**PROOF BY INDUCTION**

Homework problems are due in writing on Monday, June 23rd.

**IN-CLASS EXERCISES**

Prove all of the following statements using induction.

1. The sum of the first \( n \) positive odd integers is \( n^2 \).

2. \( 1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n + 1)(2n + 1) \) for all natural numbers \( n \).

3. For all \( n \in \mathbb{N} \), the number \( 6^n + 4 \) is divisible by 5.

4. \( n^2 > 2n + 1 \) for all natural numbers \( n \geq 3 \).

5. \( 2^n > n^2 \) for all natural numbers \( n \geq 5 \).

**HOMEWORK PROBLEMS**

6. Find a natural number \( k \) such that for all natural numbers \( n \geq k \), the inequality \( 2^n > n^3 \) holds. Prove your answer.

7. Show that \( 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} \leq 2\sqrt{n} \) for all natural numbers \( n \).