching of Mathematics

What beliefs about the learning and teaching of mathematics are exhibited by a secondary mathematics teacher as she participates in a professional learning community?

Beliefs that Hinder Development

While attempting to move Jeanne more towards implementing reform teaching methods in her classroom, it became apparent that she had some strongly-held beliefs that were impeding her progress towards this goal. The results from this study demonstrated that Jeanne believed immediate recall of answers, memorizing isolated facts, and having the ability to quickly respond were important qualities of mathematical learning that
should be present in her students. Recall in session 16 that Jeanne shared: “They have to
do it quick… ‘Okay, \( \sin \frac{\pi}{6} \) is?’ Boom! You’ve got to give it to me right then and there…”

Results from the same session also demonstrate that Jeanne’s professed teaching
practices were closely linked to this belief as she supposed that in order for her students
to learn how to evaluate trigonometric functions, it would be necessary “to hammer
them…” and “to drill them…”.

An important question to try and investigate, therefore, is why did Jeanne believe
so deeply that building quick recall in her students through repetitive practice was
necessary? Results from the study reveal that one reason appears to be that she felt that
students needed to “just know” the answers quickly when they got to Calculus, so it was
her goal to get her students prepared by requiring that they do the same in Precalculus. It
was Jeanne’s opinion that it would take too long for a student to solve calculus problems
if they had to reconstruct the \( \sin \frac{\pi}{6} \) each time they were required to evaluate it. Although
this may be true at the Calculus level, it is important to note that the students she was
trying to elicit quick responses from were learning how to evaluate these functions for the
very first time. As was described while addressing the previous research question, Jeanne
appeared to come to the realization that she had made an incorrect assumption about her
students’ prior knowledge. She grew through her discussions in the PLC and,
specifically, with Mary Beth, a Math Topics 5/6 teacher, that students at her school were
not being taught some of the concepts on which she was basing her instruction.
The study reveals that Jeanne’s effort to speed up the learning process for her students corresponded to her not appreciating the time, effort, and challenges they faced. Through her work in the PLC investigating student thinking, Jeanne saw that learning the concepts that she had set out to teach was a daunting task for many of her students. It was enlightening for Jeanne to see how difficult learning foundational concepts such as those involved with fractions were for students. By reflecting on how much time it took to learn these concepts herself and after having her beliefs challenged by other members of the PLC, Jeanne became more receptive to moving away from such a strong emphasis on speed and memorization in her teaching practices. For instance, recall in session 18 when she stated:

> Then there is no reason to do the memory tests on these… If recalling it really quickly isn’t that important, then there is no reason to do those things, and we are wasting our time with those. …I tell my students to basically memorize what’s on here. The question now is why are we asking them to memorize them all?

The study reveals that trying to get Jeanne to change her teaching practices in one year of PLC implementation was a daunting task especially considering the strong beliefs that she expressed. These findings corroborate a similar finding reported by Cooney (1994) who illustrated through a case study that it was not easy for an experienced and good teacher—certified so by students, co-workers, parents, and administrators—to admit that other approaches toward learning had merit. He found that for the teacher to admit such required a shift away from a teacher-centered classroom in which the teacher felt good, was comfortable, and had “success”. In Cooney’s study, the paradigm shift that
occurred in the teacher required dismantling much of what she had previously valued and believed about the learning and teaching of mathematics by confronting her with what her students were actually learning. Furthermore, Sowder (2005) has suggested that before change can occur in a teacher’s classroom practices, his/her core beliefs may need to be challenged if they run contrary to what is required to teach mathematics for understanding.

Other beliefs that hindered Jeanne’s willingness and ability to teach using reform techniques included her perception that she didn’t have enough time in class, there were too many concepts and skills to teach, and the state standardized assessment forced her to cover the material necessary for her students to pass the examination. The study reveals that Jeanne distinguished quite often between the “honors” and the “regular” students. I believe her reason for doing so was that she believed that many of the students lacked the persistence needed to be successful, and, at the same time, held that not all of her students were capable of learning the more challenging concepts. For instance, she stated in her post-PLC interview that she: “…always thought fractions were easy, and that certain kids just don’t get it.” Another example was when a PLC activity was being developed in session 19, Jeanne spoke of making the more thought-revealing and thought-intensive questions extra credit or offering bonus points to students who did them. The facilitator/researcher pointed out to Jeanne that doing so could give the students the impression that not all of them were expected to think hard about those items or to work to come up with a solution.
Finally, the study’s results revealed that there were instances when Jeanne’s beliefs about the learning and teaching of mathematics were in conflict with each other. The study illustrated that Jeanne believed that for meaningful understanding of mathematics to occur, one needed to concentrate on the systematic use of general thought processes. At other times, results were evident that she believed that it was important to concentrate on memorizing isolated facts and algorithms. Likewise, the study revealed that Jeanne believed that for meaningful understanding of mathematics to occur, one needed to reconstruct new knowledge in one’s own way, and, yet, she also frequently purported that students must memorize knowledge as given. Similarly, the study’s results provided support for Jeanne possessing professed teaching practices that counteracted her beliefs. There was evidence to support that Jeanne believed that for effective mathematics instruction to occur, a teacher needed to focus on student understanding of principle concepts and ideas. However, she oftentimes focused her classroom practices on developing procedures and quick computation in her students. Jeanne alleged to believe that building upon students’ prior knowledge and personal experience was how students learn best, and, yet, she frequently presented material without regard to individual meaning and sensemaking. Illustrations of the above mentioned conflicts were plentiful in the data and were shared in the Summary of Results found in Chapter 6.

Discussion

Based on the results of this study, I offer possible explanations for the inconsistencies. I conclude that Jeanne possibly:
was unaware of the inconsistency that existed between her beliefs, and/or between her beliefs and teaching practices,
did not know how to remedy the problem, or
did not see the need or importance in addressing the inconsistencies that she knew existed.

Consequently, I recommend that added emphasis be placed on having teachers reflect upon their beliefs and practices in an effort to challenge the teachers to confront the inconsistencies that may exist and work towards greater alignment of them.

Furthermore, I put forth that it should be an elevated goal of PLCs to increase the level of participants’ implementation of the teaching practices that research has demonstrated to be beneficial for developing student understanding. As was revealed with the case of Jeanne, it is possible for a teacher to believe strongly in curriculum that is designed to build students’ conceptual understanding, and, yet, maintain and practice teacher-centered instruction. In other words, reform curriculum does not necessarily go hand-in-hand with reform instruction. Therefore, I maintain that it may be necessary for the professional learning community to focus first on convincing participating teachers to utilize reform curriculum or conceptual-based problems in their classrooms. A follow-up goal may then be to attempt to move teachers towards employing reform instructional techniques as they learn the value of doing so.

These suggestions are in alignment with prior research that made the case that some mathematics educators are not conscious of what they believe or where they stand on mathematical issues making them incapable of making informed decisions about how
to align their beliefs with their teaching practices (Schoenfeld, 1989; Sosniak, Ethington, & Varelas, 1991). Cooney (1999) has suggested that efforts to improve instruction should begin with the teacher reflecting on what mathematics means to him/her and how he/she envisions the teaching of the subject. The findings also support the prominence that Ernest (1994a) has placed upon teacher reflection including the teacher’s: (a) awareness of having adopted specific views and assumptions as to the nature of mathematics and its teaching and learning, (b) ability to justify these views, (c) awareness of viable alternatives, (d) context sensitivity in choosing and implementing situationally appropriate strategies in accordance with his/her own views, and (e) being concerned enough to reconcile and integrate classroom practices with beliefs.

**Beliefs that Support Development**

In this section, I share some of Jeanne’s beliefs about the learning and teaching of mathematics that supported her professional development. As was portrayed in Tables 9 and 17, Jeanne alleged to believe that for effective mathematics instruction to occur, a teacher needs to build upon students’ prior knowledge and personal experience and to develop individual meaning and sensemaking. This was evidenced by Jeanne’s desire to incorporate real-world applications into her classroom instruction. It appears that from her experience as a homemaker and as a result of being 40 years of age at the time she started teaching, Jeanne came back to the discipline of mathematics with a different vantage point. She openly questioned why mathematics was generally presented in a contextual void and primarily focused on building skills that, in her opinion, were not important. Recall Jeanne’s self-reflection, “… I didn’t start teaching until I was 40, and I
come in here and look at the kids’ faces and think, ‘How can I justify this? How can I justify teaching this to kids? It just won’t work with my kids. What’s the point?’ Well, there is no point. You know what? Let’s not do it—you know—let’s just not do it! …I started looking at it with fresh eyes.”

This finding lends support to the research results found by others. Ernest (1994) found that teachers are framed and shaped by their life experiences, many of which happen long before they formally enter the world of mathematics education. Furthermore, research done by Adamson et al. (2003) found that teachers’ experiences of life and educational background were instrumental in how they chose to teach their current mathematics classes. One teacher in the study with twenty-five years of experience mentioned that her college major, Nutrition, was instrumental in developing her strongly-held and sustained belief that “real-world” applications were instrumental and fundamental for student understanding of mathematical concepts. The teacher professed to believe that because she learned mathematics best by tapping into what she had personally experienced in life, she chose to incorporate this notion into her teaching.

Finally, Jeanne’s belief in reform curriculum supported her efforts to develop conceptual knowledge. Results reveal that she had grown in conceptual knowledge as she taught using reform material at both the Calculus and Precalculus levels. In the study, she readily admitted that she had not had a deep understanding of many important aspects of the function concept until she taught her students using conceptually-focused curriculum. Another consequence of the growth she experienced from using the reform curriculum was that she had come to believe that effective mathematics instruction should be built on
conceptual understanding and sensemaking. She took action by eliminating many of what she perceived to be out-dated rules, procedures, and techniques (e.g. Descartes’ rule of signs, the rational root theorem, etc.) from the mathematics department curriculum and placing a heavier emphasis on focusing her teachers’ attention on developing conceptual understanding in students.

**Behavioral Traits**

This study offers compelling evidence that Jeanne’s behavioral traits emerged as powerful factors in her professional development and decision-making processes. These traits have the potential to either promote or hinder growth in her classroom practices. The following conclusions are made:

1. Jeanne communicated that she had hoped to become more student-centered in the classroom. However, she cited her impatience with the learning process and desire for quick results as being primary obstacles to her being able to achieve this goal. An illustration of this is found when Jeanne admitted: “I know that I should have the students work more in groups, to communicate, to reflect, but the reason for me personally not doing it, quite frankly, is impatience. I don’t go around and ask the kids and engage them… because I am too impatient to wait… I don’t force them to participate, and I know I need to. That’s a problem… It’s my personality that gets in the way.” The results of the study also revealed that by the end of the year Jeanne had begun to challenge this behavioral trait and to consider making the effort to overcome it: “We need to quit being impatient with kids not getting it and do something else…I don’t know what, yet. I don’t know what. It is just hard.”
(2) Jeanne’s preference for being the one in authority as department chair may have manifested itself in a teacher-centered classroom. In the PLC sessions, she most often referred to what occurred in her classroom in the first person (e.g. I tell them, I show them, I go to the board) and rarely spoke of what the students were doing. This was also supported by the findings from the two classroom visits made during the year. Jeanne appears to find it hard to relinquish control of the classroom by allowing her students to take more active roles in their learning.

(3) Jeanne’s preference for being comfortable with what transpired in her classroom was an additional roadblock to her trying new teaching techniques or classroom activities. Jeanne stated in her mid-year reflection that she hadn’t felt comfortable with the activities that had been developed, so she was hesitant to implement them. She noted that her tendency to be a loner when it comes to teaching made it difficult for her to feel good about using an activity that was created by others and planned in advance of instruction.

(4) Jeanne was very self-assured in her beliefs and abilities which made it difficult to encourage and facilitate change in her classroom practices. Results from one of her interviews reveal extreme self-confidence in her sense of direction and certainty about what can be taught and how it should be taught. The strength of this behavioral trait is evidenced in her statement, “… I would do what I wanted to do, and I wouldn’t care what they said. I would teach how I want to teach.” I conclude that when teachers are this firm in their stance (regardless of what that may be), it becomes a real challenge to move them to a different position.
(5) Other related behavioral traits that were found to be obstacles to her professional development were her desire to react to students in the moment of instruction and the value that she placed on her independence as an educator. Although these are not necessarily negative characteristics, it appears that they had led Jeanne to become an overly reactionary instructor. Mason and Spence (1999) suggest that in order to effectively act in the moment, a teacher needs to intentionally prepare for the moment. The lack of importance that Jeanne placed upon producing well-thought-out classroom interventions had consequences for her development. This practice did not lend itself well to Jeanne thinking deeply about mathematical concepts, studying obstacles to learning, and planning effective instructional methods which are all goals for participants’ professional development in a PLC. In addition, these traits hindered Jeanne’s implementation of the PLC classroom activity. Remember that despite Jeanne having been instrumental in the development of the PLC-designed activity and demonstrating that she had learned much about the conceptions students have, she only made the decision to use the activity at the last minute. As a result, it did not go well, and she became frustrated with the students’ lack of success.

Question 5 – Professed Teaching Practices

What self-reported classroom practices does a secondary mathematics teacher display as she participates in a professional learning community?

Although research is quite clear that affecting change in a teacher’s classroom practice is a monumental task, there were some significant findings with regard to this
aspect of professional growth. Based on the evidence presented in chapter 6, this study draws the following conclusions:

(1) Jeanne’s classroom teaching practices could be described as being teacher-centered and primarily teacher-directed. In PLC sessions, Jeanne described what went on in her classroom most frequently by speaking of what it was that she was doing to get students to learn rather than what students were doing. From the two classroom observations of Jeanne completed by the facilitator/researcher, additional evidence was found to support this. The observed lessons did not begin with recognition of students’ prior knowledge and conceptions, nor did they attempt to engage students as members of a learning community. Furthermore, there was not much time allocated to student investigation of mathematics or student-to-student discussion.

(2) Jeanne was hesitant to accept change in her classroom teaching practices, and this was partially due to her behavioral traits. These include Jeanne’s impatience, independence, and self-assurance, as well as her desire to be in control, the one in authority, and to be comfortable with what transpires in the classroom.

(3) Jeanne professed to want to use activities that got students investigating mathematics and to be “exploring and touching and experimenting”. The study revealed that Jeanne held that there were factors other than her behavioral traits that hindered her from altering her practice to include more actively-involved students. Perceived time constraints she faced were problematic due to the vast amount of material to cover and the requirements to prepare students for standardized assessments and future mathematics courses.
(4) Although it appeared that Jeanne conceptually understood the \[ \sin \left( \frac{11\pi}{6} \right) \] and set the educational goal to develop her students’ conceptual knowledge and ability to make sense of mathematical concepts, her pedagogical choices were such that they didn’t promote this type of knowledge. The role that Jeanne’s beliefs played here was significant. Jeanne demonstrated that she was a teacher who believed she needed to “drill-and-kill”, demand quick recall, and require students to memorize facts. As a result, the conflict that existed between her conceptual knowledge, goals, beliefs, and classroom practices appeared to contribute (albeit inadvertently) to her not electing to focus on conceptual and meaningful learning in her students.

(5) Despite the fact that there were obstacles to Jeanne’s improving her classroom practices, there were also factors and experiences that led to Jeanne beginning to address the need to change. When asked to share what it was that she used to teach evaluating trigonometric functions to her students, Jeanne described a situation in which she was at the front of the class quizzing students on their memorization of similar types of problems (e.g. \( \sin \left( \frac{5\pi}{3} \right) \), \( \cos \left( \frac{3\pi}{4} \right) \), etc.) She required them to provide answers in a short timeframe and purposefully created an environment that encouraged stress and anxiety in students. Based on what Jeanne had learned in the PLC about students’ knowledge of fraction concepts, Jeanne was instrumental in developing a classroom activity that led students sequentially through steps that were designed to build a conceptual understanding for evaluating trigonometric functions using the unit circle.
(6) Results from the study also revealed that Jeanne began to admit that change must occur as she investigated student’s thinking. Jeanne learned that students generally did not have a solid understanding of the fraction concept, and as a result, became more receptive to considering a change in her classroom teaching practices. Jeanne appeared to come to see that studying student thinking and basing classroom instruction upon it are important parts of being an effective teacher. Jeanne also theorized that the root cause for the challenges that the students experience was not that the students were unintelligent or unmotivated, but, rather, it was how these concepts had been taught to them. Jeanne reacted to the Lamon (1999) research article the PLC participants had read and studied about students’ conceptualizations of the fraction concept by sharing in session 17:

I was really impressed by that article!…The point is that it’s not that they (the students) are bad; it’s that we (teachers) are bad for not teaching them correctly so that they have a better conception of what is going on. I told my kids just today, “It’s not your fault; it’s our fault. We are not teaching right…”

Much evidence was found to support these conclusions and the profound impact that focusing on student thinking had on Jeanne. Another instance of the affect this had on Jeanne was when she shared in her post-PLC interview how she had come to see that the learning process was difficult for students and that her teaching needed to change as a result. Recall when she commented:

…that whole fraction thing (cycle of investigation) that we did was great! I am looking at my little sophomores and thinking, “Holy cow! They don’t have a clue about fractions. How can I teach them anything, and it is not their fault.” I always
thought fractions were easy, and that certain kids just don’t get it. No, fractions are fundamentally hard. They are hard! It must be hard considering that even I didn’t get them until I had gone through a whole bunch of math. I started wondering, “How can I make them see the connection?” I learned that things are more difficult than we think they are. That is the problem with math teachers; they think, “How can they not get this?” I don’t know, but it’s not because they are stupid. It’s because it’s hard!

(7) The findings from this study support the conclusions made by many current researchers who agree that for meaningful and lasting changes in classroom practices to occur, teachers need to engage in practical inquiry—to move back and forth among a variety of settings to learn about new instructional strategies, to try them out in their own classrooms, and to reflect on what they observed in a collaborative setting (Stipek et al., 2001; Borko et al., 1997; Franke et al., 1998; Kagan, 1992; Peterson et al., 1989; Wood et al., 1991). Considering this model for change, it is clear that appropriate research design for a professional learning community must start with the assumption that beliefs and practices are closely linked, and emphasis on either one without considering the other is likely to fail to provide the expected movement in teachers’ classroom practices.

Professional Learning Community Theoretical Framework

For this study, a framework for understanding a precalculus professional learning community was used as a lens by which participant professional development could be analyzed. The emergent framework includes three major components of growth including conceptual knowledge, pedagogical content knowledge, and beliefs about the learning
and teaching of mathematics. The behavioral traits of teachers have been added to the framework as an important factor to consider in a PLC to reflect the findings of the case study of Jeanne. The remainder of this section is a brief description of these important aspects of teacher development concluding with a presentation of the framework in its current form. (A full description of the framework and its subdimensions was presented in Chapter 3.)

**Conceptual Knowledge**

Teacher conceptual knowledge is at the forefront of the knowledge base for teachers of mathematics (Bransford et al., 1999; Ball, 2005; Sowder, 2005; NCTM, 2000). It has been revealed that shallow subject knowledge restricts teachers’ capacity to promote conceptual learning among students (Ma, 1999; Stigler & Hiebert, 1999). How well teachers know mathematics conceptually is central to their capacity to use instructional materials wisely, to assess students' progress, and to make sound judgments about presentation, emphasis, and sequencing (Ball, 2003). Teachers need to understand the big ideas of mathematics and be able to represent mathematics as a coherent and connected activity (Ma, 1999; Cooney, 1999; Mason & Spence, 1999). Research shows that teachers need to understand the different representations of a concept, the relative strengths and weaknesses of each, and how they are related to one another in order to facilitate the same understandings in their students (Wilson et al., 1987; NCTM, 2000).

Given the significance of teachers’ understanding of mathematical concepts and its effect on instruction, providing a professional growth opportunity that focuses on developing teachers’ conceptual knowledge is of utmost importance. For this present
study, expanding conceptual knowledge required the careful development of a well-designed professional learning community that made a deliberate and sustained attempt to identify the conceptual knowledge needed for teaching mathematics and on understanding its specific uses in teaching. With this understanding, the PLC structure and cycles of investigation were purposefully designed to advance the teachers’ conceptual knowledge.

**Pedagogical Content Knowledge**

Mathematics educators need an understanding of learning processes and the role of mathematical tasks in the learning process. Mathematics educators also need to design and use mathematical tasks to promote mathematical conceptual learning. Recent studies have reported that many mathematics teachers do not have deep pedagogical content knowledge (Ball, 2005; Sowder, 2005; Ma, 1999; Lewis 2002). This type of knowledge entails understanding the intricacies of teaching mathematics and ways of representing and formulating mathematical concepts that make them comprehensible to others. This would include “knowing” such things as the common conceptions students have, how to structure and represent concepts for teaching to students, how to use effective analogies, metaphors, or examples, and having knowledge of specific teaching strategies that can be implemented for the enhancement of student learning (Shulman, 1986, 1987).

As a consequence of this lack of knowledge in many teachers, the professional learning community in this study was not designed simply for the purpose of producing individuals who knew more mathematics. The goal was to improve students' learning which dictated that the teachers' opportunities to learn in the PLC had to equip them with
the mathematical knowledge and skill that would enable them to teach mathematics more effectively. This dissertation study established that the primary focus of the professional learning community was on what students were thinking rather than on teaching. The facilitators anticipated, and the results bear out, that what the teachers learned from investigating student thinking had an impact on their own conceptual knowledge, beliefs, and classroom practices. The facilitators, therefore, attempted to place the emphasis in the PLC sessions upon working as a community to determine how to engage students in the consideration of essential content in ways that help them develop a deep understanding of that content. PLC-developed activities were directed by the facilitators towards building a conceptual understanding of important mathematical concepts and away from merely skill development.

Beliefs about the Learning and Teaching of Mathematics

Schoenfeld (1998) has argued that what a teacher decides to do in the classroom and why he/she decides to do it is dependent on the teacher’s knowledge, goals, and beliefs. The teacher’s beliefs about what is important about the subject matter, about learning, and about the students play a critical role in shaping what the teacher does in his/her classroom. Knowledge of content and pedagogy is important, but it alone is not enough to account for the differences between mathematics teachers’ classroom practices. As has been offered in this dissertation study, the beliefs that a teacher holds about the learning and teaching of mathematics must be a strong consideration while constructing and implementing the professional learning community.
The results from this study suggest that the importance of a participant’s beliefs necessitates that a professional learning community confront and challenge the beliefs a teacher holds about learning and teaching mathematics. The behavioral traits that Jeanne demonstrated also were important factors in determining the level of professional growth that she experienced and were categorized as a subset of her beliefs. It was not known at the outset of this study that these characteristics would be such powerful factors to consider.

**Emergent Framework for Analyzing Participants in a PLC**

In this section, I present the framework that emerged in this study as a tool for the analysis of participants’ professional growth in future professional learning communities. The framework that evolved was based upon the body of research literature and includes key aspects of teaching and learning mathematics. It was extended and refined by the findings of the study regarding the PLC structure and the case study of Jeanne. The framework is characterized by components and subcomponents of conceptual knowledge, pedagogical content knowledge, beliefs about the learning and teaching of mathematics, and behavioral traits and is displayed in Table 18. More details regarding the framework and its components was provided in Chapters 3 and 6.
Table 18

*Framework for Analyzing Participants in a PLC*

<table>
<thead>
<tr>
<th>Learning</th>
<th>Ongoing action, perpetual curiosity, examination of data, and a continuous effort to deepen one’s knowledge base and refine instructional practice with the goal of enhancing student understanding (DuFour &amp; Eaker, 1998; Hord, 2004; Cox, 2004; NCTM, 2000)</th>
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<tbody>
<tr>
<td><strong>Learning 1 (L1): Conceptual Knowledge</strong></td>
<td></td>
</tr>
<tr>
<td>L1.1</td>
<td>recognize the central and peripheral ideas of mathematics and their relative importance (Schifter, 1999; Sowder, 2005; Shulman, 1987),</td>
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<tr>
<td>L1.2</td>
<td>provide reasoned responses and justify thought processes when problem solving (NCTM, 2000),</td>
</tr>
<tr>
<td>L1.3</td>
<td>build a concept upon earlier ideas, formulate connections within and across domains, and create further extensions (Ball, 2005; Santos-Trigo, 1998; Ma, 1999; Sowder, 2005),</td>
</tr>
<tr>
<td>L1.4</td>
<td>understand the different representations of a concept, the relative strengths and weaknesses of each, and how they are related to one another (NCTM, 2000; CBMS, 2001; Wilson, Shulman, &amp; Richert, 1987; Ma, 1999; Cooney, 1999; Mason &amp; Spence, 1999),</td>
</tr>
<tr>
<td>L1.5</td>
<td>articulate contextual meaning (NCTM, 2000; Gravemeijer &amp; Doorman, 1999) and exhibit sense-making regarding a concept (Adamson, 2005; Simon &amp; Tzur, 2004), and</td>
</tr>
<tr>
<td>L1.6</td>
<td>use knowledge flexibly by appropriately applying or adapting what is learned in one setting in another (NCTM, 2000).</td>
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<tr>
<td><strong>Learning 2 (L2): Pedagogical Content Knowledge</strong></td>
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<td>--------------------------------------------------</td>
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<tr>
<td>Pedagogical content knowledge is characterized by having the ability to–</td>
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<tr>
<td>L2.1 imagine what it looks like for someone else to have an understanding of a concept (Silverman, 2005),</td>
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<tr>
<td>L2.2 articulate/reveal common conceptions students have regarding a particular concept (Shulman, 1987; NCTM, 2000; NRC, 2005),</td>
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<tr>
<td>L2.3 plan a roadmap for how others might acquire understanding of a concept by building upon their existing knowledge base (Silverman, 2005; Ball, 2003; NRC, 2005),</td>
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<tr>
<td>L2.4 use powerful and effective analogies, metaphors, representations, models, verbal descriptions, and examples in facilitating and enhancing the learning experience of others without taking over the process of thinking for them and thus eliminating the challenge (Shulman, 1987; NCTM, 2000),</td>
<td></td>
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<tr>
<td>L2.5 utilize both real-world experiences of students and purely mathematical contexts that are intriguing, challenging, and invite speculation and hard work by those with varied levels of expertise (NCTM, 2000), and</td>
<td></td>
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<tr>
<td>L2.6 accommodate and integrate students’ culture and language into the learning process and equitably provide effective learning experiences for all (NCTM, 2000).</td>
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</tbody>
</table>
Learning 3 (L3): Beliefs about Learning and Teaching Mathematics

L3.1 Learnability: (Carlson, 1997)
   a. Mathematics is learnable by anyone willing to make the effort rather than by a few isolated people.
   b. Achievement depends more on persistent effort than on the influence of the teacher or textbook.

L3.2 Critical Thinking: (Carlson, 1997)
   For meaningful understanding of mathematics, one needs to—
   a. concentrate more on the systematic use of general thought processes rather than on memorizing isolated facts and algorithms,
   b. examine situations in many ways, and not feel intimidated by committing mistakes rather than follow a single approach from an authoritative source,
   c. look for discrepancies in one’s own knowledge instead of just accumulating new information, and
   d. reconstruct new knowledge in one’s own way instead of memorizing it as given.

L3.3 Personal Relevance: (Carlson, 1997)
   a. Mathematics and related technology are relevant to everyone’s life rather than being of exclusive concern of mathematicians.
   b. Mathematics should be studied more for personal benefit than for just fulfilling curriculum requirements.

L3.4 Instructional Philosophy:
   For effective mathematics instruction to occur, a teacher needs to—
   a. focus primarily on student understanding of principle concepts and ideas instead of on procedures and quick computation (Sowder, 2005; NRC, 2005),
   b. build upon students’ prior knowledge and personal experience rather than presenting material without regard to individual meaning and sensemaking (Dewey, 1933; von Glaserfeld, 1990; Boaler, 1993; Gravemeijer & Doorman, 1999; Adamson, 2005),
   c. purposefully provide tasks that offer diverse and appropriate challenges rather than striving to alleviate the struggle and ambiguity of doing mathematics (Santos-Trigo, 1998; Cooney, 1993),
   d. design instruction that requires active participation by students and encourages curiosity, creativity, investigation, collaboration, and healthy questioning rather than only expecting that students listen intently, take good notes, follow directions and examples, behave themselves, and do the assigned work (Lawson et al., 2002; Ernest, 1991; Cooney, 1993),
   e. require students to communicate their thinking with others in a variety of ways rather than simply providing to the teacher correct written solutions to problems that they individually arrive at (NCTM, 2000; NRC, 2005), and
   f. facilitate students’ self-monitoring by teaching them to critically reflect, estimate, and assess their own understanding rather than looking only to an authoritative source for verification (NRC, 2005).
Learning 4 (L4): Behavioral Traits (BT)

Important factors that affect decisions regarding teaching practices include the participant’s–

BT1: Self-identity
   a. Role of Authority – the level at which the teacher desires to be in charge and the central focus of the classroom.
   b. Self-Assurance – the level at which the teacher feels confidence in the “correctness” or “effectiveness” of their teaching methods.
   c. Independence – the level at which the teacher desires to make classroom instructional decisions on his/her own.

BT2: Personality Characteristics
   a. Impatience – the willingness of a teacher to wait and let the learning process take place.
   b. Comfort Level – the willingness of a teacher to experiment, accept change, work hard, and strive for continual improvement.

Limitations of the Study

This study reveals much about the role that a professional learning community plays in providing participating teachers the opportunity to develop professionally and supporting them in their efforts to do so. Also, this study indicates that, as a result of experiencing collaborative discussions about learning and teaching mathematics and delving into mathematical content, teacher growth can be positively impacted. However, this dissertation study does have limitations. In this section, I discuss three limitations: the role of the researcher as the facilitator of the PLC, the selection of one teacher as the investigative focus of the study, and the nature of the teacher self-reported data.

In this study, the researcher also served as the primary facilitator for creating, developing, and implementing the professional learning community. In reporting the results of the study, I largely omitted any mention of the role that the facilitator may have
played in impacting teacher growth and the establishment of the community. Rather, I focused on the way in which the supportive PLC environment, the participant discourse, research materials, and the development of classroom activities and tasks may have aided in positively impacting Jeanne’s growth in the areas of study. I acknowledge that the role of the facilitator in any group setting is very important. An important future study, connected to this study, could include the careful investigation of the development and implementation of another PLC by a different facilitator. In doing so, one might find some similarities in results, thus validating this study or one might find new results, thus adding to the body of research knowledge. Note that in this study, the role that the facilitator played (though mentioned sparingly) fell into the category of structuring the professional learning community. That is, I make the claim that the establishment of the community and how it functions is largely controlled and affected by the actions of the PLC facilitator.

Furthermore, the facilitator also played the role of researcher and conducted the individual teacher interviews. One must wonder if the outcome of these interviews would have been different if the interviewer was not actively involved in the PLC. It would seem reasonable to assume that some participants might have been hesitant to openly share their opinions, beliefs, and thoughts with the researcher knowing that the same person was the one meeting with her and her colleagues each week. It is noted that this did not seem to be the case, however, as the year unfolded. I hold that the participants were truthful and honest in all dealings especially as the bonds of trust and respect were built throughout the year.
Another limitation of this study is the choice I made to closely examine only one of the six participating teachers. Since the participants in this dissertation study were secondary mathematics teachers from the same school, they did by default have many characteristics and school cultural elements in common. However, it was noted during data analysis that a different account could have been told about each teacher considering the fact that each had her own level of conceptual and pedagogical content knowledge, beliefs about the learning and teaching of mathematics, and teaching practices. An interesting follow-up study would be to closely analyze each of the other participating teachers to determine if similar results could be found. Likewise, it would be important to highlight the differences that exist between the teachers in an attempt to illuminate the potential reasons for attaining differing results. This type of study could be done to better equip future facilitators for implementing the PLC sessions and for more effectively designing the structural components of the PLC.

The final limitation I discuss is that of the nature of some of the data collected and reported in this dissertation study. Much of the data related to teacher beliefs and classroom practices were self-reported by teachers. As is the case with most data of this sort, caution should arise in the researcher when considering its proper usage. For instance, it is important to consider the validity of Jeanne’s claim that she desired to use reform teaching techniques more frequently in her classes but was held back in her efforts because of the lack of class time available to her, the large volume of material to cover, and her impatience with the learning process. Further research could be done to provide more evidence of the validity of each of these claims and to study the possible root causes
for their existence. In addition, research could be done to determine if the factors that Jeanne suggests are insurmountable obstacles or if they could be overcome by the collaborative effort of professional colleagues in the context of a PLC.

Recommendations for Future Research

This dissertation study suggests several areas for future research. The teachers in this study reported not having available to them the opportunity to participate in many professional development programs that specifically address their interests and needs. It would be informative to conduct a study where teachers experience multiple models of professional development in addition to the professional learning community described in this study. What model components would the teachers note as being most advantageous to them and for being the most supportive to their growth in both teacher knowledge and pedagogy? An interesting follow-up study would be with the teachers that participated in this PLC as they continue through future iterations. How much growth would the participants demonstrate longitudinally over a period of years? Replicating this study with another group of secondary precalculus teachers from the same school district but one that does not utilize reform curriculum may be enlightening too. How much more difficult would it be to facilitate change and encourage teachers to embrace teaching methods in alignment with what is called for in the research literature? It would also be interesting to see how this same group of teachers functions with a different facilitator. This would help to determine the extent to which the results of this study were dependent on my particular facilitation style and the choices I made.
As follow-up work with Jeanne, a study could be set up to determine what is needed to help her address the behavioral traits that emerged as being a hindrance to her becoming more student-centered in terms of classroom instruction. Furthermore, studies that are designed to specifically address the other areas that Jeanne mentioned as being barriers to her (and the teachers in her department) professional development could be conducted to help her (and them) overcome them.

One could also conduct similar studies of professional learning communities where the mathematical focus was on a topic other than the function concept, was not at the precalculus level, or took place with teachers with less teaching experience or knowledge. How does the fact that the study was conducted with teachers that possessed a relatively good knowledge base and most had many years of teaching experience effect the PLC development and implementation? How would the results be different with novice teachers or those with a lesser knowledge base? Would the PLC discussions need to move from being focused more on pedagogical content knowledge and student thought processes to a heavier emphasis on building teacher conceptual knowledge, developing pedagogical techniques, or improving classroom management strategies? My classroom teaching experiences and the work I have done with teachers at all levels of development suggest that the professional learning community structure described in this dissertation would be well-suited to accommodate the needs of any group of mathematics teachers. As a result, as I attempted to do in this study, the PLC model could be adapted and used to address the stated needs of the individual teachers that comprise a community. Novice teachers may, in fact, need to focus their collective attention initially on deepening their
own conceptual knowledge and improving classroom management. It would seem that having beginning and experienced teachers together in such a PLC would be beneficial to all involved.

A study could involve investigating how to focus participants’ attention even more closely on student thought processes and finding evidence of support for assessment. Reflecting on my own teacher preparation program and from what I know to exist in many current undergraduate programs of study, I believe that most secondary mathematics teachers do not have the ability to analyze their students’ thinking nor do they recognize the value in doing so. It is my opinion that further studies should work to train teachers in PLCs to look more closely at student thinking and to self-reflect using more “formal” analytical tools including:

- written case studies,
- student interviews,
- videotape of students problem solving,
- students’ written work (homework, quizzes, tests, projects, etc.),
- classroom questioning strategies, and
- alternative and wide-ranging assessment techniques.

Another suggestion for further study is for more investigation to be done on the role and characteristics of the PLC facilitator. Gamoran et al. (2000) pointed out that one of the fundamental changes needed for promoting teacher improvement is partnerships between professional learning communities and outside agents such as university-based researchers. Research groups are seen as an invaluable link that can provide the support
necessary to help with the development and sustenance of professional learning communities. The facilitator’s unique position is of paramount importance as the establishment of a PLC attempts to link the collegiate world of research with classroom practice. In a related issue, the results of this study revealed that there was an element of distrust and disrespect that existed on the part of the participating teachers toward collegiate personnel. A researcher coming in from outside the participants’ school culture must be cognizant and take note of these feelings. Lack of mutual respect is a hindrance to establishing workable relationships and can thwart the effort to develop and nurture an effective PLC. Additional research on this topic would be beneficial.

This dissertation study also suggests that more work needs to be done in order to be successful in creating a model of professional development that is self-sustaining over time and continues to make a positive impact on the teachers who participate. To this end, I recommend that the goal for future PLC iterations should be to create a community of teachers that:

- have a clear vision as to what it is that they are collectively striving to build in a PLC,
- believe each session is a worthwhile endeavor,
- assume increasing authority of the PLC with regards to what, when, and how to investigate mathematical concepts,
- are expected to adapt their teaching practices to enhance student understanding and to try out new ideas and techniques in their classrooms,
• celebrate positive results and make them public to students, parents, administrators, and other mathematics educators,

• secure ongoing support from the school administrators,

• expand their network of professional colleagues ranging from the local school to multiple school sites and levels (elementary, junior high, community college, and university), and

• have ongoing support, material, tools, and encouragement from the supporting university.

Finally, it is recommended that the professional learning community structure be joined with a course designed to focus primarily on the mathematical content that teachers need to teach their subjects well. This notion was supported by Jeanne in her post-PLC interview: “I would love to have courses called ‘Teaching Precalculus for Teachers’, ‘Teaching Functions for Teachers’, ‘Teaching Trigonometry for Teachers’, and teach me the content.” The lack of a more marked change in Jeanne’s conceptual knowledge can be partly attributed to the time constraints that the PLC had to work within (one-hour weekly sessions and a total of thirty hours over the year) and the importance placed on each of the six aspects of professional development that a cycle of investigation incorporates. Without a significant increase in the amount of time that PLCs meet, it will be difficult to effectively address this issue. The results from this study suggest that the PLC is a powerful and effective vehicle for building community and linking participants’ learning directly to their students’ thinking and classroom practices.
I also believe that additional benefits may be reaped by connecting the PLC to a course designed to focus more intensely on building the knowledge base required by teachers.

Concluding Remarks

Reflecting upon my twenty years of teaching experience and countless hours of professional development, I am convinced that the model outlined in this dissertation study holds renewed hope for transforming mathematics education. I believe that the PLC structure described in the previous pages incorporates the most important aspects of learning and teaching mathematics and links research with practice in such a way that both can be used to inform the other and result in improved student learning. I also hold that the PLC model for professional development will be instrumental in helping the teaching of mathematics in secondary schools to become more in step with what is called for by the authors of *Principles and Standards* (NCTM, 2000).

To be a successful model of professional development that advances student understanding of mathematics, I am also aware that the PLC model will need to continually evolve and improve. Since the completion of this dissertation study, two other iterations of professional learning communities have taken place. I am pleased to announce that many of the changes suggested in this study have already been employed with encouraging results. For instance, heavier emphasis has been placed upon having participating teachers work collaboratively on challenging mathematics problems in recent iterations and efforts to affect change in their teaching practices have been intensified. Early responses to these changes, as measured by participants’ self-reflections, have been very positive. Additionally, a course offered by the supporting
research institution and designed to build teachers’ knowledge of important concepts of mathematics has been linked to the PLC model and required of all participants. There are early indications that this alteration also appears to be producing positive results. I am confident that these and other changes to the PLC model for professional development will continue to help secondary mathematics students learn mathematics more deeply through the improved instruction that they receive from teachers involved in this project.

Finally, I expect that, through engagement in a PLC where teachers collaborate with their colleagues and are supported by mathematics educators, the participants will deepen their conceptual knowledge, improve in pedagogical content knowledge, and align their teaching practices more closely with what has been called for in the literature. As the PLC model continues to evolve and improve, I envision building supportive communities of professionals in which the teachers participate in the arena of ideas, admit weaknesses, challenge each others’ belief systems, nurture one another, and provide excellent instructional experiences for their students. I am excited to be an integral part of the ongoing efforts to improve mathematics education and look forward to seeing a growing number of significant and transportable results.
REFERENCES


<table>
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<th><strong>Social Factors</strong></th>
<th><strong>Description</strong></th>
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<tr>
<td><strong>Location</strong></td>
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<td>Students’ views of mathematics education</td>
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APPENDIX B

TEN NECESSARY QUALITIES FOR BUILDING COMMUNITY
1. **Safety and Trust** – In order for participants to connect with each other, there must be a sense of safety and trust. This is especially true as participants reveal weaknesses in their teaching or ignorance of teaching processes or literature.

2. **Openness** – In an atmosphere of openness, participants can feel free to share their thoughts and feelings without fear of retribution.

3. **Respect** – In order to coalesce as a learning community, members need to feel as though they are valued and respected as people.

4. **Responsiveness** – Members must respond to each other, and the leader(s) must respond quickly to the other participants.

5. **Collaboration** – The importance of collaboration in consultation and group discussion on individual members’ projects and on achieving learning outcomes hinges on the group’s ability to work with and respond to each other.

6. **Relevance** – Learning outcomes are enhanced by relating the subject matter to the participants’ teaching, courses, scholarship, and life experiences.

7. **Challenge** – Expectations for the quality of outcomes should be high, engendering a sense of progress, scholarship, and accomplishment.

8. **Enjoyment** – Activities must include social opportunities to lighten up, bond, and should take place in invigorating environments.

9. **Esprit de Corps** – Sharing individual and community outcomes with colleagues in the academy should generate pride and loyalty.

10. **Empowerment** – A sense of empowerment is both a crucial element and a desired outcome of participation in a learning community.
APPENDIX C

PROFESSIONAL LEARNING COMMUNITY: 
THEORETICAL FRAMEWORK
### Professional

A professional is someone with expertise in a specialized field who has not only pursued advanced training to enter the field, but who also demonstrates adaptability by remaining current in its evolving knowledge base and is adept at coping with change. A professional has ongoing working relationships with colleagues, participates in shared practice, and displays confidence in his/her ability to meet challenges and solve problems successfully. A professional can communicate effectively, and is expected to present him or herself in a positive manner. (DuFour & Eaker, 1998; Hord, 2004; Bryson, 2005)

### Learning

Ongoing action, perpetual curiosity, examination of data, and a continuous effort to deepen one’s knowledge base and refine instructional practice with the goal of enhancing student understanding (DuFour & Eaker, 1998; Hord, 2004; Cox, 2004; NCTM, 2000)

#### Learning 1 (L1): Conceptual Knowledge

Conceptual knowledge is characterized by having the ability to –

L1.1 recognize the central and peripheral ideas of mathematics and their relative importance (Schifter, 1999; Sowder, 2005; Shulman, 1987),

L1.2 provide reasoned responses and justify thought processes when problem solving (NCTM, 2000),

L1.3 build a concept upon earlier ideas, formulate connections within and across domains, and create further extensions (Ball, 2005; Santos-Trigo, 1998; Ma, 1999; Sowder, 2005),

L1.4 understand the different representations of a concept, the relative strengths and weaknesses of each, and how they are related to one another (NCTM, 2000; CBMS, 2001; Wilson, Shulman, & Richert, 1987; Ma, 1999; Cooney, 1999; Mason & Spence, 1999),

L1.5 articulate contextual meaning (NCTM, 2000; Gravemeijer & Doorman, 1999) and exhibit sense-making regarding a concept (Adamson, 2005; Simon, 2004), and

L1.6 use knowledge flexibly by appropriately applying or adapting what is learned in one setting in another (NCTM, 2000).
Learning 2 (L2): Pedagogical Content Knowledge

Pedagogical content knowledge is characterized by having the ability to –

L2.1 imagine what it looks like for someone else to have an understanding of a concept (Silverman, 2005),

L2.2 articulate/reveal common conceptions students have regarding a particular concept (Shulman, 1987; NCTM, 2000; NRC, 2005),

L2.3 plan a roadmap for how others might acquire understanding of a concept by building upon their existing knowledge base (Silverman, 2005; Ball, 2003; NRC, 2005),

L2.4 use powerful and effective analogies, metaphors, representations, models, verbal descriptions, and examples in facilitating and enhancing the learning experience of others without taking over the process of thinking for them and thus eliminating the challenge (Shulman, 1987; NCTM, 2000),

L2.5 utilize both real-world experiences of students and purely mathematical contexts that are intriguing, challenging, and invite speculation and hard work by those with varied levels of expertise (NCTM, 2000), and

L2.6 accommodate and integrate students’ culture and language into the learning process and equitably provide effective learning experiences for all (NCTM, 2000).
Learning 3 (L3): Beliefs about Learning and Teaching Mathematics

L3.1 Learnability: (Carlson, 1997)
   a. Mathematics is learnable by anyone willing to make the effort rather than by a few isolated people.
   b. Achievement depends more on persistent effort than on the influence of the teacher or textbook.

L3.2 Critical Thinking: (Carlson, 1997)
   For meaningful understanding of mathematics, one needs to –
   a. concentrate more on the systematic use of general thought processes rather than on memorizing isolated facts and algorithms,
   b. examine situations in many ways, and not feel intimidated by committing mistakes rather than follow a single approach from an authoritative source,
   c. look for discrepancies in one’s own knowledge instead of just accumulating new information, and
   d. reconstruct new knowledge in one’s own way instead of memorizing it as given.

L3.3 Personal Relevance: (Carlson, 1997)
   a. Mathematics and related technology are relevant to everyone’s life rather than being of exclusive concern of mathematicians.
   b. Mathematics should be studied more for personal benefit than for just fulfilling curriculum requirements.

L3.4 Instructional Philosophy:
   For effective mathematics instruction to occur, a teacher needs to –
   a. focus primarily on student understanding of principle concepts and ideas instead of on procedures and quick computation (Sowder, 2005; NRC, 2005),
   b. build upon students’ prior knowledge and personal experience rather than presenting material without regard to individual meaning and sensemaking (Dewey, 1933; von Glaserfeld, 1990; Boaler, 1993; Gravemeijer & Doorman, 1999; Adamson, 2005),
   c. purposefully provide tasks that offer diverse and appropriate challenges rather than striving to alleviate the struggle and ambiguity of doing mathematics (Santos-Trigo, 1998; Cooney, 1993),
   d. design instruction that requires active participation by students and encourages curiosity, creativity, investigation, collaboration, and healthy questioning rather than only expecting that students listen intently, take good notes, follow directions and examples, behave themselves, and do the assigned work (Lawson et al., 2000; Ernest, 1991; Cooney, 1993),
   e. require students to communicate their thinking with others in a variety of ways rather than simply providing to the teacher correct written solutions to problems that they individually arrive at (NCTM, 2000; NRC, 2005), and
   f. facilitate students’ self-monitoring by teaching them to critically reflect, estimate, and assess their own understanding rather than looking only to an authoritative source for verification (NRC, 2005).
<table>
<thead>
<tr>
<th>Community</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>An environment that fosters mutual cooperation, emotional support, shared vision, and personal growth in members as they work together to achieve what they cannot accomplish alone</strong> (Cox, 2004; Hord, 2004; DuFour &amp; Eaker, 1998; Franke, Carpenter, Levi, &amp; Fennema, 2001; Wenger, 1998)</td>
</tr>
<tr>
<td><strong>C1</strong> Safety and Trust – In order for participants to connect with each other, there must be a sense of safety and trust. This is especially true as participants reveal weaknesses in their teaching or ignorance of mathematical content or pedagogy.</td>
</tr>
<tr>
<td><strong>C2</strong> Respect – In order to coalesce as a learning community, each member needs to feel as though they are valued and respected as a professional.</td>
</tr>
<tr>
<td><strong>C3</strong> Shared Values, Vision, and Mission – In order to be successful, the community needs a common set of values they work from, a common vision for which they collectively strive to achieve, and a common mission to assume.</td>
</tr>
<tr>
<td><strong>C4</strong> Relevance – Learning community outcomes are enhanced by relating the subject matter to the participants’ scholarship of learning and teaching, and life experiences.</td>
</tr>
<tr>
<td><strong>C5</strong> Collaboration – Learning community outcomes hinge on the group’s ability to consult, listen, respond, and work with each other.</td>
</tr>
<tr>
<td><strong>C6</strong> Shared Leadership/Empowerment – For generativity to occur, participants must claim ownership, responsibility, and leadership for the community’s decisions, actions, and directions.</td>
</tr>
<tr>
<td><strong>C7</strong> Shared Practice – Planning jointly, sharing student work, posing perplexing questions, challenging ideas and beliefs, and observing each others classroom instruction are essential to supporting individual and communal improvement.</td>
</tr>
<tr>
<td><strong>C8</strong> Public – The community’s work must be made public to a broader audience for the purpose of sharing results, gaining feedback, and adding to the professional body of knowledge.</td>
</tr>
<tr>
<td><strong>C9</strong> Enjoyment – In order for the learning community to be desirable, it must take place in an invigorating environment that fosters bonding, lightheartedness, and concern for one another.</td>
</tr>
<tr>
<td><strong>C10</strong> Responsibility – Individual commitment to participate fully is a necessity for a successful learning community.</td>
</tr>
</tbody>
</table>
APPENDIX D

VIEWS ABOUT MATHEMATICS SURVEY (VAMS) TAXONOMY
Epistemological
Structure:
Mathematics is a coherent body of knowledge about relationships and patterns contrived by careful investigation
- rather than a collection of isolated facts and algorithms.

Methodology:
The methods of mathematics are systematic and generic
- rather than idiosyncratic and situation specific.

Mathematical modeling for problem solving involves more
- than selecting formulas for number crunching.

Mathematicians use technology more to enhance their ways of solving problems
- than to allow them to get quick easy solutions.

Validity:
Mathematical knowledge is validated by logical proofs
- rather than by corresponding to the real world.

Mathematical knowledge is tentative and refutable
- rather than absolute and final.

Pedagogical
Learnability:
Mathematics is learnable by anyone willing to make the effort
- rather than by a few isolated people.

Achievement depends more on persistent effort
- than on the influence of teacher or textbook.

Critical Thinking:
For meaningful understanding of mathematics, one needs to:
A. Concentrate more on the systematic use of general thought processes
   - rather than on memorizing isolated facts and algorithms.

B. Examine situations in many ways, and not feel intimidated by committing mistakes
   - rather than follow a single approach from an authoritative source.

C. Look for discrepancies in one’s own knowledge
   - instead of just accumulating new information.
D. Reconstruct new knowledge in one’s own way
   - instead of memorizing it as given.

Personal Relevance:
Mathematics and related technology are relevant to everyone’s life
- rather than being of exclusive concern of mathematicians.

Mathematics should be studied more for personal benefit
- than for just fulfilling curriculum requirements.
APPENDIX E

PARTICIPANT PRE-INTERVIEW PROTOCOL
1. Why did you decide to become a math teacher?

2. Let’s suppose for a moment that you could teach your math class any way you choose. It is totally up to you what you teach and how you teach it. Describe this class in terms of what you would teach and how you would teach it.

3. Describe how the math courses you took in high school and college were run. What kind of impact, if any, has this had on your teaching?

4. Describe how the math methods courses you took in college were run. What kind of impact, if any, has this had on your teaching?

5. Describe your mentoring teacher when you student taught. What kind of impact, if any, has this person had on your teaching?

6. What does it mean to be a successful precalculus teacher?

7. What would be the best way to prepare future high school math teachers?

8. (Administer the NOM and NOME surveys as interview protocols.)

General Comments:
Teachers have differing beliefs about the nature of mathematics. For each of the following pairs of statements (#1 - #9), darken the circle that best shows how closely your own beliefs fall on the continuum. Please darken only one circle for each set.

<table>
<thead>
<tr>
<th>Statement 1</th>
<th>Circle 1</th>
<th>Circle 2</th>
<th>Circle 3</th>
<th>Circle 4</th>
<th>Statement 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. “Mathematics is an objective, absolute, certain, and consistent body of knowledge.”</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“Mathematics is a process of inquiry, and a coming to know.”</td>
</tr>
<tr>
<td>2. “The body of mathematics is based on human intuition and shared meaning.”</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“The body of mathematics is based on deductive logic/reasoning.”</td>
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<tr>
<td>3. “Current mathematical knowledge is not open to future revision and change.”</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“Current mathematical knowledge is open to future revision and change.”</td>
</tr>
<tr>
<td>4. “Mathematics consists of objects and patterns that have no existence outside of the mind.”</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“Mathematics consists of objects and patterns that are inherent in real objects and natural phenomena.”</td>
</tr>
<tr>
<td>5. “As new mathematics arises, its primary value is in furthering yet more mathematics.”</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“As new mathematics arises, its primary value is in describing real objects and modeling natural phenomena.”</td>
</tr>
<tr>
<td>6. “Mathematics is a dynamic and continually expanding field.”</td>
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<td></td>
<td></td>
<td></td>
<td>“Mathematics is an accumulated set of facts, rules, skills, and procedures.”</td>
</tr>
<tr>
<td>7. “Mathematics is created/produced.”</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“Mathematics is discovered.”</td>
</tr>
<tr>
<td>8. “The elegance and beauty of mathematics is expressed when complex problems are solved using symbolic notation, language, and representation.”</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“The elegance and beauty of mathematics is expressed in the patterns of nature.”</td>
</tr>
<tr>
<td>9. “Mathematical claims are true when a formal proof is given and accepted by the community of mathematical experts.”</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>“Mathematical claims are true when sufficient supporting evidence has been found.”</td>
</tr>
</tbody>
</table>

Key - Nature of Mathematics (NOM) Survey for Teachers

1. 1, 2, 3, 4, 5  
2. 5, 4, 3, 2, 1  
3. 1, 2, 3, 4, 5  
4. 1, 2, 3, 4, 5  
5. 1, 2, 3, 4, 5  
6. 5, 4, 3, 2, 1  
7. 5, 4, 3, 2, 1  
8. 1, 2, 3, 4, 5  
9. 1, 2, 3, 4, 5

If the average score is 4 – 5, then the person has a more applied (fallibilist) view of the nature of mathematics.

If the average score is 3, then the person has an integrated view of the nature of mathematics.

If the average score is 1 – 2, then the person has a more pure (absolutist) view of the nature of mathematics.
APPENDIX G

NATURE OF MATHEMATICS EDUCATION SURVEY FOR TEACHERS
Teachers have differing beliefs about the nature of how mathematics should be taught. For each of the following pairs of statements (#10 - #14), darken the circle that best shows how closely your own beliefs fall on the continuum. Please darken only one circle for each set.

<table>
<thead>
<tr>
<th></th>
<th>Statement 1</th>
<th>Statement 2</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.</td>
<td>“I see my role mainly as a facilitator by providing opportunities and resources for students to discover or construct concepts for themselves.”</td>
<td>“I see my role mainly as a transmitter of knowledge by telling students what they need to know and do.”</td>
<td>5, 4, 3, 2, 1</td>
</tr>
<tr>
<td>11.</td>
<td>“The most important part of instruction is the content of the curriculum. The content is what students need to know and do.”</td>
<td>“The most important part of instruction is that it encourages ‘sense-making’ or thinking among students. Covering the content is secondary.”</td>
<td>1, 2, 3, 4, 5</td>
</tr>
<tr>
<td>12.</td>
<td>“My job as a mathematics Instructor is teaching students to calculate and answer problems without using technology.”</td>
<td>“Technology, including graphing calculators and computers, is an integral part and an invaluable tool in mathematics instruction in today’s math classroom.”</td>
<td>1, 2, 3, 4, 5</td>
</tr>
<tr>
<td>13.</td>
<td>“Students working together with others when solving math problems learn the mathematics more deeply than when they do so on their own.”</td>
<td>“Students working math problems with other students in groups do not achieve the depth of learning as when working on their own.”</td>
<td>5, 4, 3, 2, 1</td>
</tr>
<tr>
<td>14.</td>
<td>“When preparing mathematics lessons, I generally follow the textbook and/or the prescribed curriculum.”</td>
<td>“When preparing mathematics lessons, I generally modify or supplement the textbook with additional problems and/or activities.”</td>
<td>1, 2, 3, 4, 5</td>
</tr>
</tbody>
</table>

**Key - Nature of Mathematics Education (NOME) Survey for Teachers**

10. 5, 4, 3, 2, 1  11. 1, 2, 3, 4, 5  12. 1, 2, 3, 4, 5  13. 5, 4, 3, 2, 1  14. 1, 2, 3, 4, 5

If the average score is 4 – 5, then the person has a more **social constructivist** view of the nature of mathematics education.

If the average score is 3, then the person has an **integrated** view of the nature of mathematics education.

If the average score is 1 – 2, then the person has a more **authoritarian** view of the nature of mathematics education.
APPENDIX H

PARTICIPANT POST-INTERVIEW PROTOCOL
**Interview #1 Questions**

**Baseline Data:**

1. I’d like to start out by learning a little more about what brings you to teaching. What made you decide to become a math teacher?

(Probe their intellectual interests and perspective. Try to discover what the person enjoys about mathematics.)

2. Describe your “formal” training in mathematics. How well do you think it prepared you for teaching math to high school students?

**Beliefs (Conceptual Knowledge)**

3. Describe to me someone you perceive to be good at mathematics.

**Beliefs (Pedagogical Content Knowledge)**

4. It has been said that teachers need to "know the math" and others have said that they need to "know how to teach the math." Do you see these 2 as different? Explain.

5. Think of someone who you consider to be an "excellent" math teacher. Describe that person to me, what makes them an "excellent" math teacher?

6. Think of someone who you consider to be an "ineffective" math teacher? Describe that person to me, what makes them an "ineffective" math teacher?

7. Do you have an image of what it would look like for you to be an "ideal" math teacher? What are the primary obstacles that prevent you from reaching that "ideal" you have for yourself? What would help you to reach toward that "ideal?"

   a. How could a PLC help you achieve the “ideal”?
Professional Development Questions:

8. What types of professional development have you participated in the past? Describe how these experiences have or have not impacted and translated into your teaching in the mathematics classroom.

9. Describe other opportunities you have had to have discussions with the other teachers in your school, department, or district in order to revise and improve your personal teaching practices.

10. How has the PLC compared with the other modes of professional development you have experienced? How has the PLC been beneficial to you as a secondary teacher? Be as specific as you can.

11. As a teacher of secondary mathematics, what are your thoughts regarding mathematics education research literature or materials?

   (Probe: If not a positive response: What would make research literature/materials more appealing to you as a teacher of secondary mathematics? If a positive response: What has made it appealing?)

12. It was mentioned in the PLC that “math education research just gives the problems we know we have names, what I want are answers.” Do you agree with this statement? Explain.

13. What would be the most beneficial thing(s) that universities/professional development organizations could do to help you grow as a high school math teacher?

   (Probe: If not mentioned: If the research community provided you with some video clips and/or written transcripts of how typical students struggle with some of the important topics of precalculus and then provide some possible teacher interventions, can you see any benefits to you as a classroom teacher? How?

   (Probe: If not mentioned: How about ongoing professional learning communities similar to the one you just participated in?)

14. If this PLC were to continue next fall here at HHS, what suggested changes and improvements would you have in terms of engaging each other in more beneficial discussions of mathematics and in teaching practices?

15. Is there anything else that you would like to share with me regarding your experiences this year in the PLC?
Interview #2 Questions

Beliefs:

1. What factors affect how you make decisions regarding the pacing of your precalculus (mathematics) courses? Do you feel pressure to “cover the material”? If so, why? If not, why not? How do you judge whether you have “covered the material”?

2. Imagine that you have just received notice that the XXXX School District is transferring you from XXXX High School to another high school. You have no say in the matter and must go. What is your reaction? What will you miss most about XHS? What will you not be looking forward to? What would you be excited about?

3. The group developed four norms to live by. They were: do your part with a little bit of humor, don’t waste time, strive for equal participation in conversations, and check for understanding of new ideas.

   What are your thoughts regarding these norms that the group functioned with? Did they work? Were they sufficient? What would you add or change next time?

4. In your opinion, was the PLC worthwhile? If so, in what ways? In what ways could it be improved?

5. In one professional development model, colleagues study together a concept and how to teach it for an extended period of time with multiple revisions of the lesson taking place after seeing how students perform.

   Do you believe that there is any merit to studying extensively one math concept and how to teach it effectively? Explain.

6. It seems that getting students to solve math problems quickly is valued by some teachers. What are your thoughts about getting your students to quickly solve mathematics problems? Is this important to you? Why or why not?
Conceptual:

1. A concept map is a drawing or diagram that shows connections between aspects of a concept or major concepts in a course (see sample). Construct a concept map for a course in precalculus. (Note: make books available, use numbered post-its)

2. (Solve selected PCA content problems using think aloud protocol)

3. Do you believe the PLC provided a mechanism for you to:
   a. deepen your precalculus conceptual knowledge? If so, what aspects of the PLC were most effective in this respect? If not, why not?

Pedagogical Content Knowledge:

1. Here’s a section from an algebra textbook on function composition. Take a few minutes to look it over, and then we’ll talk.
   a. What are your initial reactions to this textbook section?
   b. Are there things you think are quite good in here?
   c. Are there things you think are weaknesses or flaws? Why?

2. Here’s a section from another text series on function composition. Compare the two textbook excerpts. Does anything seem different?
   a. Which do you prefer? Why?

3. In your opinion, what do you believe it is that students need to understand or be able to do before they start learning function composition?
   a. Why is __________ important for learning this?

4. How would you approach function composition if you were teaching? Don’t feel that you have to stick to any of this textbook material if you have another way you’d want to work with your class, but you can use it if you choose.
   (Probes to elicit concrete and specific descriptions):
   a. Can you give me an example?
   b. If I were in your classroom, what would I see?
c. “I’d try to connect it to something the students can relate to”: Could you give me an example?

d. “I’d do a few examples on the board”: For example? What would I hear you saying or see you doing?

e. Why would you do it this way?

f. How did you come up with this idea/approach?

g. (Give out the PLC composition activity) Does the PLC activity we created this year on composition add anything to the way you have taught composition before? Do you plan to use it in the future when you teach composition? Explain how you plan to use it or why not if you don’t think you will.

5. Here is a PCA question on function composition. Let’s assume that a student, Lynn, got the problem incorrect by choosing letter D. Take some time to look it over and then let’s talk about what you make of Lynn’s work.

   a. What do you think is going on here with Lynn? What is your hunch about why she got this problem wrong?

   b. Okay, imagine that Lynn is a student of yours. How would you respond to this? What would you do next? Why?

      If the person says she would see if others in the class were having the problem and maybe “go over it” with the whole class say:

   c. How would you decide whether or not to do something with the whole class?

6. (If time:) What aspect(s) of the PLC were most helpful to you in terms of improving how you teach precalculus mathematics to your students? Explain.
APPENDIX I

REFORMED TEACHING OBSERVATION PROTOCOL (RTOP)
I. BACKGROUND INFORMATION

Name of teacher ___________________________  Announced Observation? ___________________________

(yes, no, or explain)

Location of class ____________________________ (district, school, room)

Years of Teaching ___________________________  Teaching Certification ____________________________

(K-8 or 7-12)

Subject observed ___________________________  Grade level ____________________________

Observer ___________________________  Date of observation ____________________________

Start time ___________________________  End time ____________________________

II. CONTEXTUAL BACKGROUND AND ACTIVITIES

In the space provided below please give a brief description of the lesson observed, the classroom setting in which the lesson took place (space, seating arrangements, etc.), and any relevant details about the students (number, gender, ethnicity) and teacher that you think are important. Use diagrams if they seem appropriate.
### III. LESSON DESIGN AND IMPLEMENTATION

<table>
<thead>
<tr>
<th></th>
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<th>Never Occurred</th>
<th>Very Descriptive</th>
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</thead>
<tbody>
<tr>
<td>1)</td>
<td>The instructional strategies and activities respected students’ prior knowledge and the preconceptions inherent therein.</td>
<td>0 1 2 3 4</td>
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<tr>
<td>2)</td>
<td>The lesson was designed to engage students as members of a learning community.</td>
<td>0 1 2 3 4</td>
<td></td>
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<tr>
<td>3)</td>
<td>In this lesson, student exploration preceded formal presentation.</td>
<td>0 1 2 3 4</td>
<td></td>
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<tr>
<td>4)</td>
<td>This lesson encouraged students to seek and value alternative modes of investigation or of problem solving.</td>
<td>0 1 2 3 4</td>
<td></td>
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<tr>
<td>5)</td>
<td>The focus and direction of the lesson was often determined by ideas originating with students.</td>
<td>0 1 2 3 4</td>
<td></td>
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</table>

### IV. CONTENT

**Propositional knowledge**

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<tr>
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<th>Never Occurred</th>
<th>Very Descriptive</th>
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<tbody>
<tr>
<td>6)</td>
<td>The lesson involved fundamental concepts of the subject.</td>
<td>0 1 2 3 4</td>
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<tr>
<td>7)</td>
<td>The lesson promoted strongly coherent conceptual understanding.</td>
<td>0 1 2 3 4</td>
<td></td>
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<tr>
<td>8)</td>
<td>The teacher had a solid grasp of the subject matter content inherent in the lesson.</td>
<td>0 1 2 3 4</td>
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<tr>
<td>9)</td>
<td>Elements of abstraction (i.e., symbolic representations, theory building) were encouraged when it was important to do so.</td>
<td>0 1 2 3 4</td>
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<tr>
<td>10)</td>
<td>Connections with other content disciplines and/or real world phenomena were explored and valued.</td>
<td>0 1 2 3 4</td>
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**Procedural Knowledge**

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<tbody>
<tr>
<td>11)</td>
<td>Students used a variety of means (models, drawings, graphs, concrete materials, manipulatives, etc.) to represent phenomena.</td>
<td>0 1 2 3 4</td>
<td></td>
</tr>
<tr>
<td>12)</td>
<td>Students made predictions, estimations and/or hypotheses and devised means for testing them.</td>
<td>0 1 2 3 4</td>
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<tr>
<td>13)</td>
<td>Students were actively engaged in thought-provoking activity that often involved the critical assessment of procedures.</td>
<td>0 1 2 3 4</td>
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<tr>
<td>14)</td>
<td>Students were reflective about their learning.</td>
<td>0 1 2 3 4</td>
<td></td>
</tr>
<tr>
<td>15)</td>
<td>Intellectual rigor, constructive criticism, and the challenging of ideas were valued.</td>
<td>0 1 2 3 4</td>
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## V. CLASSROOM CULTURE

### Communicative Interactions

<table>
<thead>
<tr>
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<th>Never Occurred</th>
<th>Very Descriptive</th>
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<tbody>
<tr>
<td>16)</td>
<td>Students were involved in the communication of their ideas to others using a variety of means and media.</td>
<td>0 1 2 3 4</td>
</tr>
<tr>
<td>17)</td>
<td>The teacher’s questions triggered divergent modes of thinking.</td>
<td>0 1 2 3 4</td>
</tr>
<tr>
<td>18)</td>
<td>There was a high proportion of student talk and a significant amount of it occurred between and among students.</td>
<td>0 1 2 3 4</td>
</tr>
<tr>
<td>19)</td>
<td>Student questions and comments often determined the focus and direction of classroom discourse.</td>
<td>0 1 2 3 4</td>
</tr>
<tr>
<td>20)</td>
<td>There was a climate of respect for what others had to say.</td>
<td>0 1 2 3 4</td>
</tr>
</tbody>
</table>

### Student/Teacher Relationships

<table>
<thead>
<tr>
<th></th>
<th>Never Occurred</th>
<th>Very Descriptive</th>
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</thead>
<tbody>
<tr>
<td>21)</td>
<td>Active participation of students was encouraged and valued.</td>
<td>0 1 2 3 4</td>
</tr>
<tr>
<td>22)</td>
<td>Students were encouraged to generate conjectures, alternative solution strategies, and ways of interpreting evidence.</td>
<td>0 1 2 3 4</td>
</tr>
<tr>
<td>23)</td>
<td>In general the teacher was patient with students.</td>
<td>0 1 2 3 4</td>
</tr>
<tr>
<td>24)</td>
<td>The teacher acted as a resource person, working to support and enhance student investigations.</td>
<td>0 1 2 3 4</td>
</tr>
<tr>
<td>25)</td>
<td>The metaphor “teacher as listener” was very characteristic of this classroom.</td>
<td>0 1 2 3 4</td>
</tr>
</tbody>
</table>

Additional comments you may wish to make about this lesson.
APPENDIX J

SAMPLE CODING FORM
<table>
<thead>
<tr>
<th>Date/Session:</th>
<th>Person: Quote</th>
<th>Facilitator Comments:</th>
<th>Code(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/20/2004 PLC Meeting 16</td>
<td>Jeanne: I want to drill this (the trig function values of special angles) in to them so they can recall them. I’ll put the random number generator on and call on a student, give them 3 seconds…count to three (while she banged on the desk as she spoke to the PLC) you’re done, and then go on to someone else. Everyone gets a turn when it comes around. They have to do it quick and you know how hard that is…$\sin \frac{\pi}{6}$…boom, you’ve got to give it to me right then and there. Everybody’s watching…</td>
<td>F/R: I don’t know if she actually did that in class or not but WOW! O/R: I was thinking that too! F/R: When you are asking a question that does take a lot of thought, putting the pressure of time constraints is very detrimental. I think we need to try and address why do they encourage speed so much?</td>
<td>L3.2a L3.2d L3.4a L3.4b</td>
</tr>
<tr>
<td>5/10/2004 Post Baseline Interview</td>
<td>I'd like to start out by learning a little more about what brings you to teaching. What made you decide to become a math teacher? I like math, I enjoyed it, and I thought I could teach it. A lot of it is personality because I like being in charge. I enjoy the kids and I enjoy the topic. I think I can do things. I think I can effect changes in kids.</td>
<td>F/R: She likes to be “in charge”. This definitely effects her teaching style.</td>
<td>BT1a</td>
</tr>
</tbody>
</table>
APPENDIX K

PLC SAMPLE AGENDA
Professional Learning Community
______ High School

Agenda #17

Time: 2:40 p.m. – 3:40 p.m.
Room: 1270
Tuesday, January 27th

Norms:
- Do your part with a little bit of humor.
- Don’t waste time.
- Strive for equal participation in conversations.
- Check for understanding of new ideas.

I. Business- Stipend

II. Cycles of Investigation:
1. Concept Investigation
2. Practitioner and Research Knowledge: The objective of this meeting is to look at ways to help students understand or work with radians and what research says about learning and teaching the concept.
3. Activity Development & Implementation
4. Assessment and Refinement of Activity
5. Reflect on the implementation (Done when all have tried the activity)

III. Homework:
1. What connections did you make between the article and your students’ struggles?
2. How did your students do with the fraction problems?

IV. Research:
1. Why do students struggle with fractions (continued)
2. What are radians?

V. What do we want students to be able to do/know about radians?
1. Convert to degrees
2. Identify place on unit circle
3. Recall trigonometric values?

VI. Narrow topic of focus
1. What are the concepts/ideas we’ve talked about
2. Do we want to do 2 or 3 small activities?
3. Choose topic(s) of focus

VII. Assignment:
1. Write a ½ page reflection on our past 2 meetings in which we have focused on the concept of radians. Have you learned anything new? For yourself or about your students? Do you have any new questions you are wondering about? Are you thinking any differently about how you are going to teach your trig unit?
Note: If you haven’t finished the radian packet given out yesterday or compiled (in writing) your students’ fraction work, please do that before reflecting.
2. List the top 2 things you want students to “get” about radians. Be specific.
APPENDIX L

PARTICIPANT CONSENT FORM
From: Trey Cox
trey.cox@cgcmail.maricopa.edu

February 2nd, 2004

To the participant, _____________________________________:
(Participant name)

My name is Trey Cox. I am a doctoral student of Curriculum & Instruction in Math Education at Arizona State University. You are invited to participate in a research study that focuses on determining what factors are influential in assisting secondary teachers to create a mathematical learning community in their classroom.

The results of this study may be published, but your names will not be used. The collected data will be used for a Ph.D. dissertation study entitled: Developing and Investigating a Secondary Precalculus Professional Learning Community. The major advisor for this study is Marilyn Carlson in the mathematics department at Arizona State University.

The commitment in time is the following:
• Initial interviews of individual teachers. Approximately 90 minutes.
• Each teacher involved in the study will be observed while teaching two or three of their regular classes. Approximately 3 hours for one semester.
• Teachers will be asked to do weekly readings, create an activity or assessment, and/or solve a math problem(s). 1 hour per week for one semester.
• On a voluntary basis, the teachers may at any time share thoughts they may have with regard to their teaching, mathematical understanding, or curricular issues with the researcher.
• Participate in weekly learning and planning sessions (LAPS) meetings to discuss mathematical content, plan effective methods of instruction, and analyze lessons. 1 hour per week for one semester.
• After all data collection, the researcher will conduct final interviews with each teacher. Approximately 2 hours.

Other math education researchers may view the collected data to help with analysis. In this case, participants will be informed of the situation and of the viewers. The participant may request that their data not be shown in order to protect their privacy. Once all of the research is complete, the interview transcripts and audiotapes will be destroyed. Participants’ employer/principal will not get information except for regular access to any published paper.

The possible benefits of your participation will be to add to the body of mathematics teaching knowledge and improved understanding of how school districts and teacher preparation programs can support teachers. Your participation is voluntary. For any reason, you may opt out of the study.

If you have any questions regarding this research and your part in it, please contact me at 480-988-8125, trey.cox@cgcmail.maricopa.edu or Dr. Marilyn Carlson at (480) 965-6168.

Sincerely,
Trey Cox
Researcher

Please Sign
I give consent to participate in the above study. I understand that the taped information will be used for research purposes only and that they will be erased upon completion of the dissertation.

__________________________________________  ________________________  __________________________________
Date    Print Name             Signature
Frank Everett Cox III (Trey) was born in Shelbyville, Illinois, on March 17, 1964. He received his elementary education at Vine Street School and his secondary education at Shelbyville High School. In 1982, Trey entered Concordia University Wisconsin, Mequon, Wisconsin, majoring in Secondary Education (Mathematics). Upon graduation in 1986, he began his teaching career at Luther High School North in Chicago, Illinois, and also taught high school mathematics at Chester High School in Chester, Illinois, and Chandler High School in Chandler, Arizona until 2000. Currently, Trey is a residential faculty member at Chandler-Gilbert Community College in Chandler, Arizona. In January 2000, Trey entered the Graduate College at Arizona State University to pursue a doctorate in Curriculum and Instruction. From 2001-2005 while teaching full-time at Chandler-Gilbert, he served as a research assistant for the Center for Research on Education in Science, Mathematics, Engineering, and Technology (CRESMET) while pursuing his doctorate. In his role as researcher, Trey was supported in his efforts by two grants from the National Science Foundation.