Theorem 1 (Whitney, 4.2.8) A graph is 2-connected if and only if it has an ear decomposition. Moreover the first cycle of the decomposition can be chosen to be an arbitrary cycle in $G$.

Proof. (If there is an ear decomposition then $G$ is 2-connected).
We will prove it by induction on the number of ears, $t$. If $t = 0$ then $G$ is a cycle and so it is 2-connected (does not have a cut vertex). Suppose $G$ has $t$ ears, $P_0, P_1, \ldots, P_t$. Then $G = H \cup P_t$ where $H = \bigcup_{i<t} P_i$ and by inductive assumption $H$ is 2-connected. $P_t$ is a path of length at least one with endpoints $x, y \in H$ and with internal vertices not in $H$. We shall prove that $G$ does not have cut vertices. To that end let $z \in V(G)$.

Case 1: $z$ is an internal vertex of $P_t$. Then every other vertex of $P_t$ is connected to one of the endpoints $x$ or $y$ of $P_t$. Since $H$ is connected there is path between $x$ and any other vertex in the graph (including $y$) and there is a path between $y$ and any vertex of $H$.

Case 2: $z \in V(H)$. $H - z$ is connected as $H$ is 2-connected. Thus for $w \in \{x, y\} - \{z\}$ and any vertex $v$ in $V(H)$ there is a $v, w$-path in $H - z$. If $z \neq x$ then there is a path from every internal vertex of $P_t$ to $x$. If $z = x$ then there is a path from every internal vertex of $P_t$ to $y$.

(If $G$ is 2-connected then $G$ has an ear decomposition starting at any cycle). Note that $G$ contains a cycle as $\delta(G) \geq 2$. Let $C$ be a cycle in $G$. We will show by induction on $n$ that:

If $n \leq |E(G)|$ then there is a subgraph $H$ of $G$ with at least $n$ edges which has an ear decomposition starting at $C$.

Base Step. $n = 0$. Take $C$.

Inductive Step. Let $n \leq |E(G)|$. By the inductive assumption there is a
subgraph $H$ of $G$ with at least $n - 1$ edges which has an ear decomposition $P_0, P_1, \ldots, P_l$ starting at $C$.

- If $|E(H)| \geq n$ then we are done.

- If there is an edge $xy \in E(G) \setminus E(H)$ with $x, y \in H$ then we add $xy$ to the decomposition of $H$ to get a subgraph on $n$ edges.

- Finally, if no such edge exists then $H$ is an induced subgraph of $G$ and so $[V(H), V \setminus V(H)]$ is an edge-cut in $G$ as $|V(H)| < n$. Let $e = xy$ be such that $x \in V(H)$ and $y \in V \setminus V(H)$. As $G$ is 2-connected there is a path $P$ in $G$ from $y$ to $V(H) - x$. Add $P_{t+1} := xyP$ to $H$. Then the new graph has at least $n$ edges and has decomposition $P_0, \ldots, P_l, P_{l+1}$.