Review for Final Exam, 12/13, 10:00-11:50

December 7, 2006

1. Echelon forms
   - Elementary row operations.
   - Back Substitution (free variables, leading variables).
   - Gaussian elimination, Gauss-Jordan elimination.

2. Homogeneous systems.

   - Addition, scalar multiplication, multiplication.
   - Inverse of a matrices:
     - $2 \times 2$ matrices formula.
     - Method for finding $A^{-1}$.

4. Determinants of $2 \times 2$ matrices.
   - Transpose of a matrix and relation to determinants.
   - Solving systems using Cramer’s Rule.

5. Higher-order determinants.
   - $ij$th minor, $M_{ij} =$ determinant of a submatrix obtained by deleting $i$th row and $j$th column.
   - $ij$th cofactor, $A_{ij} = (-1)^{i+j} M_{ij}$. 
• Cofactor expansions of determinants.

\[ \text{det} A = a_{i_1} A_{i_1} + a_{i_2} A_{i_2} + \cdots + a_{i_n} A_{i_n} \]

\[ \text{det} A = a_{1j} A_{1j} + a_{2j} A_{2j} + \cdots + a_{nj} A_{nj}. \]

6. Determinants and the inverse of a matrix

• Properties of determinants.
• Adjoint matrix.
• Inverse of a matrix.

7. Vector Spaces.

• Abstract vector spaces and subspaces.
• Solution spaces.
• Linear combinations and \( \text{span}(S) \).
• Linear independence.

8. Vector spaces associated with a matrix \( A \).

• Row space. Finding a basis of \( \text{Row}(A) \).
• Column space. Finding a basis of \( \text{Col}(A) \).
• Null space. Finding a basis of \( \text{Null}(A) \).
• Rank of a matrix: \( \text{rank}(A) + \text{dim}(\text{Null}(A)) = n. \)


• Scalar product, length of a vector, Cauchy-Schwarz inequality.
• Orthogonal and orthonormal bases.
• \( V^\perp \) and properties.
• \( \text{Row}(A)^\perp = \text{Null}(A), \text{Null}(A)^\perp = \text{Row}(A) \).
• Finding a basis of \( V^\perp \).

• Finding the least squares solution of $Ax = b$.
• Finding a projection of vector $b$ on $V$.
• Finding a projection of $b$ on $V$ in the case we have an orthogonal basis of $V$.
• Finding an orthogonal basis of $V$. Finding an orthonormal basis of $V$.

11. Eigenvalues and eigenvectors.

• Finding eigenvalues and eigenvectors.
• Diagonalization.