Suggested Problems

Set 5

A. Czygrinow

Department of Mathematics
Arizona State University
Problem 1

Show that the intersection of any family of convex sets is convex.
Problem 2

Write the dual to the following problems:

1. **maximize** $x_1 + 3x_2 + 4x_3$
   subject to
   
   $x_1 + x_2 + 2x_3 \leq 1$
   $x_1 - x_2 + 4x_3 \leq -1$
   $x_1 - 2x_3 \leq 4$
   $x_1, x_2, x_3 \geq 0$

2. **maximize** $3x_1 + x_2$
   subject to
   
   $x_1 + x_2 \leq -1$
   $-x_1 + 4x_2 \leq 3$
   $2x_1 + 6x_2 \leq 2$
   $x_1, x_2 \geq 0$
Problem 3

Write the primal to the following problems:

1. \textbf{minimize} \; y_1 - 3y_2 \\
   \text{subject to} \\
   \begin{align*}
y_1 + y_2 & \geq 1 \\
y_1 - 2y_2 & \geq 4 \\
2y_1 + y_2 & \geq 2 \\
y_1, y_2 & \geq 0
\end{align*}

2. \textbf{minimize} \; y_1 + y_2 + 3y_3 + 4y_4 \\
   \text{subject to} \\
   \begin{align*}
y_1 + y_2 + y_3 + y_4 & \geq 1 \\
y_1 + 2y_2 + y_3 & \geq 2 \\
2y_1 + y_2 + y_4 & \geq 1 \\
y_1, y_2, y_3, y_4 & \geq 0
\end{align*}
Problem 4

Let \((x^*_1, x^*_2, \ldots, x^*_n)\) be a feasible solution to a primal and let \((y^*_1, y^*_2, \ldots, y^*_m)\) be a feasible solution to the dual. Show that if \(\sum_{j=1}^{n} c_j x^*_j = \sum_{i=1}^{m} b_i y^*_i\) then both solutions are optimal.