MAT 310, Fall 2003, Zandieh
Test 2
50 points

Do not write your answers on this page. Use the blank paper that is provided.
However, do put your name on this page and staple it to the top of your work when you are done.

1. (6 point) Explain the central (or point) symmetry transformation. As part of your explanation, describe in some detail what happens to a point on the line and what happens to a point not on the line.
   (4 point) Explain why this transformation has the same result as the half-turn transformation on the surface of the plane or sphere.

2. (8 points) List the symmetries of a lune on the sphere; include any intrinsic rotation, reflection, rigid-motion or central symmetries. Make sure to state the exact location of any relevant points or lines and specific any relevant amounts. Also note that by lune I am referring to a figure on the sphere with two vertices (at antipodal points) and two sides, each the length of one-half of a great circle.

3. (6 points) Prove that the two angles in a lune are congruent to each other. As part of your answer, clearly state the definition of angle and the definition of what it means for two angles to be congruent that are being used in your proof.

4. (6 points) State a formula for the area of a lune with interior angle of measure θ. Use S to indicate the surface area of the sphere on which the lune is located. Explain why this is the correct formula.

5. A. (6 points) Given the two congruent triangles on the next page. Precisely describe the isometries needed to move triangle ABC onto triangle A′B′C′.
   B. (2 points) Would these same steps work no matter how I had positioned two congruent triangles on the page?

6. A. (4 points) Choose two of the small triangle definitions below that you think are equivalent and prove that they are equivalent.
   B. (4 points) Choose two of the small triangle definitions below that you think are NOT equivalent and prove that they are not equivalent.
   C. (4 points) Choose another pair of small triangle definitions below that you think are NOT equivalent (different than your answer to B) equivalent and prove that they are not equivalent.

For this problem a **triangle** on a plane or sphere is defined as follows: A triangle is a figure formed by taking 3 non-collinear points and connecting each pair by a single straight line segment (or great circle segment on the sphere). We have agreed to only allow triangles with an interior and exterior which have non-zero area.

Small triangle definitions:
- Definition 1 states: A small triangle is a triangle, as defined in the triangle definition, such that each of its sides is less than half a great circle.
- Definition 2 states: A small triangle is a triangle, as defined in the triangle definition, such that each of its angles is less than 180 degrees.
- Definition 3 states: A small triangle is a triangle, as defined in the triangle definition, such that the sum of its interior angles is less than 540 degrees.

- Definition 4 states: A small triangle is a triangle, as defined in the triangle definition, which can fit in a hemisphere of the sphere (NOT including the boundary).
- Definition 5 states: A triangle on a sphere is a small triangle if it is not intersected by every great circle.

A.