MAT 210. BRIEF CALCULUS

Rules for finding derivatives

- **Derivative of a constant.** For any constant \( C \),
  \[
  \frac{d}{dx}[C] = 0.
  \]

- **Derivative of a constant times a function.** For any constant \( C \) and differentiable function \( f(x) \),
  \[
  \frac{d}{dx}[Cf(x)] = C \frac{d}{dx}[f(x)].
  \]

- **Sum rule.** For differentiable functions \( f(x) \) and \( g(x) \),
  \[
  \frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)].
  \]

- **Product rule.** For differentiable functions \( f(x) \) and \( g(x) \),
  \[
  \frac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot \frac{d}{dx}[g(x)] + g(x) \cdot \frac{d}{dx}[f(x)].
  \]

- **Quotient rule.** For differentiable functions \( f(x) \) and \( g(x) \),
  \[
  \frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2},
  \]
  provided \( g(x) \neq 0 \).

- **Chain rule.** For differentiable functions \( y = f(u) \) and \( u = g(x) \),
  \[
  \frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x),
  \]
  or
  \[
  \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.
  \]

- **Power rule.** For any real number \( n \),
  \[
  \frac{d}{dx}[x^n] = nx^{n-1}.
  \]

- **General power rule.** For any real number \( n \) and differentiable function \( f(x) \),
  \[
  \frac{d}{dx}[(f(x))^n] = n(f(x))^{n-1}f'(x).
  \]
• Exponent rule (1).
\[ \frac{d}{dx}[e^x] = e^x. \]

• Exponent rule (2). For \( f(x) \) which is differentiable,
\[ \frac{d}{dx}[e^{f(x)}] = e^{f(x)} f'(x). \]

• General exponent rule. For constant \( a \) and \( f(x) \) which is differentiable,
\[ \frac{d}{dx}[a^{f(x)}] = e^{f(x)} f'(x) \ln a. \]

• Logarithm rule (1). For \( x > 0 \),
\[ \frac{d}{dx} \ln x = \frac{1}{x}. \]

• Logarithm rule (2). For any differentiable function \( f(x) \) such that \( f(x) > 0 \),
\[ \frac{d}{dx} \ln f(x) = \frac{f'(x)}{f(x)}. \]

• Logarithm rule (3). For any differentiable function \( f(x) \) such that \( f(x) \neq 0 \),
\[ \frac{d}{dx} \ln |f(x)| = \frac{f'(x)}{f(x)}. \]

• Derivatives of \( \sin x \) and \( \cos x \).
\[ \frac{d}{dx} \sin(x) = \cos(x), \]
\[ \frac{d}{dx} \cos(x) = -\sin(x). \]