Mat 243 Exam 1 Review

OBJECTIVES  (Review problems: on next page)

1.1 Distinguish between propositions and non-propositions.
   Know the truth tables (i.e., the definitions) of the logical operators \( \neg, \land, \lor, \oplus, \rightarrow \) and \( \leftrightarrow \).
   Write truth tables for compound propositions.
   Negate mathematical inequalities.
   Translate variously phrased conditional statements from English into formal statements and vice versa,
   (including "necessary", "sufficient", "only if" and "unless" type statements).
   Recognize differently worded English language conditional statements as logically equivalent.
   Understand and produce variations of conditional statements (inverses, converses, contrapositives) and
   know which of these four statements equivalent or non-equivalent.
   Know the order of precedence for logical operators (hence understand the compound expressions written
   with the minimal set of pairs of parentheses)

1.3 Know the definition of tautology, contradiction, contingency, logical equivalence (state and use).
   Use algebraic laws of logic to simplify compound propositions, identify them as tautologies, contradictions or contingencies, and to prove that two Boolean expressions are logically equivalent.

1.4 & 1.5 Translate English sentences involving "for all.." or "for every.." into formal statements by defining
   appropriate predicates and selecting appropriate quantifiers (\( \exists \) and \( \forall \)) and logical connectives.
   Translate formal statements involving quantifiers and predicates into (natural!) English sentences.
   Express domain restricted universally or existentially quantified statements as the correct unrestricted
   Quantified statement.
   Identify variables as bound or free.
   Identify the scope of a quantifier.
   Negate quantified statements.
   Recognize statements that involve universal and existential quantification and conjunction and
disjunction as equivalent or nonequivalent
   Determine the truth of mathematical statements involving single quantifier or nested quantifiers.

1.6 Break down arguments into their elementary argument forms.
   Understand the definition of valid argument and identify argument forms as valid or invalid.
   Identify valid argument forms (aka rules of inference) by name.
   Prove the validity of an argument using the rules of inference by providing a formal proof.

1.7 Define the words "theorem", "proposition", "lemma", "corollary", axiom".
   Define the concepts of odd and even, rational and irrational numbers.
   Write direct and indirect (by contradiction or contraposition) proofs of simple theorems involving "for all" and
   "there exists" type statements for odd and even numbers and rational and irrational numbers.
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1. Fill in the blank in the statements below:
   (a) Two propositions are logically equivalent if and only if _____________________________

   (b) A tautology is a _____________________________

   (c) A contradiction is a _____________________________

   (d) The negation of "if p then q" is _____________________________

   (e) "r is a necessary condition for s" means: if _____ then ______

   (f) "r is a sufficient condition for s" means: if _____ then ______

   (g) "A unless B" means: if _____ then ______

2. Given the conditional: “I go to the beach and I go out dancing in the evening, if I am done with my homework,”

   (a) state the contrapositive of this statement:

   (b) state the converse of this statement:

   (c) state the inverse of this statement:

   (d) state the negation of this statement:

3. Write the following statements in symbolic form. Identify the propositional functions (i.e., the predicates) (if needed), and the universe of discourse you are using:

   (a) Anne likes Dave but Dave does not like anyone.

   (b) For every real number there exists a larger real number.
4. Write the following statements in “If ___ then ___” form:
   (a) It is necessary that \( n \) is even for \( n^2 \) to be even.

   (b) I get an A in my English class only if I get an A on the final.

5. Express the negation of the following statements without using the symbol for the negation operator.
   (a) \( \forall x \exists y (y > 0 \rightarrow (-2 \leq x < 6)) \)

   (b) \( \exists x \forall y (y > 0 \rightarrow \exists z \left( \frac{\sin y}{1+z^2} < x \right) \)

6. For each of the following statements, choose the symbolic form that represents it:
   (a) Every ASU student who takes discrete mathematics will learn valid arguments.
      Domain of discourse: all ASU students. \( D(x) \): \( x \) takes discrete math course. \( A(x) \): \( x \) learns valid arguments.
      - \( \forall x (D(x) \land A(x)) \)  True or False or not a statement. Explain.
      - \( \forall x D(x) \rightarrow A(x) \)  True or False or not a statement. Explain.
      - \( \forall x (D(x) \rightarrow A(x)) \)  True or False or not a statement. Explain.

   (b) There is an ASU student who takes discrete mathematics and knows valid arguments.
      Domain of discourse: all ASU students. \( D(x) \): \( x \) takes discrete math course. \( A(x) \): \( x \) knows valid arguments.
      - \( \exists x D(x) \land A(x) \)  True or False or not a statement. Explain.
      - \( \exists x (D(x) \rightarrow A(x)) \)  True or False or not a statement. Explain.
      - \( \exists x (D(x) \land A(x)) \)  True or False or not a statement. Explain.

   (c) Pat loves exactly one person.
      Universe of discourse: all the people. \( L(x,y) \): \( x \) loves \( y \)
      - \( \exists y L(Pat, y) \land \forall z(L(Pat, z) \rightarrow z=y) \)  True or False or not a statement. Explain.
      - \( \exists y \forall z (L(Pat, y) \land L(Pat, z) \land z=y) \)  True or False or not a statement. Explain.
      - \( \exists y (L(Pat, y) \land \forall z(L(Pat, z) \rightarrow z=y)) \)  True or False or not a statement. Explain.
      - \( \exists y \forall z(L(Pat, y) \land (L(Pat, z) \rightarrow z=y)) \)  True or False or not a statement. Explain.
7. True or False:  
\[ \exists x \in \mathbb{Z} \ (\sqrt{3} = x) \]
\[ \forall x \in \mathbb{R} \ (x = \sqrt{x}) \]
\[ \forall x \\exists y \ (x + y^2 = 10) \quad \text{universe} = \mathbb{R} \]
\[ \forall x \\exists y \ (x + y^3 = 10) \quad \text{universe} = \mathbb{R} \]
\[ \exists x \\forall y \ (x > 0 \rightarrow x + y = 20) \quad \text{universe} = \mathbb{R} \]

8. Fill in the blank in the statements below:

(a) An argument is valid if and only if __________________________________________________

(b) The argument with premises p and p → q and conclusion q is called ______________________

(c) If premises q and p → q are given and someone concludes p is called __________________________

(d) A corollary is ____________________________________________________________________

(e) When the conditional statement p → q needs to be proved, state what needs to be assumed and what needs to be shown to prove the statement:

(i) using direct proof:

(ii) using proof by contraposition:

(iii) using proof by contradiction:

9. Use the rules of inference to prove \( p \land s \) given the following premises. Give a formal proof, i.e, write your solution as a numbered sequence of statements; identify each statement as either a premise, or a conclusion that follows according to a rule of inference from previous statements; in the latter case, state the rule of inference and refer by number to the previous statements that the rule of inference used.

(1) \( \neg r \)
(2) \( s \)
(3) \( q \lor r \)
(4) \( q \rightarrow p \)
10. Prove or disprove the following statement.
   There exists a real number \( x \) such that \( x^2 > x + 9 \).

11. Let \( m \) be an odd integer and \( n \) be an even integer. Prove that \( m^2 - 3n \), is odd.

12. Use proof by contraposition to show that if \( mn \) is even, then \( m \) is even or \( n \) is even.

13. Use proof by contradiction to show that \( \sqrt{3} \) is irrational. (Hint: You may use the (known) fact that "If \( k^2 \) is a multiple of 3, then \( k \) is a multiple of 3."

14. Using the Laws of Logical Equivalence, show that each of the following pairs are logically equivalent:
   (a) \( p \rightarrow q \land r \) and \( (p \rightarrow q) \land (p \rightarrow r) \)
   (b) \( s \land p \lor \neg s \land q \) and \( (s \rightarrow p) \land (\neg s \rightarrow q) \)

15. Consider the implication "If \( n \) is odd and \( m \) is even, then \( m^2 - 3n \) is odd".
   (a) Write the converse, inverse and contrapositive of this implication in English.
   (b) Give a complete direct proof to the given implication.

16. Negate the following quantified expression. Your final answer should only negate the predicates (i.e., you can’t have "\( \neg(p \rightarrow q) \)" but you can have "\( \neg P(x) \)".
   \( \forall s(P(s) \land \exists t(R(s, t) \rightarrow P(t))) \)

17. Consider the implication "If \( m^2 + 7n \) is even, then \( m \) is odd or \( n \) is odd.”
   (a) Write the converse, inverse and contrapositive of this implication in English.
   (b) Give a complete indirect proof by contraposition to the given implication.