Linear Programming

Lecture 20

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Caterer problem

Caterer must provide fresh napkins over a period of \( n \) days. Let \( d_j \) denote the number of napkins required on the \( j \)th day. Caterer has the following choices:

- buy new napkins - \( a \) cents apiece
- launder napkins using fast service - napkins are returned \( q \) days later. Cost is \( b \) cents apiece.
- launder napkins using slow service - napkins are returned \( p \) days later. Cost is \( c \) cents apiece.

Assume \( p > q, a > b > c \).
How to solve it?

Consider the network with nodes for

- demands of napkins
- store (with many fresh napkins)
- nodes that correspond to laundered napkins (one for each day with supply of the demand for that day)
- inventory (with unused napkins)

and arcs from store to demand-nodes, from laundered-napkins-nodes to demand-nodes. The costs on arcs are defined $a$, $b$ or $c$ accordingly to what type of arc it is.
so how to solve it?

Once, we have the network, we can apply the network simplex algorithm. There is a small problem though.

The solution can have values which are not integers. (Clearly caterer cannot buy 1/10 of a napkin).
Integrality Theorem

**Theorem 1**  Consider a transshipment problem: 
minimize $cx$  subject to $Ax = b$, $x \geq 0$  
such that all components of $b$ are integers. If the problem has at least one feasible solution then it has an integer-valued feasible solution. If it has an optimal solution then it has an integer-valued optimal solution.
Application

Theorem 2  If in a set of $n$ girls and $n$ boys every girl knows $k$ boys and every boy knows exactly $k$ girls then $n$ marriages can be arranged with everybody knowing her or his spouse.