ANSWERS TO EVEN ASSIGNED REVIEW PROBLEMS FROM CH. 12

2. As in Exercise 1., we have \( m = n = 3 \) and \( \Delta A = 1 \). Using the contour map to estimate the value of \( f \) at the center of each subsquare, we have

\[
\iiint_R f(x, y) dA \approx \Delta A(f(5.5) + f(5.1.5) + f(5.2.5) + f(1.5.1.5) \\
+ f(1.5.2.5) + f(2.5.1.5) + f(2.5.2.5)) \\
\approx 1 \cdot (1.2 + 2.5 + 5 + 3.2 + 4.5 + 7.1 + 5.2 + 6.5 + 9.0) \\
\approx 44.2
\]

10. The region \( R \) is enclosed by the lines \( x = 4 - y \), \( x = y - 4 \) and the \( x \)-axis. Setting up the integral as a type II region, gives

\[
\iiint_R f(x, y) dA = \int_0^4 \int_{4-y} f(x, y) dx dy.
\]

14. The region \( D \) is shown below. The integral is set up as a type II region. Reversing the order of the limits and setting up the integral as a type I gives:

\[
\int_0^1 \int_0^{x^2} ye^x \, dy \, dx = \int_0^1 \frac{e^x}{x} \left[ y^2 \right]_0^1 \, dx = \int_0^1 \frac{1}{2} xe^x \, dx \\
= \frac{1}{2} \left[ \frac{e^x}{2} \right]_0^1 = \frac{1}{4} (e - 1)
\]

18. The region \( D \) is shown below. Setting up the integral as a Type I region we have:

\[
\iiint_D \frac{1}{x^2 + 1} \, dA = \int_0^1 \int_x^{1/x^2} \frac{1}{x^2 + 1} \, dy \, dx \\
= \int_0^1 \left[ \frac{1}{x^2 + 1} - \frac{x}{x^2 + 1} \right] \, dx \\
= \left[ \arctan x - \frac{1}{2} \ln(x^2 + 1) \right]_0^1 \\
= \arctan 1 - \frac{1}{2} \ln 2 - (\arctan 0 - \frac{1}{2} \ln 1) \\
= \frac{\pi}{4} - \frac{1}{2} \ln 2
\]

22. The limits for \( \theta \) are \( 0 \leq \theta \leq \frac{\pi}{2} \) (first quadrant) and the limits for \( r \) are \( 1 \leq r \leq \sqrt{2} \) (between the circles of radius 1 and \( \sqrt{2} \)). We then have

\[
\iiint_D rdA = \int_0^{\pi/2} \int_1^{\sqrt{2}} r^2 \cos \theta \, dr \, d\theta \\
= \int_0^{\pi/2} \cos \theta \left[ \frac{r^3}{3} \right]_1^{\sqrt{2}} \, d\theta \\
= \frac{2(\sqrt{2} - 1)}{3} \int_0^{\pi/2} \cos \theta \, d\theta \\
= \frac{(2(\sqrt{2} - 1))}{3} \sin \theta \bigg|_0^{\pi/2} = \frac{(2(\sqrt{2} - 1))}{3}
\]