MAT 300, Fall 2003, Zandieh
Midterm, October 9, 2003, 100 points
Calculators allowed; No notes.
Starting time: 10-11:30 am. Ending time: Within 2 hours after your start.
Do not write your answers on this page. Use the included pages.

1. (16 points) Let P, Q, and R be statements.
Complete a Truth Table for the statement: (P **and** Q) $\Rightarrow \neg$R. Please make columns for P, Q, R, $\neg$R, P **and** Q, (P **and** Q) $\Rightarrow \neg$R. The order of your columns may vary.

For the remainder of the problems, let A, B, and C be subsets of the set universal set, U.

2. (12 points) Prove or disprove the associative property for set difference, i.e. prove or disprove that $A\backslash(B\backslash C) = (A\backslash B)\backslash C$.

3. (12 points) Fill in the blank to make a true statement from the choices below. You do NOT have to prove or explain your answer.
Recall that Δ means symmetric difference.
Choices: $A\cup B$, $A\cap B$, $A\backslash B$, $B\backslash A$, $\varnothing$, U.
   a. If $A\cap B=\varnothing$, then $A\Delta B$ = …………..
   b. If A=B, then $A\Delta B$ = …………..
   c. If $A\subseteq B$, then $A\Delta B$ = …………..
   d. If $A\neq B$ and $B\backslash A=\varnothing$, then $A\Delta B$ = …………..

4. (16 points) One of DeMorgan's Laws from Logic states that $\neg$(P **or** Q) = $\neg$P **and** $\neg$Q.
   a. Establish the truth of this law using Truth Tables. Make sure that you have columns for P, Q, $\neg$P, $\neg$Q, $\neg$P **and** $\neg$Q, P **or** Q, $\neg$(P **or** Q). The order of your tables may vary. Please state what we should notice about the table in order to establish the truth of the statement.
   b. State and prove the corresponding DeMorgan's Law from Set Theory using DeMorgan's Law from Logic.

5. (16 points) Prove the following statement: $A\subseteq B$ if and only if $A\cap B=A$.

6. (16 points) Prove the following statement by Contradiction or by Contraposition:

$$A \subseteq B\cap C \Rightarrow A\backslash B \cup A\backslash C = \varnothing.$$

7. (12 points) Prove that $C\backslash A \cup C\backslash B = C\backslash (A\cap B)$. You are encouraged to use results we have proven in class or in homework. Make sure you state which results you are using.