ZERMELO-FRAENKEL AXIOMS

(ZF1) **Axiom of extensionality.** Two sets are equal if and only if they have the same elements.

(ZF2) **Axiom of the Empty Set.** There is a set with no elements.

(ZF3) **Axiom of Pairing.** Given any sets $A$ and $B$ there exists a set whose elements are $A$ and $B$.

(ZF4) **Set-Union Axiom.** Let $\{A_i|i \in I\}$ be a family of sets. There is a set which has as its elements all elements that belong to at least one of $A_i$’s.

(ZF5) **Power Set Axiom.** For any set $A$ there is a set which contains as elements all subsets of $A$.

(ZF6) **Axiom of separation.** Given any well-formed formula $F(x)$ and any set $A$ there is a set which consists of elements $x$ of $A$ for which $F(x)$ holds.

(ZF7) **Axiom of replacement.** Given any well-formed formula $F(x,y)$ which determines a function and any set $A$ there is a set $B$ consisting of all elements $y$ such that $F(x,y)$ holds for some $x$ from $A$.

(ZF8) **Axiom of infinity.** There exists a set $A$ such that empty set is in $A$ and such that if $x$ is in $A$ then $x \cup \{x\}$ is in $A$.

(ZF9) **Axiom of regularity.** Every nonempty set is disjoint from at least one of its elements.

(AC) **Axiom of choice.** Let $A$ be any set. Let $\{X_a|a \in A\}$ be a family of nonempty subsets indexed by $A$. There is a function $F : A \rightarrow \bigcup_{a \in A} X_a$ such that for each $a \in A$, $F(a) \in X_a$.

**Comments:**
- (AC) is not a part of Zermelo-Fraenkel axioms.
- (ZF6) follows from (ZF7).
- (ZF2) can be concluded from (ZF8) and (ZF6).

**References**
