A Parallel Block Iterative Method for the Hydrodynamic Device Model

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Abstract—Block iterative methods are applied to hydrodynamic simulations of a one-dimensional (1-D) submicrometer semiconductor device. We show that block successive under-relaxation (SUR) converges with a fixed relaxation factor $\omega = 0.13$ for simulations at 300 K and $\omega = 0.04$ at 77 K. To demonstrate the robustness of the block iterative method, we present numerical simulations of a steady-state electron shock wave in Si at 300 K for a 0.1- μm channel and at 77 K for a 1.0- μm channel. The block SUR method is parallelizable if each diagonal block solve can be done efficiently in parallel. Using chaotic relaxation and the preconditioned conjugate gradient method for the parallel diagonal block solves, we obtain a parallel speed up of approximately 2.5 on 10 processors of a Butterfly GP-1000. The 1-D simulations serve as a numerical laboratory for developing methods that will be essential for computational efficiency in two-dimensional (2-D) problems.

I. INTRODUCTION

THIS investigation applies block iterative methods to both subsonic and transonic one-dimensional (1-D) hydrodynamic model computations. We show that block successive under-relaxation (SUR) converges with a fixed relaxation factor $\omega = 0.13$ for simulations at 300 K and $\omega = 0.04$ at 77 K. The block SUR method is parallelizable if each diagonal block solve can be done efficiently in parallel. Using chaotic relaxation and the preconditioned conjugate gradient method for the parallel diagonal block solves, we obtain a parallel speedup of approximately 2.5 on 10 processors of a Butterfly GP-1000, a nonuniform memory access (NUMA) computer. Scaling arguments indicate that the block iterative method will be much more efficient than banded or sparse direct methods in two dimensions (2 D), and will parallelize much more efficiently. The 1-D simulations presented here serve as a numerical laboratory for developing methods that will be essential for computational efficiency in 2-D problems.

To demonstrate the robustness of the block iterative method, we present numerical simulations of subsonic flow and of a steady-state electron shock wave in a submicrometer semiconductor device, using a steady-state

second upwind method. For the $n^+ - n - n^+$ diode (which models the channel of a MOSFET), the shock wave is fully developed in Si (with a 1-V bias) at 300 K for a 0.1- μm channel and at 77 K for a 1.0- μm channel [1].

We give a brief introduction to the hydrodynamic model in Section II. In Section III, we describe the 1-D $n^+ - n - n^+$ diode problem, and discuss the second upwind discretization of the 1-D steady-state hydrodynamic equations. The parallel block iterative method is described in Section IV. In Section V, we review central difference and second upwind computations for steady-state subsonic flow of electrons in the $n^+ - n - n^+$ diode. In Section VI, we review transonic electron flow computations in the $n^+ - n - n^+$ diode using the second upwind method. Section VI-6.1 presents a steady-state Mach 1.6 electron shock wave in Si at 300 K, and Section VI-6.2 presents a steady-state Mach 4.1 electron shock wave in Si at 77 K. We make some concluding remarks in Section VII.

II. THE HYDRODYNAMIC MODEL

The hydrodynamic model plays an important role in predicting the behavior of electrons in submicrometer semiconductor devices, since it exhibits hot carrier effects missing in the standard drift-diffusion model. The propagation of electrons in the semiconductor is approximated as the flow of a compressible charged fluid. This hydrodynamic approximation should be valid for devices with active regions $\geq 0.1 \mu\text{m}$, since at this length scale simulations of the hydrodynamic model are in excellent agreement with Monte Carlo simulations of the Boltzmann equation [2]–[4].

The hydrodynamic transport equations were derived by Blötekjær [5] from the Boltzmann equation:

$$\frac{\partial n}{\partial t} + \nabla \cdot (nv) = 0 \quad (1)$$

$$\frac{\partial p}{\partial t} + v \nabla \cdot p + p \cdot \nabla v = -enE - \nabla(nT) + \left(\frac{\partial p}{\partial t} \right)_c \quad (2)$$

$$\begin{aligned} \frac{\partial W}{\partial t} + \nabla \cdot (vW) = -env \cdot E - \nabla \cdot (vnT) \\ + \nabla \cdot (\kappa \nabla T) + \left(\frac{\partial W}{\partial t} \right)_c \end{aligned} \quad (3)$$

Manuscript received July 3, 1990. The work of C. L. Gardner was supported in part by the National Science Foundation under Grant DMS-8905872. The work of D. J. Rose was supported in part by the Office of Naval Research under Contract N00014-85-K-0487. This paper was recommended by Guest Editor M. Law.

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IEEE Log Number 9101061.

where n is the electron density, v is the velocity, p is the momentum density, $e(>0)$ is the electronic charge, E is the electric field, T is the temperature in energy units, W is the energy density, κ is the thermal conductivity, and the subscript c indicates collision terms. We will assume that the energy bands are parabolic:

$$p = mnv, \quad W = \frac{3}{2} nT + \frac{1}{2} mnv^2 \quad (4)$$

where m is the effective electron mass.

In addition to the transport equations (1)–(3), we have Poisson's equation for the electric field

$$\nabla \cdot (\epsilon \nabla \phi) = -e(N_D - N_A - n), \quad E = -\nabla \phi \quad (5)$$

where N_D is the density of donors and N_A is the density of acceptors. Equations (1)–(5) determine the variables n , p , W , and ϕ .

The nonlinear conservation laws (1)–(3) are the Euler equations [6] for a gas of charged particles in an electric field, with the addition of a heat conduction term. The polytropic gas constant $\gamma = 5/3$, as is appropriate for a monatomic gas, and the soundspeed $c = \sqrt{T/m}$ [7].

Equations (1)–(3) are in conservation form, and may be written in terms of the variables n , v , T , and ϕ . These variables represent the simplest choice for upwind methods.

III. THE $n^+ - n - n^+$ DIODE PROBLEM

The $n^+ - n - n^+$ diode models the channel in a MOSFET, and clearly exhibits hot carrier effects at submicrometer scales. The diode begins with an n^+ "source" region, is followed by an n "channel" region, and ends with an n^+ "drain" region.

We use the Baccarani-Wordeman models [8] for the collision terms and for the thermal conductivity:

$$\left(\frac{\partial p}{\partial t}\right)_c = \frac{-p}{\tau_p} \quad \tau_p = m \frac{\mu_{n0}}{e} \frac{T_0}{T} \quad (6)$$

$$\left(\frac{\partial W}{\partial t}\right)_c = \frac{-(W - \frac{3}{2} nT_0)}{\tau_w} \quad \tau_w = \frac{m}{2} \frac{\mu_{n0}}{e} \frac{T_0}{T} + \frac{3}{2} \frac{\mu_{n0}}{ev_s^2} \frac{TT_0}{T + T_0} \quad (7)$$

$$\kappa = \kappa_0 \frac{\mu_{n0}}{e} nT_0 \quad (8)$$

where T_0 is the ambient temperature, $\mu_{n0} = \mu_{n0}(T_0, N_D + N_A)$ is the low field electron mobility, and $v_s = v_s(T_0)$ is the saturation velocity.

The simplest boundary conditions for the simulations are $n = N$ and $T = T_0$ at $x_{\min}$ and $x_{\max}$, and $e\phi(x_{\min}) = T \ln(n/n_i)$ and $e\phi(x_{\max}) = T \ln(n/n_i) + eV$, where V is the bias across the diode and n_i is the intrinsic electron concentration.

The variables n , T , and ϕ are defined at the grid points $i = 0, 1, \dots, N-1, N$, while the velocity v is defined at the midpoints of the elements $l_{i-1/2}$ ($i = 1, \dots, N$)

connecting grid points $i-1$ and i . The boundary conditions specify n , T , and ϕ at $i = 0$ and $i = N$. Equations (1), (3), and (5) are enforced at the interior grid points $i = 1, \dots, N-1$, while (2) is enforced at the midpoints of the elements $l_{i-1/2}$, $i = 1, \dots, N$. The discrete steady-state hydrodynamic equations are derived by using central differences (for subsonic flow) or the second upwind method (for subsonic or supersonic¹ flow).

The second upwind method is a conservative discretization which captures the essential physical transport property of supersonic flow, that advected quantities directly influence the solution only downstream. For subsonic flow, the second upwind method corresponds to the Scharfetter-Gummel discretization for strong electric fields; for weak electric fields the Scharfetter-Gummel discretization corresponds to central differences [9].

The steady-state hydrodynamic equations in 1 D have the form

$$\begin{bmatrix} f_n \\ f_v \\ f_T \\ f_\phi \end{bmatrix} = \frac{d}{dx} \begin{bmatrix} vg_n \\ vg_v \\ vg_T \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ h_v \\ h_T \\ h_\phi \end{bmatrix} + \begin{bmatrix} 0 \\ s_v \\ s_T \\ s_\phi \end{bmatrix} = 0 \quad (9)$$

where

$$g_n = n, \quad g_v = mnv, \quad g_T = \frac{5}{2} nT + \frac{1}{2} mnv^2 - en\phi \quad (10)$$

$$h_v = -en \frac{d\phi}{dx} + \frac{d}{dx} (nT), \quad h_T = -\frac{d}{dx} \left(\kappa \frac{dT}{dx} \right) \quad (11)$$

$$s_v = \frac{mnv}{\tau_p} \quad s_T = \frac{\frac{1}{2} mnv^2 + \frac{3}{2} n(T - T_0)}{\tau_w} \quad (12)$$

In the second upwind method, the advection terms $d(vg)/dx$ in (9) are discretized using second upwind differences

$$\frac{d}{dx} (vg)_i \approx (v_{i+(1/2)} g_R - v_{i-(1/2)} g_L) \Delta x \quad (13)$$

where

$$g_R = \begin{cases} g_i & (v_{i+(1/2)} > 0) \\ g_{i+1} & (v_{i+(1/2)} < 0) \end{cases} \quad (14)$$

$$g_L = \begin{cases} g_{i-1} & (v_{i-(1/2)} > 0) \\ g_i & (v_{i-(1/2)} < 0) \end{cases}$$

and central differences are used for h_v , h_T , and h_ϕ .

¹The central difference method is unstable for supersonic flow.

IV. BLOCK ITERATIVE METHODS

We use Newton's method to linearize the discrete hydrodynamic equations (9):

$$J \begin{bmatrix} \delta n \\ \delta v \\ \delta T \\ \delta \phi \end{bmatrix} = - \begin{bmatrix} f_n \\ f_v \\ f_T \\ f_\phi \end{bmatrix} = -f \quad (15)$$

where J is the Jacobian and f is the 4 by 1 residual. The new solution is obtained by setting

$$\begin{bmatrix} n \\ v \\ T \\ \phi \end{bmatrix} \leftarrow \begin{bmatrix} n \\ v \\ T \\ \phi \end{bmatrix} + \tau \begin{bmatrix} \delta n \\ \delta v \\ \delta T \\ \delta \phi \end{bmatrix} \quad (16)$$

where τ is a damping factor [10] between 0 and 1, chosen to insure that the norm of the residual f decreases monotonically.

The discretized Jacobian has the following block structure:

$$J = \begin{bmatrix} \frac{\delta f_n}{\delta n} & \frac{\delta f_n}{\delta v} & 0 & 0 \\ \frac{\delta f_v}{\delta n} & \frac{\delta f_v}{\delta v} & \frac{\delta f_v}{\delta T} & \frac{\delta f_v}{\delta \phi} \\ \frac{\delta f_T}{\delta n} & \frac{\delta f_T}{\delta v} & \frac{\delta f_T}{\delta T} & \frac{\delta f_T}{\delta \phi} \\ \frac{\delta f_\phi}{\delta n} & 0 & 0 & \frac{\delta f_\phi}{\delta \phi} \end{bmatrix}. \quad (17)$$

Each block is diagonal, bidiagonal, or tridiagonal. Newton's equation (15) may be solved directly by banded or sparse matrix techniques, or by block iterative methods.

Write Newton's equation (15) as

$$J_k x = -f_k, \quad z_{k+1} = z_k + t_k x \quad (18)$$

where $k = 1, 2, \dots$ labels the Newton iterations, $x = (\delta n, \delta v, \delta T, \delta \phi)^T$, $z = (n, v, T, \phi)^T$, and t_k is a damping factor between 0 and 1. The form of the Jacobian suggests solving Newton's equation by block iterative methods.

In 1 D, solving the discrete version of Newton's equation by banded methods is faster than using block iterative methods. However, in 2 D, block iterative methods will be essential, especially as the number of PDE's increases. Block iterative methods are ideally suited for the hydrodynamic model, since in 2 D the number of PDE's is $K = 4N_{\text{species}} + 1$, where N_{species} may be as large as four in GaAs (upper and lower valley electrons and holes). Solving K PDE's on an $N \times N$ grid by sparse direct methods requires $O(K^3 N^3)$ operations, while block iterative methods require $O(KN^3) + N_{\text{iterations}} \cdot O(KN^2 \ln(N))$ operations as N and K become large. The crossover from sparse

direct to block iterative methods becomes favorable if $N_{\text{iterations}} \ll K^2 N / \ln(N)$.

The number of "inner" iterative loops (versus "outer" Newton loops) is optimized by using the methods of [10]. We define

$$\alpha_{k,n} = \|f_k + J_k x_n\| \|f_k\|^{-1} \quad (19)$$

where $n = 1, 2, \dots$ labels the inner iterations. If the iterative method converges, $\alpha_{k,n} \rightarrow 0$ as $n \rightarrow \infty$. We stop the inner iterations when

$$\alpha_{k,n} \leq \alpha_0 \|f_k\| / \|f_0\| \quad (20)$$

where $\alpha_0 \in (0, 1)$. This criterion guarantees quadratic convergence of the Newton method. To actually attain quadratic convergence, the number of inner iterations must in general be squared at each successive Newton iteration. In practice, we place an upper bound on the number of inner iterations in exchange, possibly, for additional Newton steps.

One of our goals in this investigation is to find an efficient parallel algorithm for solving the hydrodynamic system (15) of equations. Chaotic relaxation would be an ideal parallel algorithm, since it gives linear speedup. (Chaotic relaxation is an algorithm based on the classical Jacobi and Gauss-Seidel relaxation schemes in which the solution point that is relaxed and the solution iterate used in the relaxation are arbitrary [11].) For the chaotic relaxation method to converge, the point Jacobi iteration

$$\mathcal{D}_k(x_n - x_{n-1}) = -(J_k x_{n-1} + f_k) \quad (21)$$

must converge, where $\mathcal{D}_k$ is the diagonal of J_k and n denotes the Jacobi iteration number. The point Jacobi iteration, however, does not converge for the 1-D steady-state hydrodynamic equations.

We found that block SUR does converge. Block SUR may be described by

$$(D_k - \omega L_k)(x_n - x_{n-1}) = -\omega(J_k x_{n-1} + f_k) \quad (22)$$

where D_k is the block diagonal of J_k , L_k is the block lower triangular part of J_k , and ω is the relaxation factor. If $\omega < 1$, the iteration is called under-relaxation.

In general (22) is computed sequentially and is not suited for parallel computers. A block solve must be computed of the form

$$D_i^k x_i^n = r(x_{j>i}^{n-1}, x_{j<i}^n) \quad (23)$$

for every diagonal block D_i^k , where the subscripts i and j label the blocks, the superscript k labels the outer Newton iterations, and the superscript n labels the inner SUR iterations. In [12], Lanzkron showed that if each diagonal block solve can be done efficiently in parallel, sufficient speedup may be attained to make the whole iteration perform well. We then have two tasks: 1) to determine a blocking for J_k for which block SUR converges, and 2) to parallelize the resulting diagonal block solves.

The equations provide a natural blocking. We found that the ϕ and T equations in (22) may be separately blocked, but that the n and v equations must be consid-

TABLE I
TIMINGS ON A SUN-4 IN MINUTES: SECONDS FOR $N = 60, 120$, AND 240
GRID INTERVALS USING BANDED AND BLOCK ITERATIVE SOLVES. ALL
BLOCK ITERATIVE COMPUTATIONS WERE DONE WITH $\omega = 0.13$ FOR 300 K
AND $\omega = 0.04$ FOR 77 K . α_0 WAS SET EQUAL TO $1/2$ AND THE MAXIMUM
NUMBER OF INNER ITERATIONS WAS SET EQUAL TO $1/2 KN$.

Subsonic Flow	60	120	240
banded	0:39	1:27	3:11
block iterative	1:11	3:39	7:29
300 K shock	60	120	240
banded	0:45	1:41	4:01
block iterative	1:43	4:41	12:15
77 K shock	60	120	240
banded	2:18	4:51	10:38
block iterative	7:02	13:16	36:00

ered as one large block. (No relaxation factor can be found to force convergence if the n , v , ϕ , and T equations are blocked separately.) This blocking is natural in that the n and v equations form a hyperbolic subsystem for the time dependent problem, while the T and ϕ equations are parabolic and elliptic respectively.

In Table I, we show that the block SUR method converges for both subsonic and transonic problems, and compare block SUR with a banded matrix solve. Note that in 1 D the equations may be re-ordered so that an 11 band system arises. The timings indicate that block SUR is competitive with banded solves in 1 D. This may seem counter intuitive, but there are two factors that make the block SUR computations relatively efficient: 1) the iterations need not solve the equations exactly² (see [10]); 2) only some of the rows in J_k actually have 11 elements (others have only 4), and thus many extra operations are performed in the band factorization.

Efficient parallel solutions of the diagonal blocks are more difficult to obtain. The T and ϕ equations are elliptic in steady state, and yield positive semi-definite systems. These equations converge under chaotic relaxation, and one can expect linear speedup for large systems. The n - v block corresponds to a hyperbolic/elliptic system in steady-state, and various attempts at classical iterative methods have not yielded a parallelizable solution to this system.

The preconditioned conjugate gradient (PCG) method is a parallelizable method that converges for the n - v block. Conjugate gradient (CG) is a semi-iterative technique for solving $Ax = b$ that attempts to minimize the quadratic form $(1/2)x^T Ax - b^T x$. There are several CG methods for nonsymmetric systems, e.g., CGS and BICG [13]. These methods are slow to converge or may even diverge if no preconditioning is performed. However, the standard incomplete LU factorization preconditioning for CGS does not lend itself to parallelization on NUMA architectures.

²In some cases, the number of Newton iterations required for global convergence increases.

Our approach is to modify the system and solve the symmetric system $A^T Ax = A^T b$. As a preconditioner we use the diagonal of $A^T A$. Note that we do not need to explicitly form $A^T A$ since only matrix multiples are performed in CG. The diagonal of $A^T A$ can be formed in parallel. For the small systems we have considered up to this point, we have obtained speedups of approximately 2.5 on 10 processors of the Butterfly GP-1000.

V. COMPUTATIONS FOR SUBSONIC FLOW

In this section and the next, we review the subsonic and transonic problems [1] simulated in Table I. The set of simulations is generic in that steady-state electron flow in the $n^+ - n - n^+$ diode may either involve shocks or not, and the two most important temperatures for semiconductor devices are 300 and 77 K.

For the subsonic computations, we take a diode consisting of a $0.1\text{-}\mu\text{m}$ source, a $0.4\text{-}\mu\text{m}$ channel, and a $0.1\text{-}\mu\text{m}$ drain. In the n^+ region, the doping density $N = 5 \times 10^{17}\text{ cm}^{-3}$, while in the n region $N = 2 \times 10^{15}\text{ cm}^{-3}$. The ambient device temperature $T_0 = 300\text{ K} = 0.0259\text{ eV}$. The saturation velocity $v_s = 1.03 \times 10^7\text{ cm/s}$.

To validate the second upwind method for subsonic problems, we performed simulations (with 120 grid intervals) of the $n^+ - n - n^+$ diode at $V = 1\text{ V}$ using the hydrodynamic model.

Fig. 1 compares the temperature of the electron gas for the central difference and second upwind methods. Note that the temperature increases as the electrons flow from left to right across the channel. Also note the slight cooling of the electron gas as the electrons enter the channel. This cooling results from the electrons having to overcome a small potential barrier at the junction. In Fig. 2 we compare the electron velocity for the central difference and second upwind methods. Note the velocity overshoot phenomenon $v > v_s$ throughout most of the channel. Typical Mach numbers for these flows are ≤ 0.7 .

VI. COMPUTATIONS FOR TRANSONIC FLOW

Higher electron velocities can be obtained in a semiconductor device by making the active device length shorter. Since the electron soundspeed $c = \sqrt{T/m}$, higher electron Mach numbers may also be obtained by lowering the ambient temperature.

We present two parameter regimes for the $n^+ - n - n^+$ diode in which there is a transition from subsonic to supersonic electron flow just to the right of the $n^+ - n$ junction and from supersonic to subsonic flow to the left of the $n - n^+$ junction. In analogy with gas dynamical flow in a converging-diverging Laval nozzle, a shock wave develops at the transition from supersonic to subsonic flow. The $n^+ - n - n^+$ doping of the diode corresponds to the converging-diverging geometry of the Laval nozzle. For further analysis of the shock waves, see [1].

The transonic computations use the second upwind method. (The central difference method becomes unstable

Fig. 1. Electron temperature in electronvolts for $V = 1$ V, $0.4\text{-}\mu\text{m}$ channel, 300 K: central difference (solid) versus second upwind (dotted). x is in $0.1\text{ }\mu\text{m}$ for all figures.

Fig. 2. Electron velocity in 10^7 cm/s for $V = 1$ V, $0.4\text{-}\mu\text{m}$ channel, 300 K: central difference (solid) versus second upwind (dotted).

just as the maximum Mach number of the flow slightly exceeds 1.) The electron shock waves in Section VI-6.1 and 6.2 were originally simulated by Gardner [1] using the steady-state second upwind method, and have been reproduced by Fatemi, Gardner, Jerome, Osher, and Rose using a time-dependent nonoscillatory higher order upwind method in [14].

6.1. Electron Shock Wave at 300 K

For the transonic computations at 300 K, we take a diode consisting of a $0.1\text{-}\mu\text{m}$ source, a $0.1\text{-}\mu\text{m}$ channel, and a $0.1\text{-}\mu\text{m}$ drain. In the n^+ region, the doping density $N = 5 \times 10^{17} \text{ cm}^{-3}$, while in the n region $N = 2 \times 10^{15} \text{ cm}^{-3}$.

Figs. 3 and 4 present a simulation of a Mach 1.6 electron shock wave at $V = 1$ V (with 120 grid intervals). Since the simulation is steady state, we are in the rest frame of the shock. The shock profile develops at $x_s \approx 0.035\text{ }\mu\text{m}$ and is spread over about $5\Delta x$. The electron flow is subsonic behind the shock profile and supersonic ahead of the shock profile. Note that the shock wave is a sharp profile in v and M . The electron temperature is elevated well into the drain.

6.2. Electron Shock Wave at 77 K

For the transonic computations at 77 K, we take a diode consisting of a $0.1\text{-}\mu\text{m}$ source, a $1.0\text{-}\mu\text{m}$ channel, and a

Fig. 3. Electron velocity in 10^7 cm/s and electron Mach number (dotted) for $V = 1$ V, $0.1\text{-}\mu\text{m}$ channel, 300 K.

Fig. 4. Electron temperature in electronvolts for $V = 1$ V, $0.1\text{-}\mu\text{m}$ channel, 300 K.

Fig. 5. Electron velocity in 10^7 cm/s and electron Mach number (dotted) for $V = 1$ V, $1\text{-}\mu\text{m}$ channel, 77 K.

$0.1\text{-}\mu\text{m}$ drain. In the n^+ region, the doping density $N = 10^{18} \text{ cm}^{-3}$ (at 77 K), while in the n region $N = 10^{15} \text{ cm}^{-3}$ (at 77 K). The ambient device temperature $T_0 = 77 \text{ K} = 0.00665 \text{ eV}$. The saturation velocity $v_s = 1.26 \times 10^7 \text{ cm/s}$.

Figs. 5 and 6 present a simulation of a Mach 4.1 electron shock wave at $V = 1$ V (with 240 grid intervals). The shock profile develops at $x_s \approx 0.3\text{ }\mu\text{m}$, and is spread over about $10\Delta x$. Note that the shock wave is a much sharper profile in v and M than the Mach 1.6 shock wave.

Fig. 6. Electron temperature in electronvolts for $V = 1$ V, 1- μ m channel, 77 K.

VII. CONCLUSION

We have demonstrated that block SUR converges for the hydrodynamic model for a 1-D submicrometer semiconductor device with a fixed relaxation factor $\omega = 0.13$ for simulations at 300 K and $\omega = 0.04$ at 77 K. The same fixed relaxation factor works for both subsonic and transonic electron flow. This fact is surprising since in general the relaxation factor must change not only with each problem, but with each outer Newton iteration.

Parallel block iterative methods will be essential for computational efficiency in 2-D problems. For the 1-D $n^+ - n - n^+$ diode, we obtained a parallel speedup of approximately 2.5 on 10 processors of a Butterfly GP-1000 using chaotic relaxation and the PCG method for the parallel diagonal block solves. There is far more communication between processors for PCG than for the chaotic relaxation method. We expect much better results for 2-D problems, since the hyperbolic/elliptic block will contain 2-D equations for n and v , and the ratio of communication to computation will fall.

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Carl L. Gardner, for a photograph and biography, please see page 651 of the May 1991 issue of this TRANSACTIONS.

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