The revised simplex method

Let $A$ be an $m$ by $m + n$ matrix of $a_{ij}$'s (recall $\sum_{j=1}^{n} a_{ij}x_j \leq b_i$) with and appended identity matrix of dimension $m$ by $m$ (which corresponds to slack variables). Let $b = [b_1, \ldots, b_m]^T$ and let $x = [x_1, \ldots, x_{n+m}]^T$ ($x_{n+1}, \ldots, x_{n+m}$ are slack variables). Finally let $c = [c_1, \ldots, c_n, 0, \ldots, 0]$ be a vector with $m + n$ entries. Then the LP problem can be expressed as follows:

**maximize** $cx$

**subject to:** $Ax = b$

$x \geq 0$
Basic Matrix

Every basic solution $x^*$ partitions the set of variables into a set of $m$ basic variables and a set of $n$ nonbasic variables. Let $B$ ($A_N$) be a matrix obtained from $A$ by considering only columns that correspond to basic (nonbasic) variables.

$B$ is called a basic matrix.
In a similar way define $x_B$, $x_N$ and $c_B$, $c_N$.

Fact 1  Matrix $B$ is nonsingular.
Dictionary

We have:
\[ x_B = B^{-1}b - B^{-1}A_Nx_N \]
\[ z = c_B B^{-1}b + (c_N - c_B B^{-1}A_N)x_N \]
i.e. we can easily describe a dictionary using only the knowledge of what are the basic variables (and of course original matrix \( A, b, \) and \( c \)).

**Question:** How to obtain a new solution from an old solution?
Revised simplex algorithm

- Solve system $y_B = c_B$
- Choose an entering column (any column $a$ of $A_N$ such that $ya$ is less than the corresponding coordinate of $c_N$). If there are none then the current solution is optimal.
- Solve $Bd = a$
- Find the largest $t$ such that $x_B^* - td \geq 0$. If there is no such $t$ then the problem is unbounded. Otherwise at least one component of $x_B^* - td$ equals zero and the corresponding variable is the leaving variable.
Revised simplex algorithm

- Set the value of the entering variable to $t$ and replace the other values of $x_B^*$ by $x_B^* - td$. Replace the leaving column of $B$ by the entering column.