Linear Programming

Lecture 15

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Feasibility

Question: When the following problem is feasible?

\[
\text{maximize } \sum_{j=1}^{n} c_j x_j \\
\text{subject to:} \\
\sum_{j=1}^{n} a_{ij} x_j \leq b_i, \quad (i \in I) \\
\sum_{j=1}^{n} a_{ij} x_j = b_i, \quad (i \in E)
\]

nonnegativity constraints will be viewed as special cases of \( \sum a_{ij} x_j \leq b_i \).
Inconsistent systems

System
\[
\begin{align*}
\sum_{j=1}^{n} a_{ij} x_j & \leq b_i, \quad (i \in I) \\
\sum_{j=1}^{n} a_{ij} x_j &= b_i, \quad (i \in E)
\end{align*}
\]

is called inconsistent if there exist numbers \(y_1, \ldots, y_m\) such that

- \(y_i \geq 0, i \in I\)
- \(\sum_{i=1}^{m} a_{ij} y_i = 0, j = 1, \ldots, n\)
- \(\sum_{i=1}^{m} b_i y_i < 0\)
Theorem

Theorem 1  A system of linear inequalities and equations is unsolvable if and only if it is inconsistent.