Linear Programming

Lecture 10

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The slackness theorem

**Theorem 1** Let $x_1^*, \ldots, x_n^*$ be a feasible solution to the primal and $y_1^*, \ldots, y_m^*$ be a feasible solution to the dual. Necessary and sufficient conditions for simultaneous optimality of $x_1^*, \ldots, x_n^*$ and $y_1^*, \ldots, y_m^*$ are

1. $\sum_{i=1}^{m} a_{ij} y_i^* = c_j$ or $x_j^* = 0$, $j = 1, \ldots, n$
2. $\sum_{j=1}^{n} a_{ij} x_j^* = b_i$ or $y_i^* = 0$, $i = 1, \ldots, m$
Another version

**Theorem 2** A feasible solution $x_1^*, \ldots, x_n^*$ of the primal is optimal if and only if there exist numbers $y_1^*, \ldots, y_m^*$ such that

1. • if $x_j^* > 0$ then $\sum_{i=1}^{m} a_{ij} y_i^* = c_j$
   • if $\sum_{j=1}^{n} a_{ij} x_j^* < b_i$ then $y_i^* = 0$

2. • $\sum_{i=1}^{m} a_{ij} y_i^* \geq c_j$, $j = 1, \ldots, n$
   • $y_i^* \geq 0$, $i = 1, \ldots, m$
Unique solution of (1)

**Theorem 3** If $x_1^*, \ldots, x_n^*$ is a nondegenerate basic feasible solution of the primal then the system in (1) of Theorem 2 has a unique solution.