Feasibility

Question: When the following problem is feasible?

\[
\begin{align*}
\text{maximize} & \quad \sum_{j=1}^{n} c_j x_j \\
\text{subject to:} & \quad \sum_{j=1}^{n} a_{ij} x_j \leq b_i, \quad (i \in I) \\
& \quad \sum_{j=1}^{n} a_{ij} x_j = b_i, \quad (i \in E)
\end{align*}
\]

nonnegativity constraints will be viewed as special cases of \( \sum a_{ij} x_j \leq b_i \).
Inconsistent systems

System
\[ \sum_{j=1}^{n} a_{ij} x_j \leq b_i, \quad (i \in I) \]
\[ \sum_{j=1}^{n} a_{ij} x_j = b_i, \quad (i \in E) \]
is called *inconsistent* if there exist numbers \( y_1, \ldots, y_m \) such that

- \( y_i \geq 0, \ i \in I \)
- \( \sum_{i=1}^{m} a_{ij} y_i = 0, \ j = 1, \ldots, n \)
- \( \sum_{i=1}^{m} b_i y_i < 0 \)
Theorem

Theorem 1  A system of linear inequalities and equations is unsolvable if and only if it is inconsistent.