MAT 170 Project: Finding the sine and cosine values of 36°.

You learned in MAT 170 that there are exact algebraic expressions for the sine and cosine values of 0, 30°, 45°, 60° and 90°. In this project, you will find exact algebraic expressions for sin(36°) and cos(36°). Consider the following isosceles triangle:

1. A small isosceles triangle ATC has been created inside the large isosceles triangle ABC, as indicated by the dotted line. Use this assumption, and the fact that angles in a triangle add up to 180°, to show that this small triangle has the same angles as ABC, which means that ABC and ATC are similar.

2. Use the similarity of these triangles to justify the following equation: \( \frac{x}{1} = \frac{1-x}{x} \).

3. Solve the equation. There are two solutions, but only one is valid. Which?

4. Read [http://en.wikipedia.org/wiki/Golden_ratio](http://en.wikipedia.org/wiki/Golden_ratio) and explain how the golden ratio \( \varphi \) is related to the equation in 2. Confirm explicitly that \( x = 1/ \varphi \).

5. With the value you found for \( x \), use triangle ABT to determine \( \cos(36°) \). Rationalize the denominator.

6. Now use the pythagorean theorem to calculate \( \sin(36°) \), and \( \tan(36°) \).

7. Explain how, in principle, you could find exact formulas for \( \sin \), \( \cos \) and \( \tan \) of every angle that is an integer multiple of 3° using the half-angle formula, the difference formulas and values for certain angles that you already know.

8. BONUS Problem: Find explicit algebraic expressions for \( \sin \) and \( \cos \) of 18°, 15°, 6° and 3°.