MAT 598 Homework 9

Suppose we have the scalar initial value problem $u_t - u_x = 0$ for $x \geq 0$. Consider the Leap Frog scheme:

$$w_j^{n+1} = w_j^{n-1} + \frac{\Delta t}{\Delta x} (w_{j+1}^{n} - w_{j-1}^{n}),$$

with numerical boundary conditions:

$$w_0^{n+1} = 2w_1^{n} - w_2^{n-1}.$$

1. Determine whether or not the scheme is stable.

2. Solve the initial value problem with $u(x, 0) = e^{-32(x-\frac{1}{4})^2}$ with inflow $u(1, t) = 0$ up until time $t = .3$. (Make sure that you choose the number of points $N$ large enough so that the initial conditions satisfy the boundary condition.) Choose the number of points $N$ large enough to resolve the problem and $\Delta t$ small enough to maintain stability. Does your solution improve as $N$ increases?