Diet problem 1

Goal is to spend as little money as possible on food and get all the necessary

- Energy - at least 2000 kcal.
- Protein - at least 55 g.
- Calcium - at least 800 mg.
## Diet problem 2

Choices are:

<table>
<thead>
<tr>
<th>Food</th>
<th>Size</th>
<th>Energy</th>
<th>Protein</th>
<th>Calcium</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oatmeal</td>
<td>28g</td>
<td>110</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Chicken</td>
<td>100g</td>
<td>205</td>
<td>32</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>Eggs</td>
<td>2 large</td>
<td>160</td>
<td>13</td>
<td>54</td>
<td>13</td>
</tr>
<tr>
<td>Milk</td>
<td>237cc</td>
<td>160</td>
<td>8</td>
<td>285</td>
<td>9</td>
</tr>
<tr>
<td>Pie</td>
<td>170g</td>
<td>420</td>
<td>4</td>
<td>22</td>
<td>20</td>
</tr>
<tr>
<td>Pork</td>
<td>260g</td>
<td>260</td>
<td>14</td>
<td>80</td>
<td>19</td>
</tr>
</tbody>
</table>
Diet Problem 3

Restrictions:

- Oatmeal - at most 4 servings per day.
- Chicken - at most 3 servings per day.
- Eggs - at most 2 servings per day.
- Milk - at most 8 servings per day.
- Pie - at most 2 servings per day.
- Pork - at most 2 servings per day.
Linear Functions

If \( c_1, c_2, \ldots, c_n \) are real numbers then

\[
f(x_1, x_2, \ldots, x_n) = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n = \sum_{j=1}^{n} c_j x_j
\]

is called a linear function,

\[
f(x_1, x_2, \ldots, x_n) = b
\]

where \( b \) is a real number is called a linear equation,

\[
f(x_1, x_2, \ldots, x_n) \leq b
\]

\[
f(x_1, x_2, \ldots, x_n) \geq b
\]

are called linear inequalities.
Standard form

LP problems of the following form:

\[
\text{maximize } \sum_{j=1}^{n} c_j x_j \\
\text{subject to } \sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad (i = 1, \ldots, m) \\
x_j \geq 0 \quad (j = 1, \ldots, n)
\]

are called LP problems in the standard form.
More notation

- The last \( n \) of \( n + m \) constraints are called nonnegativity constraints.
- The linear function that is to be maximized (or minimized depending on a problem) is called an objective function.
- Numbers \( x_1, \ldots, x_n \) that satisfy all the conditions of an LP problem are said to constitute a feasible solution.
- Solution that maximizes the objective function (or minimizes it) is called an optimal solution.
Remarks

1. LP problem can have more than one optimal solution. For example:

\[
\text{maximize } x_1 + x_2 \\
\text{subject to } \begin{align*}
    x_1 + x_2 & \leq 1 \\
    x_1, x_2 & \geq 0
\end{align*}
\]

2. LP can have no optimal solutions at all.
   - there are no feasible solutions at all (problem is called \textit{infeasible})
   - it is not possible to find maximum as the values tend to infinity (problem is \textit{unbounded})