Mat 342 Review for Final Exam

Part 1, Computations

• System of Linear Equations and Gaussian elimination
• Inverse of a matrix and singular matrices
• Finding the determinant of a matrix
• Finding the adjoint of a matrix
• Cramer’s formulae
• Checking if vectors are linearly independent
• Checking if vectors form a spanning set
• Finding a basis and dimension of a vector space \( \text{span}(S) \)
  \( S \) is a set of vectors in \( \mathbb{R}^n \), matrices, polynomials, functions.
• Finding the transition matrix
  – In \( \mathbb{R}^n \), \( S = U^{-1}V \).
  – If \( V = P_n \) or \( C[a, b] \) set up the system of equations for coefficients.
• Finding a basis and dimension of \( N(A), R(A), R(A^T) \)
• Linear Transformations
  – Checking if a given function is a linear transformation.
  – Finding the kernel.
• Matrix representation of a linear transformation
• Finding a basis and dimension of \( \text{span}(S)^\perp \)
• The least squares solutions and the normal equation
• Checking if a given function defines an inner product
• Checking if a given function defines a norm
• Orthogonality
  – Checking if vectors (polynomials, functions) are orthogonal or orthonormal.
  – Gram-Schmidt procedure for finding an orthonormal basis.

• Eigenvalues
  – Finding eigenvalues, eigenspaces.
  – Finding the sum and the product of all eigenvalues.
  – Diagonalization of a matrix.

Part 2, Theory
• Singular matrices
• Equivalent conditions for an $n \times n$ matrix to be nonsingular
• Definition of the null space of a matrix

• Determinants
  – Definition of the determinant (Laplace expansion)
  – Basic properties of the determinant: elementary matrices, the transpose of a matrix.
  – If $A, B$ are $n \times n$ then $\det(AB) = \det(A)\det(B)$.

• Definition of the adjoint of a matrix

• Linearly independent vectors and spanning sets
  – Definitions.

• A basis and dimension
  – Definitions.
  – Facts about spanning sets and independent sets when the size of the set is equal to the dimension of the vector space.

• Row space, column space, null space
– Definitions. The rank and the nullity of a matrix.
– The rank-nullity theorem.
– Correspondence between row spaces (column spaces) of equivalent matrices.
– The dimension of the column space in relation to the dimension of the row space.
– **The null space is the orthogonal complement of the row space.**
  The null space of the transpose is the orthogonal complement of the column space.

**Linear Transformations**
– Definition of a linear transformation.
– Every linear transformation $L$ from $\mathbb{R}^n$ to $\mathbb{R}^m$ has the form $L(x) = Ax$.
  – The kernel and the image.
  – Similarity of matrices.

**Orthogonality**
– Orthogonality of vectors. Orthogonal subspaces. The orthogonal complement of a subspace. The set of orthogonal and orthonormal vectors.
  – Properties of the orthogonal complement of $S$.

**Inner product spaces and normed vector spaces**
– Definition of an inner product. Definition of a norm.
– Important examples of inner products and norms.
– Orthonormal basis.

**Eigenvalues**
– Definitions of eigenvalues, eigenvectors, eigenspaces.
– Properties of eigenvectors, trace.
– Diagonalization.