EQUIVALENCE RELATIONS

Homework problems are due in writing on Wednesday, June 18th.

IN-CLASS EXERCISES

1. Let \( X = \{1, 2, 3, 4\} \). Are the following relations equivalence relations on \( X \)? If yes, what are the equivalence classes? If no, which conditions are violated? Justify your answers.
   a. \( R = \{(1, 2), (2, 1)\} \)
   b. \( R = \{(1, 1), (2, 2), (1, 2), (2, 1)\} \)
   c. \( R = \{(1, 1), (2, 2), (3, 3), (4, 4), (2, 3), (3, 4)\} \)
   d. \( R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 4), (4, 1), (2, 4), (4, 2)\} \)

2. Are the following equivalence relations on the real numbers?
   a. \( x \sim y \) iff \( \exists n \in \mathbb{Z} \) such that \( x, y \in [n, n + 1) \). If yes, describe the equivalence classes. If no, say why.
   b. \( x \sim y \) iff \( \exists n \in \mathbb{Z} \) such that \( x, y \in [n, n + 1] \). If yes, describe the equivalence classes. If no, say why.
   c. \( x \sim y \) iff \( \exists n \in \mathbb{Z} \) such that \( x, y \in (n, n + 1) \). If yes, describe the equivalence classes. If no, say why.

3. Is the following relation an equivalence relation on the points of the plane \( \mathbb{R}^2 \)? Two points are equivalent if and only if their distance is less than 1.

HOMEWORK PROBLEMS

4. Prove that the following is an equivalence relation: two natural numbers \( a, b \) are equivalent if and only if there are natural numbers \( p, q \) such that \( a^p = b^q \).

5. Prove that the following is an equivalence relation: two real numbers are equivalent if and only if their difference is rational. Give an example of an equivalence class. Are there countably or uncountably many equivalence classes?