Part 1, Computations

• Finding a basis and dimension of a vector space $span(S)$
  $S$ is a set of vectors in $\mathbb{R}^n$, matrices, polynomials, functions.

• Finding the transition matrix
  – If $V = \mathbb{R}^n$ then $S = U^{-1}V$.
  – If $V = P_n$ or $C[a,b]$ set up the system of equations for coefficients.

• Finding a basis and dimension of $N(A), R(A), R(A^T)$

• Linear Transformations
  – Checking if a given function is a linear transformation.
  – Finding the kernel.

• Matrix representation of a linear transformation

• Finding a basis and dimension of $span(S)^\perp$

• The least squares solutions and the normal equation

• Checking if a given function defines an inner product

• Checking if a given function defines a norm

• Orthogonality
  – Checking if vectors (polynomials, functions) are orthogonal or orthonormal.
  – Gram-Schmidt procedure for finding an orthonormal basis.
Part 2, Theory

• **A basis and dimension**
  – Definitions.
  – Facts about spanning sets and independent sets when the size of the set is equal to the dimension of the vector space.

• **Row space, column space, null space**
  – Definitions. The rank and the nullity of a matrix.
  – The rank-nullity theorem.
  – Correspondence between row spaces (column spaces) of equivalent matrices.
  – The dimension of the column space in relation to the dimension of the row space.
  – The null space is the orthogonal complement of the row space. The null space of the transpose is the orthogonal complement of the column space.

• **Linear Transformations**
  – Definition of a linear transformation.
  – Every linear transformation \( L \) from \( \mathbb{R}^n \) to \( \mathbb{R}^m \) has the form \( L(x) = Ax \).
  – The kernel and the image.
  – Similarity of matrices.

• **Orthogonality**
  – Orthogonality of vectors. Orthogonal subspaces. The orthogonal complement of a subspace. The set of orthogonal and orthonormal vectors.
  – Properties of the orthogonal complement of \( S \). Direct sum of subspaces.

• **Inner product spaces and normed vector spaces**
– Definition of an inner product. Definition of a norm.
– Important examples of inner products and norms.
– Cauchy-Schwarz inequality.
– Orthonormal basis. Parseval’s identity.