Linear Programming

Lecture 1

A. Czygrinow

Department of Mathematics
Arizona State University
Diet problem 1

Goal is to spend as little money as possible on food and get all the necessary

- Energy - at least 2000 kcal.
- Protein - at least 55 g.
- Calcium - at least 800 mg.
## Diet problem 2

Choices are:

<table>
<thead>
<tr>
<th>Food</th>
<th>Size</th>
<th>Energy</th>
<th>Protein</th>
<th>Calcium</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oatmeal</td>
<td>28g</td>
<td>110</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Chicken</td>
<td>100g</td>
<td>205</td>
<td>32</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>Eggs</td>
<td>2l</td>
<td>160</td>
<td>13</td>
<td>54</td>
<td>13</td>
</tr>
<tr>
<td>Milk</td>
<td>237cc</td>
<td>160</td>
<td>8</td>
<td>285</td>
<td>9</td>
</tr>
<tr>
<td>Pie</td>
<td>170g</td>
<td>420</td>
<td>4</td>
<td>22</td>
<td>20</td>
</tr>
<tr>
<td>Pork</td>
<td>260g</td>
<td>260</td>
<td>14</td>
<td>80</td>
<td>19</td>
</tr>
</tbody>
</table>
Diet Problem 3

Restrictions:

- Oatmeal - at most 4 servings per day.
- Chicken - at most 3 servings per day.
- Eggs - at most 2 servings per day.
- Milk - at most 8 servings per day.
- Pie - at most 2 servings per day.
- Pork - at most 2 servings per day.
Linear Functions

If $c_1, c_2 \ldots, c_n$ are real numbers then

$$f(x_1, x_2, \ldots, x_n) = c_1x_1 + c_2x_2 + \ldots c_nx_n = \sum_{j=1}^{n} c_jx_j$$

is called a **linear function**,

$$f(x_1, x_2 \ldots, x_n) = b$$

where $b$ is a real number is called a **linear equation**,

$$f(x_1, x_2, \ldots, x_n) \leq b$$

$$f(x_1, x_2, \ldots, x_n) \geq b$$

are called **linear inequalities**.
Standard form

LP problems of the following form:

\[
\text{maximize } \sum_{j=1}^{n} c_j x_j \\
\text{subject to } \sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad (i = 1, \ldots, m) \\
x_j \geq 0 \quad (j = 1, \ldots, n)
\]

are called LP problems in the standard form.
More notation

- The last $n$ of $n + m$ constraints are called *nonnegativity constraints*.
- The linear function that is to be maximized (or minimized depending on a problem) is called an *objective function*.
- Numbers $x_1, \ldots, x_n$ that satisfy all the conditions of an LP problem are said to constitute a *feasible solution*.
- Solution that maximizes the objective function (or minimizes it) is called an *optimal solution*.
Remarks

1. LP problem can have more than one optimal solution. For example:

   maximize $x_1 + x_2$
   subject to
   
   $x_1 + x_2 \leq 1$
   $x_1, x_2 \geq 0$

2. LP can have no optimal solutions at all.
   - there are no feasible solutions at all (problem is called infeasible)
   - it is not possible to find maximum as the values tend to infinity (problem is unbounded)