MAT 310, Fall 2003, Zandieh
Final Exam
150 points

Do not write your answers on this page. Use the blank paper that is provided.

0. (4 point) Please put your name on this page and all other pages. Please leave margins around and between your work as spaces for grader comments. Please do not write where you will be stapling the paper. Full credit for any honest attempt to comply. ☺

1. (10 points) Imagine that you are standing in the Midwest on the 40 degree latitude line. You use a compass to point yourself directly east. Then you throw away the compass. You now move in a straight line directly east for 20 miles, using any method we have described in class to determine straightness. Please state an appropriate method for determining straightness, and answer the following: After 20 miles, are you now on the 40 degree latitude circle or north or south of it? Justify your answer.

2. (20 points) **State** and **prove** the vertical angle theorem using any proof except the (measurement) proof where angle sums are compared. State clearly and in full the definition of angle and of angle congruence you are using.

3. (20 points) Use the holonomy formula to calculate the holonomy of a 90-90-90 triangle on a sphere. Then, use the definition for holonomy of a small triangle on a sphere and a drawing of a 90-90-90 triangle to illustrate in detail why this answer should be correct for that specific triangle.

4. (20 points) Suppose x and x' are parallel transports along geodesic t on the *sphere*. Draw a picture that represents this situation making sure to label x, x' and t. Prove that any line that does not go through the midpoint of the lune will not parallel transport x and x’. Make sure to include a clear statement of your definition of “midpoint of the lune” and show how you are using that in your proof.

5. (20 points) State whether each of the following theorems in true on the plane. If it is not true, provide a clear, detailed counterexample and state a set of triangles for which it is true. Find one of the statements that is true (with or without modification for the set of triangles) and prove it on the plane.
A. Side-Side-Side (SSS). Are two triangles congruent if the two triangles have congruent corresponding sides?
B. Angle-Side-Side (ASS). Are two triangles congruent if an angle, an adjacent side, and the opposite side of one triangle are congruent to an angle, an adjacent side, and the opposite side of the other?
C. Side-Angle-Angle (SAA). Are two triangles congruent if one side, an adjacent angle, and the opposite angle of one triangle are congruent respectively to if one side, an adjacent angle, and the opposite angle of the other triangle?

6. A. (10 points) Given the statements listed below in Aa-Ff pick two of them such that your first statement choice implies your second. Do the proof involved in showing the implications.
Aa. The sum of the interior angles of a triangle is equal to 180°, i.e. a straight angle.
Bb. A quadrilateral with three right angles must have all right angles.
Cc. Non-intersecting lines are parallel transports along every transversal.
Dd. If two lines on the plane are parallel transports of each other along some transversal, then they are parallel transports along any transversal.

Ee. On a plane, non-intersecting lines remain a fixed distance apart.
Ff. Given lines m and n are parallel transports along line L. If I then parallel transport L along m, prove that the angle between lines L and n will also remains fixed.
B. (10 points) Use EFP, PPP or any ONE of the statement Aa-Ff above to prove that there is a counterexample to the angle-angle-angle triangle congruence theorem on the plane. Make sure to clearly state what the AAA theorem says and how you have proven the existence of a counterexample to it.

7. (16 points) Draw a strip pattern for each of the parts A-C below such that the strip has the characteristics listed in each part. Any additional characteristics that the figure does or does not have are optional. The grader will expect and ignore minor inaccuracies in the drawing. You may explain your drawing if you wish, but it is not necessary.
   A. Has: translation symmetry along a line parallel to the edge of the strip, glide reflection symmetry; Does Not Have: any pure reflection symmetries, half-turn rotation symmetry.
   B. Has: translation symmetry along a line parallel to the edge of the strip, reflection symmetry perpendicular to the edge of the strip, half-turn rotation symmetry; Does Not Have: a reflection symmetry parallel to the edge of the strip.

8. (20 Points) For each of the attached figures by M.C. Escher, assume that the design continues infinitely in all directions, i.e. that it is a wallpaper pattern. Also you should ignore any differences due to color change, i.e. assume that everything is the same color for the purpose of noting the symmetries. State the following symmetries of each figure:
   A. Describe and/or label on the figure three directions of translation symmetry.
   B. Describe and/or label on the figure any lines of reflection symmetry (make sure to note all that are different from each other in the sense of going through different parts of the images in the figures.
   C. Describe and/or label on the figure any lines of glide reflection symmetry (that are not already lines of regular reflection symmetry).
   D. Describe the angles of rotational symmetry between 0 and 360 and mark on the figure an example of each different type of location for the center of the rotation. Locations are different if they occur on a different part of the images within the figure. Make sure to clearly describe or label which points on the figure match with which angles of rotation.