MAGIC SQUARES

Let a **Magic Square** be a square matrix that has the property that the row sums, column sums and the two diagonal sums are all equal to the same number. *(Usually, the definition of a magic square also requires that the entries are all distinct integers.)*

1. Explain why the set of magic squares of a given size is a vector space.

2. Assign variable names $a_1 \cdots a_9$ to the entries of a $3 \times 3$ magic square. Call the (unknown) constant sum $a_{10}$. Now write all the dependency equations between these variables in the form $Ax = 0$ and enter $A$ into MATLAB. $A$ should be an $8 \times 10$ matrix.

3. Transform $A$ into reduced row echelon form. Use that to find the null space of $A$ manually (to display numbers in rational format, use `format rat`). Write the null space explicitly as the span of three basis vectors. Now execute

   ```matlab
   >> N = null(A,'r')
   ```

   to let MATLAB compute the null space $N$. Why is the null space of $A$ interesting?

4. Compute a linear combination of your choice of the basis vectors of the null space of $A$ (without retyping the vectors! Use only $N$ and matrix-vector multiplication). Then rearrange the first 9 elements of that vector into a new matrix. Confirm that that matrix is a magic square. Repeat this computation several times, with different linear combinations. What condition will ensure that all entries are integers? Can you find coefficients that will produce only positive entries?

5. Describe, in practical terms, why it is useful to know that the solution space for a problem is a vector space, and to have a basis for it.