Four billionaires – Burns, Gates, Scrooge and Jones – play the following game:

In each round, Burns gives 10% of his money to Gates, 21% to Scrooge, and 19% to Jones. Gates gives 25% of his money to Burns, 43% to Scrooge, and 12% to Jones. Scrooge gives 28% of his money to Burns, 18% to Gates and 32% to Jones. Jones gives 41% of his money to Burns, 17% to Gates and 19% to Scrooge.

All of this happens simultaneously, so all the percentages are based on what they have at the beginning of each round.

A. Suppose \( x = (x_1, x_2, x_3, x_4)^T \) is what Burns, Gates, Scrooge and Jones have at the beginning of a round, and \( y = (y_1, y_2, y_3, y_4)^T \) is what they have after they are finished exchanging money. The matrix \( A \) that describes this transition, i.e. \( Ax = y \)

\[
A = \begin{pmatrix}
0.5 & 0.25 & 0.28 & 0.41 \\
0.1 & 0.2 & 0.18 & 0.17 \\
0.21 & 0.43 & 0.22 & 0.19 \\
0.19 & 0.12 & 0.32 & 0.23 \\
\end{pmatrix}
\]

B. Using the MATLAB command \texttt{eig} , find both \( D \) (the diagonalized version of \( A \)) and the diagonalizing matrix \( U \):

\[
>> [U, D] = \text{eig}(A)
\]

C. Extract the first column of \( U \) into a vector \( x_1 \) and confirm that \( x_1 \) is an eigenvector by computing \( Ax_1 \).

Why would one call \( x_1 \) and its multiples the equilibrium states of the game?

D. Consider the second, third and fourth columns of \( U \). Do they describe valid states of the game?

E. Recall that if \( x \) is an initial wealth distribution, \( A^n x \) is the wealth distribution after \( n \) rounds of the game. We saw in the first part of this project that no matter what \( x \) is, as \( n \) increases to infinity, \( A^n x \) will settle into a steady state, which is always a multiple of \( x_1 \). Use your knowledge of eigenvalues and eigenvectors to explain this phenomenon.

F. If the sum of the initial wealth was $100,000, what is the steady state wealth distribution? Generalize to an arbitrary initial sum \( W \).

Hint: remember the \texttt{sum} command.