Probability

1. Find the probability of getting exactly two heads when a fair coin is flipped five times.

2. Find the probability of getting at least two heads when a fair coin is flipped five times.

3. We roll two fair dice. Find the probability that the sum of numbers on dice is at least 5.

4. We roll two fair dice. Let $E$ be the event that on the first die we get the number which is at least 3, and $F$ be the event that the sum of numbers is at most 6. Find $P(F|E)$.

5. We have a fair and an unfair coin. In the unfair coin the probability of heads is $\frac{2}{3}$. We select a coin at random and flip it five times. What is the probability the coin is unfair given we observe five heads in our experiment.

6. We have coins of three types: Fair, Unfair, and Very Unfair. In the Unfair, the probability of heads is 0.6; in the Very Unfair, the probability of heads is 0.7. There are 100 Fair coins, 25 Unfair, and only 5 Very Unfair. We select one at random and flip it 5 times which results in $HHHHH$. Find the probabilities:
   - that coin is Fair given observed data;
   - that coin is Unfair given observed data;
   - that coin is Very Unfair given observed data.

Most likely which coin was used (interpret the result)?

7. Consider a random string of length 100 over the characters $A, C, G, U$ with $p_A = 0.3, p_C = 0.21, p_G = 0.26$ and $p_U = 0.23$. Find the probabilities of the following events:
   - The string contains no $U$’s.
   - The string contains 25 $U$’s.
   - The string contains at most 97 $U$’s.
• The string has 20 $A$’s and 22 $C$’s.
• The number of $U$’s in the string is between (inclusive) 45 and 60.
• The number of $U$’s plus the number of $A$’s is equal to 30.
• The string has 25 $A$’s, 25 $C$’s, 25 $G$’s and 25 $U$’s.